Maple
Introductory
Programming Guide

M. B. Monagan  K. O. Geddes  K. M. Heal
G. Labahn  S. M. Vorkoetter  J. McCarron
P. DeMarco

© Maplesoft, a division of Waterloo Maple Inc. 2005.
# Contents

## Preface
- Audience .................................................. 1
- Worksheet Graphical Interface .......................... 1
- Manual Set .................................................. 2
- Conventions .................................................. 2
  - The Maple Programming Language .................. 3
  - Using This Book ........................................ 3
  - Customer Feedback .................................... 4

## 1 Introduction to Programming in Maple 5
  - In This Chapter ........................................ 5
  - 1.1 The Maple Software ................................... 5
  - 1.2 Maple Statements ..................................... 6
    - Getting Started ....................................... 6
    - Maple Statements ...................................... 7
  - 1.3 Procedures ............................................ 9
    - Defining a Simple Procedure ....................... 9
    - Maple Library Routines, Built-In Routines, and User-Defined Procedures .......................... 13
  - 1.4 Interrupting Computations and Clearing the Internal Memory ............................................. 16
    - Interrupting a Maple Computation .................. 16
    - Clearing the Maple Internal Memory ............... 17
  - 1.5 Troubleshooting ...................................... 17
    - Unexpected End of Statement ....................... 17
Missing Operator ........................................ 18
Invalid, Wrong Number or Type of Arguments .... 19
Unbalanced Parentheses .............................. 19
Assignment Versus Equality .......................... 20
1.6 Exercises ......................................... 21
1.7 Conclusion ........................................ 21

2 Maple Language Elements 23

In This Chapter ........................................ 23
2.1 Character Set ...................................... 23
2.2 Tokens ............................................. 24
Reserved Words ....................................... 24
Programming-Language Operators ................. 25
Names .................................................. 26
Strings ............................................... 38
Natural Integers ...................................... 46
2.3 Using Special Characters ......................... 46
Token Separators ..................................... 46
Blank Spaces, New Lines, Comments, and Continuation ........................................ 46
Punctuation Marks ................................... 48
Escape Characters ................................... 51
2.4 Types and Operands .............................. 51
Integers .............................................. 52
Strings ............................................... 53
Names ............................................... 53
Concatenations ...................................... 55
2.5 Troubleshooting .................................. 55
Attempting to Assign to a Protected Name ....... 55
Invalid Left-Hand Assignment ....................... 56
Incorrect Syntax in Parse ............................ 56
White Space Characters within a Token ............ 56
Incorrect Use of Double and Single Quotes ....... 57
2.6 Exercises ......................................... 58
2.7 Conclusion ........................................ 58

3 Maple Expressions and Statements 59

In This Chapter ........................................ 59
3.1 Syntax and Semantics .............................. 59
Syntax ................................................ 59
Semantics ........................................... 60
3.2 Expressions ....................................... 61
<table>
<thead>
<tr>
<th>Contents</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constants</td>
<td>61</td>
</tr>
<tr>
<td>Operators</td>
<td>70</td>
</tr>
<tr>
<td>Data Structures</td>
<td>89</td>
</tr>
<tr>
<td>Functions</td>
<td>92</td>
</tr>
<tr>
<td>3.3 Using Expressions</td>
<td>96</td>
</tr>
<tr>
<td>Investigating the Parts of an Expression</td>
<td>96</td>
</tr>
<tr>
<td>Evaluating and Simplifying Expressions</td>
<td>100</td>
</tr>
<tr>
<td>Unevaluated Expressions</td>
<td>100</td>
</tr>
<tr>
<td>Substituting Subexpressions</td>
<td>103</td>
</tr>
<tr>
<td>Displaying Large Expressions: Labels</td>
<td>105</td>
</tr>
<tr>
<td>Structured Types</td>
<td>106</td>
</tr>
<tr>
<td>3.4 Statements</td>
<td>110</td>
</tr>
<tr>
<td>The Assignment Statement</td>
<td>111</td>
</tr>
<tr>
<td>Selection Statements</td>
<td>114</td>
</tr>
<tr>
<td>Repetition Statements</td>
<td>115</td>
</tr>
<tr>
<td>The <code>read</code> and <code>save</code> Statements</td>
<td>117</td>
</tr>
<tr>
<td>The <code>break</code> and <code>next</code> Statements</td>
<td>118</td>
</tr>
<tr>
<td>The <code>error</code> and <code>return</code> Statements</td>
<td>118</td>
</tr>
<tr>
<td>The <code>use</code> Statement</td>
<td>119</td>
</tr>
<tr>
<td>The <code>quit</code> Statement</td>
<td>119</td>
</tr>
<tr>
<td>3.5 Troubleshooting</td>
<td>119</td>
</tr>
<tr>
<td>Syntax Errors</td>
<td>120</td>
</tr>
<tr>
<td>Reserved Word Unexpected</td>
<td>120</td>
</tr>
<tr>
<td>Break or Next Not in Loop</td>
<td>121</td>
</tr>
<tr>
<td>3.6 Exercises</td>
<td>121</td>
</tr>
<tr>
<td>3.7 Conclusion</td>
<td>122</td>
</tr>
</tbody>
</table>

4 Basic Data Structures

In This Chapter | 123
4.1 Sets | 123
Manipulating a Set | 124
4.2 Lists | 126
Manipulating a List | 128
4.3 Tables | 133
An Alternate Method for Generating a Table | 135
Table Evaluation Rules | 135
Manipulating a Table | 136
4.4 arrays and Arrays | 141
Evaluation Rules for arrays | 144
Manipulating arrays and Arrays | 145
4.5 Selecting Elements from a Data Structure | 152
Contents

The Selection Operation [ ] .......................... 152
The op Command ...................................... 155
The select, remove, and selectremove Commands ... 156

4.6 Converting Between Data Structures .............. 158
Converting a Sequence to a Set or a List ............ 158
Converting Other Data Structures .................. 159

4.7 Other Maple Data Structures .................... 159
Stacks ............................................. 159
Queues ............................................ 161

4.8 Troubleshooting ................................ 162
Wrong Number of Parameters in Function .......... 163
Invalid Subscript Selector ......................... 163
Requires Range or Initialization List for Building arrays . 164
Error in array Bound or Array Index out of Range . 164

4.9 Exercises ....................................... 165

4.10 Conclusion ..................................... 166

5 Flow Control ..................................... 167
In This Chapter .................................... 167

5.1 Selection and Conditional Execution ............ 167
The if Statement .................................... 167
The ‘if’ Operator ................................... 171

5.2 Repetition ...................................... 172
The for Loop ....................................... 173
The while Loop ..................................... 177
Control within Loops ................................ 178

5.3 Looping Commands ............................. 180
The map Command .................................. 180
The select, remove, and selectremove Commands ... 181
The zip Command .................................. 182
The seq, add, and mul Commands .................. 183
Using Specialized Looping Commands .............. 185

5.4 Troubleshooting ................................ 187
Cannot Evaluate Boolean in if Statement .......... 188
Value in Loop Must Be Numeric or Character .... 188
Variable Previously Assigned .................... 189
Wrong Parameters in Function $ ................... 190

5.5 Exercises ....................................... 191

5.6 Conclusion ..................................... 191

6 Maple Procedures ................................. 193
<table>
<thead>
<tr>
<th>Section Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>In This Chapter</td>
<td>193</td>
</tr>
<tr>
<td>6.1 Defining a Procedure</td>
<td>193</td>
</tr>
<tr>
<td>Naming a Procedure</td>
<td>194</td>
</tr>
<tr>
<td>Executing a Procedure</td>
<td>194</td>
</tr>
<tr>
<td>6.2 Procedure Components</td>
<td>196</td>
</tr>
<tr>
<td>Formal Parameters</td>
<td>196</td>
</tr>
<tr>
<td>Variables</td>
<td>201</td>
</tr>
<tr>
<td>Procedure Options</td>
<td>206</td>
</tr>
<tr>
<td>Procedure Description</td>
<td>214</td>
</tr>
<tr>
<td>Procedure Body Statements</td>
<td>215</td>
</tr>
<tr>
<td>6.3 Procedure Interpretation</td>
<td>215</td>
</tr>
<tr>
<td>6.4 Procedure Return Values</td>
<td>217</td>
</tr>
<tr>
<td>Explicit Returns</td>
<td>217</td>
</tr>
<tr>
<td>Error Returns</td>
<td>219</td>
</tr>
<tr>
<td>Returning Values through Parameters</td>
<td>220</td>
</tr>
<tr>
<td>Returning Unevaluated</td>
<td>222</td>
</tr>
<tr>
<td>6.5 Documenting Your Procedures</td>
<td>225</td>
</tr>
<tr>
<td>Indicating Copyright</td>
<td>225</td>
</tr>
<tr>
<td>Formatting Code for Readability</td>
<td>225</td>
</tr>
<tr>
<td>Adding Comments</td>
<td>226</td>
</tr>
<tr>
<td>Including a Procedure Description</td>
<td>226</td>
</tr>
<tr>
<td>Creating Help Pages</td>
<td>227</td>
</tr>
<tr>
<td>6.6 Saving and Retrieving Procedures</td>
<td>227</td>
</tr>
<tr>
<td>6.7 Viewing Maple Library Source Code</td>
<td>228</td>
</tr>
<tr>
<td>Special Evaluation Rules for Procedures</td>
<td>228</td>
</tr>
<tr>
<td>Displaying Maple Library Source Code</td>
<td>229</td>
</tr>
<tr>
<td>6.8 Alternate Methods for Defining a Procedure</td>
<td>231</td>
</tr>
<tr>
<td>Functional Operators: Mapping Notation</td>
<td>231</td>
</tr>
<tr>
<td>The unapply Command</td>
<td>232</td>
</tr>
<tr>
<td>Unnamed Procedures and the map Command</td>
<td>232</td>
</tr>
<tr>
<td>Building a List of Arguments</td>
<td>234</td>
</tr>
<tr>
<td>6.9 The Procedure Object</td>
<td>236</td>
</tr>
<tr>
<td>The procedure Type</td>
<td>236</td>
</tr>
<tr>
<td>Procedure Operands</td>
<td>237</td>
</tr>
<tr>
<td>6.10 Using Data Structures to Solve Problems</td>
<td>239</td>
</tr>
<tr>
<td>Computing an Average</td>
<td>239</td>
</tr>
<tr>
<td>Testing for Membership</td>
<td>241</td>
</tr>
<tr>
<td>Performing a Binary Search</td>
<td>242</td>
</tr>
<tr>
<td>Plotting the Roots of a Polynomial</td>
<td>243</td>
</tr>
<tr>
<td>Connected Graphs</td>
<td>246</td>
</tr>
<tr>
<td>6.11 Troubleshooting</td>
<td>251</td>
</tr>
</tbody>
</table>
Missing Argument ............................................. 251
Incorrect Argument Type ................................. 252
Implicitly Declared Local Variable ................. 252
Understanding Names, Functions, and Remember Tables . 253
6.12 Exercises ............................................... 255
6.13 Conclusion ............................................. 256

7 Input and Output ............................................. 257
In This Chapter ............................................... 257
7.1 Screen and Keyboard I/O .............................. 257
Printing Output to the Screen ......................... 257
Interactive Input ........................................... 264
7.2 Standard Disk I/O ....................................... 269
Readable File Formats in Maple ....................... 270
Using Filenames in Maple Statements ............... 271
Reading and Saving Data in Maple ..................... 271
Importing Data from Files .............................. 274
Exporting Data to Files ................................... 278
7.3 Troubleshooting .......................................... 280
Syntax Error in readstat ................................. 280
Extra Characters at End of Parsed String ............ 280
Unable to Read Filename ............................... 281
7.4 Exercises ................................................. 281
7.5 Conclusion ............................................... 282

8 Debugging and Efficiency ................................. 283
In This Chapter ............................................... 283
8.1 A Tutorial Example .................................... 284
Numbering the Procedure Statements I .............. 285
Invoking the Debugger I .................................. 285
Controlling the Execution of a Procedure During Debugging I .............. 287
Invoking the Debugger II ................................ 292
8.2 Maple Debugger Commands ......................... 296
Numbering the Procedure Statements II ............. 296
Invoking the Debugger III ............................... 296
Controlling the Execution of a Procedure During Debugging II ............. 305
Changing the State of a Procedure During Debugging .............................. 306
Examining the State of a Procedure During Debugging ........................................ 309
Using Top-Level Commands at the Debugger Prompt .............................. 314
## Contents

Restrictions ........................................ 314

8.3 Detecting Errors ............................... 315
  Tracing a Procedure ............................... 315
  Using Assertions .................................. 320
  Handling Exceptions .............................. 323
  Checking Syntax .................................. 329

8.4 Creating Efficient Programs .................. 330
  Displaying Time and Memory Statistics ......... 330
  Profiling a Procedure ............................ 332

8.5 Managing Resources ........................... 334
  Setting a Time Limit on Computations .......... 334
  Garbage Collection ................................ 335
  Communicating with the Kernel ................. 336

8.6 Exercises ....................................... 336

8.7 Conclusion ...................................... 337

9 Introduction to the Maplet User Interface Customization System 339

In This Chapter ...................................... 339

9.1 Uses of Maplet Applications .................. 339
  Custom Maple Calculators ....................... 340
  Interfaces to Maple Packages and Routines .... 341
  Queries ........................................... 342
  Messages .......................................... 342

9.2 The Maplets Package ........................... 342
  Elements .......................................... 343
  Examples ......................................... 343
  Tools .............................................. 343
  Display ........................................... 343

9.3 Terminology ..................................... 343

9.4 Elements ......................................... 344
  Window Body Elements ............................ 344
  Layout Elements .................................. 353
  MenuBar Elements ................................ 354
  ToolBar Elements ................................ 355
  Command Elements ................................ 356
  Dialog Elements .................................. 357
  Other Elements ................................... 361
  Reference Options ................................ 363

9.5 Example Maplet Applications ................ 363
  Linear Algebra Subpackage ....................... 364
# Contents

Using the Example Maplet Applications .................. 364
9.6 Tools ................................................. 366
Maplet System Global Variables .......................... 367
9.7 Running a Maplet Application .......................... 367
9.8 Writing a Maplet Application .......................... 368
   Defining a Maplet Application .......................... 368
   Maplet Application Programming Style Guidelines .... 368
9.9 After Reading This Chapter ........................... 369
9.10 Conclusion ............................................. 370

Index ............................. 371
Preface

This manual introduces the basic Maple\textsuperscript{TM} programming concepts, such as expressions, data structures, looping and decision mechanisms, procedures, input and output, debugging, and Maplets.

Audience

As a Maple user, you may have only used Maple interactively, written Maple programs, or programmed in another computer language.

You should be familiar with the following:

- Maple online help introduction
- Example worksheets
- How to use Maple interactively
- The Maple User Manual

Worksheet Graphical Interface

You can access the power of the Maple computation engine through a variety of user interfaces: the standard worksheet, the command-line\textsuperscript{1} version, the classic worksheet (not available on Macintosh\textsuperscript{®}), and custom-built Maplet\textsuperscript{TM} applications. The full Maple system is available through all of these interfaces. In this manual, any references to the graphical Maple

\textsuperscript{1}The command-line version provides optimum performance. However, the worksheet interface is easier to use and renders typeset, editable math output and higher quality plots.
interface refer to the standard worksheet interface. For more information on the various interface options, refer to the ?versions help page.

Manual Set

There are three other manuals available for Maple users, the *Maple Getting Started Guide*, the *Maple User Manual*, and the *Maple Advanced Programming Guide*.²

- The *Maple Getting Started Guide* provides extensive information for new users on using Maple, and the resources available in the software and on the Maplesoft Web site (http://www.maplesoft.com).

- The *Maple User Manual* provides an overview of the Maple software including Document and Worksheet modes, performing computations, creating plots and animations, creating and using Maplets, creating mathematical documents, expressions, basic programming information, and basic input and output information.

- The *Maple Advanced Programming Guide* extends the basic Maple programming concepts to more advanced topics, such as modules, input and output, numerical programming, graphics programming, and compiled code.

In addition to the manuals, Maple has an online help system featuring examples that you can copy, paste, and execute immediately.

Conventions

This manual uses the following typographical conventions.

- **courier** font - Maple command, package name, and option name

- **bold roman** font - dialog, menu, and text field

- **italics** - new or important concept, option name in a list, and manual titles

²The Student Edition does not include the *Maple Introductory Programming Guide* and the *Maple Advanced Programming Guide*. These programming guides can be purchased from school and specialty bookstores or directly from Maplesoft.
The Maple Programming Language

Writing a Maple program can be very simple. It can be as straightforward as placing `proc()` and `end proc` around a group of Maple commands. However, using the Maple programming language, you can write Maple procedures that perform complex operations.

Writing code in Maple does not require expert programming skills. Because Maple has a large library of routines, writing useful programs from these powerful building blocks is easy. Unlike traditional programming languages, with the Maple programming language you can perform complicated tasks by using a single Maple library routine. In addition, you can use the Maple programming language to automate long or repetitive sets of instructions.

Ninety percent of the thousands of routines in the Maple language are Maple programs. You can examine these Maple programs and modify them to suit your needs, or extend them so that Maple can solve new types of problems. You can learn the fundamentals of the Maple programming language and write useful Maple programs in a few hours, rather than the days or weeks that it often takes to learn other languages. This efficiency is partly a result of the fact that Maple is interactive. This interaction makes it easier to test and correct programs.

Using This Book

Examples Perform the examples shown in the manual as you read. To strengthen your knowledge, try variations or refer to the related online help page(s).

Troubleshooting At the end of most chapters, there is a Troubleshooting section that lists common errors encountered while performing the examples in the chapter. See this section if you receive an error that you do not understand.

Exercises Develop solutions to the problems posed in the Exercises at the end of each chapter. This consolidates and extends your learning.
Customer Feedback

Maplesoft welcomes your feedback. For suggestions and comments related to this and other manuals, email doc@maplesoft.com.
Maple provides an interactive problem-solving environment, complete with procedures for performing symbolic, numeric, and graphical computations. At the core of the Maple computer algebra system is a powerful programming language, upon which the Maple libraries of mathematical routines are built.

In This Chapter
- Components of the Maple software
- Maple statements
- Procedures and other essential elements of the Maple language
- Contrasting the Maple language with traditional programming languages, which cannot perform symbolic calculations

1.1 The Maple Software
The Maple software consists of three distinct parts.
- User Interface
- Kernel
- Library
The user interface handles the input of mathematical expressions and commands, the display of output, and the control of the Maple worksheet environment options.\footnote{For more information about the Maple user interface and worksheets, refer to the Maple Getting Started Guide or Maple User Manual.}

The basic Maple system, the kernel, is a relatively small collection of compiled C code. When a Maple session is started, the entire kernel is loaded. It contains the essential facilities required to run Maple and perform basic mathematical operations. The components include the Maple programming language interpreter, arithmetic and simplification routines, print routines, memory management facilities, and a collection of fundamental functions. Its small size ensures that the Maple system is compact, portable, and efficient. In this guide, the kernel routines are referred to as built-in routines.

The library contains the majority of the Maple routines. It includes functionality related to calculus, linear algebra, statistics, graphics, and many other topics. The Maple library consists of individual routines and packages of routines. These routines are accessed and interpreted by the Maple system as required. As a result of this modularity, the computer consumes resources proportional to only the facilities that are used, enhancing the efficiency of the system. All library routines, which are implemented in the high-level Maple programming language, can be viewed and modified by users. Therefore, it is useful to learn the Maple programming language so that you can modify existing Maple code to produce customized routines.

1.2 Maple Statements

Getting Started

The Maple software runs on many different platforms. Depending on the platform, you use its specialized worksheet interface or command-line interface. In both cases, when a Maple session is started, the Maple prompt character (>) is displayed.

> 

This prompt character indicates that Maple is waiting to receive input in the form of a Maple statement.\footnote{Throughout this book, the Maple notation (or one-dimensional) input format is}
1.2 Maple Statements

There are many types of valid statements. Examples include statements that request help on a particular topic, display a text string, perform an arithmetic operation, use a Maple library routine, or define a procedure.  

Most Maple statements must have a trailing semicolon (;) or colon (:). If you enter a statement with a trailing semicolon, for most statements, the result is displayed. However, if you enter a statement with a trailing colon, the result is computed but not displayed.

```maple
> 2 + 3;
5

> 2 + 3:
```

### Getting Help

To view an online help page for a particular topic, enter a question mark (?) followed by the corresponding topic name. For example, `?procedure` displays a help page that describes how to write a Maple procedure. This type of Maple statement does not have a trailing colon or semicolon.

### Displaying a Text String

The following statement returns a *string*. The text that forms the string is enclosed in *double quotes*, and the result (the text string) is displayed because the statement has a trailing semicolon. In the second example, no result is displayed because the statement has a trailing colon.

```maple
> "Hello World";

"Hello World"

> "Hello World":
```

...used to enter Maple statements. For more information on starting a Maple session, toggling between Maple notation (the default on most platforms) and standard math notation, and managing your files, refer to the *Maple Getting Started Guide* and *Maple User Manual* or enter `?managing` at the Maple prompt.

3For more information about statements in Maple, see chapter 3.

4For more information about getting help in Maple, refer to `?help` and `?HelpGuide`.

5For more information about strings in Maple, see chapter 2 or refer to `?string`. 
Performing an Arithmetic Operation

The arithmetic operators in Maple are + (addition), - (subtraction), * (multiplication), / (division), and ^ (exponentiation). A statement can be an arithmetic operation that contains any combination of these operators. The standard rules of precedence apply.

```
> 103993/33102;

103993
33102
```

Maple displays the result—in this case an exact rational number—in the worksheet or on the terminal in use, displaying the result as closely to standard mathematical notation as possible.

You can enter statements on one line (as in the previous example) or several lines. You can even put the terminating semicolon on a separate line. It is not processed until you complete the command with a semicolon or a colon.

```
> 103993
> 33102
> ;

103993
33102
```

Assigning to a Name

By naming a calculated result or complicated expression, you can reference it. To assign to a name, use the assignment operator, :=.

```
> a := 103993/33102;

a := 103993
33102
```

```
> 2 * a;

```

---

6To enter a statement on more than one line, hold the Shift key and press Enter at the end of each line.

7For more information about commands that control printing, see Printing Output to the Screen on page 257. For information about arithmetic operators, see page 70.

8For more information about names and assignment, see chapter 2.
Using Maple Library Routines
Once a value is assigned to a name, for example, the value assigned previously to \(a\), you can use the name as if it were the assigned object. For example, you can use the Maple library routine `evalf` to compute a floating-point (decimal) approximation to \(\frac{103993}{33102}\) divided by 2 by entering the following statement.

\[
> \text{evalf}(a/2);
\]

\[
1.570796326
\]

You can use the Maple library of routines, introduced on page 6, for many purposes. For example, you can find the derivative of an expression by using the `diff` command.\(^9\)

\[
> \text{diff}(x^2 + x + 1/x, x);
\]

\[
2 x + 1 - \frac{1}{x^2}
\]

1.3 Procedures
This section formally introduces the concept of procedures in Maple. For more information about procedures, see chapter 6.

Defining a Simple Procedure
A Maple procedure (a type of program) is a prearranged group of statements processed together. The easiest way to create a Maple procedure is to encapsulate a sequence of commands, which can be used to perform a computation interactively, between the `proc(...)` and `end proc` statements.

\(^9\)For more information about the Maple library routines, refer to the *Maple User Manual* or the online help.
Entering a Procedure Definition  The following procedure generates the string “Hello World”. Enter this procedure in a Maple session by entering its definition on one line.

> hello := proc() "Hello World"; end proc;

```
    hello := proc() "Hello World" end proc
```

For improved readability, enter the procedure on multiple lines: hold Shift and press ENTER at the end of each line. Indent lines in the procedure by using the spacebar. When you enter the last line, which contains end proc, press ENTER.

> hello := proc()
   > "Hello World";
   > end proc;

```
    hello := proc() "Hello World" end proc
```

To run this procedure, enter its name followed by a set of parentheses and a semicolon. Enclose any input to the procedure—in this case none—between the parentheses and delimited (separated) by commas (,).

> hello();

```
    "Hello World"
```

The next example is a procedure that uses the evalf command.

> half := proc(x)
   > evalf(x/2);
   > end proc;

```
    half := proc(x) evalf(1/2 * x) end proc
```

This procedure requires one input, x. The procedure computes the approximation of the value of x divided by 2. A Maple procedure returns the result of the last executed statement. Since evalf(x/2) is the last calculation performed in the procedure half (in fact, it is the only calculation), the approximation of x/2 is returned.

\[^{10}\text{For more information, see } \text{Unexpected End of Statement} \text{ on page 17.}\]
The procedure is named `half` by using the `:=` notation in the same manner that you would assign any other object to a name. Once you have named a procedure, you can use it as a command in the current Maple session with the same syntax used to run a Maple library routine.

```maple
> half(2/3);
0.3333333333

> half(a);
0.5000000000 a

> half(1) + half(2);
1.500000000
```

By enclosing the `evalf(x/2)` statement between `proc()` and `end proc`, you create a procedure. In the next example, a new procedure is created that corresponds to the following two statements.

```maple
> a := 103993/33102;
a

> evalf(a/2);
1.570796326
```

The procedure definition for these statements does not explicitly require input, but it does include a `local variable`. A local variable `a` in a procedure is different from the variable `a` outside the procedure (if one exists). Thus, you can use `a` as a variable name outside of the procedure `f` without conflict.\(^\text{11}\)

```maple
> f := proc() local a;
  a := 103993/33102;
  evalf(a/2);
> end proc;
```

\(^\text{11}\)For more information about local variables, see `Variables` on page 201.
Chapter 1: Introduction to Programming in Maple

\[
f := \begin{aligned}
&\text{proc()} \text{local } a; a := 103993/33102; \text{evalf}(1/2 \times a) \text{end proc}
\end{aligned}
\]

The interpretation of this procedure definition appears immediately after the statements that define it. Examine it carefully and note the following characteristics.

- The name of this procedure (program) is \( f \).
- The procedure definition starts with \texttt{proc()}. The empty parentheses indicate that this procedure does not require input.
- Semicolons or colons separate the individual commands of the procedure.
- The \texttt{local a;} statement declares \( a \) as a local variable. As described previously, a local variable has meaning only inside the procedure definition.\(^{12}\)
- The \texttt{end proc} keywords and colon or semicolon mark the end of the procedure.
- As you enter the procedure, the commands of the procedure do not display output. The procedure definition is displayed as output only after you complete it with \texttt{end proc} and a semicolon or colon.
- The procedure definition that displays as the value of the name \( f \) is equivalent to, but not identical to, the procedure definition you enter. The commands of the procedure are simplified if possible.

The procedure definition syntax is very flexible. You can:

- Enter each statement on one or more lines
- Enter multiple statements on one line, provided they are separated by colons or semicolons
- Place extra semicolons between statements
- Omit the semicolon (or colon) from the statement preceding \texttt{end proc}

\(^{12}\)For more information about local variables, see Variables on page 201.
To suppress the display resulting from a complicated procedure definition, use a colon instead of a semicolon at the end of the definition.

```
> g := proc() local a;
> a := 103993/33102;
> evalf(a/2);
> end proc:
```

**Calling a Procedure**  The execution of a procedure is referred to as an *invocation* or a *procedure call*. When you invoke a procedure, Maple executes the statements that form the procedure body one at a time. The result of the last computed statement within the procedure is returned as the value of the procedure call.

For example, to execute procedure $f$—that is, to cause the statements that form the procedure to execute in sequence—type its name followed by parentheses and a semicolon. No input is required to run the procedure, so nothing is entered between the parentheses.

```
> f();

1.570796326
```

Only the result of the last calculation performed within procedure $f$ is returned—the result of $\text{evalf}(a/2)$. The assignment $a:=103993/33102$ is executed, but the statement result is not displayed.

**Maple Library Routines, Built-In Routines, and User-Defined Procedures**

Maple routines are implemented in one of two formats—those written in the C programming language and those written in the Maple programming language. You can easily include complicated tasks in your user-defined procedures by using the existing Maple routines instead of writing new untested code.

The routines that are written in C are compiled and built into the Maple kernel. These built-in routines are those that are generally used in computations, and those that are fundamental to the implementation of the other Maple routines. Since the kernel is compiled, it is usually faster to perform computations by using these built-in routines.\(^\text{13}\)

\(^{13}\)For more information about built-in kernel routines, see page 207 or refer to \texttt{?builtin}.
The routines in the Maple library are written in the Maple programming language. These routines exist as individual routines or as packages of routines. They are accessed and interpreted by the Maple system as required.

The code for the library routines and the definitions of user-defined procedures can be viewed and modified. However, before exploring that, it is important that you learn about evaluation rules so that you understand the code.

**Full Evaluation and Last Name Evaluation** For most named objects in Maple, such as \( e \) defined with the following command, you can obtain its value by entering its name.

\[
> \ e := 3;
\]

\[ e := 3 \]

\[
> e;
\]

\[ 3 \]

This is called *full evaluation*—each name in the expression is fully evaluated to the last assigned expression in any chain of assignments. The following statements further illustrate how full evaluation works.

\[
> c := b;
\]

\[ c := b \]

\[
> b := a;
\]

\[ b := a \]

\[
> a := 1;
\]

\[ a := 1 \]

\[
> c;
\]

\[ 1 \]
This group of statements creates the chain of assignments
\[ c \Rightarrow b \Rightarrow a \Rightarrow 1, \] and \( c \) fully evaluates to \( 1 \).

If you try this approach with a procedure, Maple displays only the name of the procedure instead of its true value (the procedure definition). For example, in the previous section, \( g \) is defined as a procedure. If you try to view the body of procedure \( g \) by referring to it by name, the procedure definition is not displayed.

\[
> g;
\]

\[
g
\]

This model of evaluation is called last name evaluation and it hides the procedure details. The reason for this approach is that procedures potentially contain many subobjects. To obtain the value of the name \( g \), use the eval command, which forces full evaluation.

\[
> \text{eval(g)};
\]

\[
\text{proc()} \text{ local } a; a := 103993/33102; \text{ evalf(1/2 * a) end proc}
\]

Viewing Procedure Definitions and Maple Library Code  You can learn about programming in Maple by studying the procedure definitions of Maple library routines. To print the body of Maple library routines, set the Maple interface variable verboseproc to 2, and then use the print command.

Example Look at the procedure definition for the Maple least common multiple routine, lcm, enter the following statements.

\[
> \text{interface(verboseproc = 2)};
> \text{print(lcm)};
\]

\[14\text{Last name evaluation applies to procedures, tables, and modules in Maple. For more information, refer to } \text{?last_name_eval}.\]

\[15\text{For more information about } \text{interface} \text{ variables, see page } 258 \text{ or refer to } \text{?interface}.\]
\begin{verbatim}
proc(a, b)
local q, t;
option remember,  'Copyright (c) 1990 by the University of Waterloo. All rights reserved.';
if nargs = 0 then 1
elif nargs = 1 then t := expand(a); sign(t) * t
elif 2 < nargs then lcm(a, lcm(op(2..nargs, [args])))
elif type(a, integer) and type(b, integer) then ilcm(a, b)
else gcd(a, b, q) ; q * b
end if
end proc

Because the built-in kernel routines are written in the C programming language and compiled, you cannot view their definitions. If you print the definition of a built-in procedure, the procedure body is comprised of the option builtin statement and a positive integer that identifies the procedure.

> print(add);

proc() option builtin; 114 end proc
\end{verbatim}

1.4 Interrupting Computations and Clearing the Internal Memory

This section introduces two important concepts: interrupting a computation and clearing the internal memory.

Interrupting a Maple Computation

To stop a computation, for example, a lengthy calculation or infinite loop, use one of the following three methods.\footnote{Maple does not always respond immediately to an interrupt request if it is performing a complex computation. You may need to wait a few seconds before the computation is halted.}

- Click the STOP icon\footnote{For more information on toolbar icons, refer to \url{worksheet,reference,toolbar}.} on the toolbar (in worksheet versions).
1.5 Troubleshooting

- Hold the CTRL key and press the C key (in UNIX and Windows command-line versions).

- Hold the COMMAND key and press the period key (.) (in Macintosh command-line and worksheet versions).

To perform a **hard** interrupt, which stops the computation and exits the Maple session, in Windows command-line Maple, hold the CTRL key and press the BREAK key.

**Clearing the Maple Internal Memory**
Clear the internal memory during a Maple session by entering the `restart` command or clicking the restart icon\(^\text{18}\) on the toolbar of the worksheet (in GUI versions). When you enter this command, the Maple session returns to its startup state; all identifiers (including variables and procedures) are reset to their initial values.\(^\text{19}\)

```maple
> restart:
```

A Maple function that can be used to free space without resetting your session is the garbage collection facility `gc`. For information on `gc`, see page 335.

---

1.5 Troubleshooting

This section provides you with a list of common mistakes, examples, and hints that will help you understand and avoid common errors. Use this section to study the errors that you may encounter when entering the examples from this chapter in a Maple session.\(^\text{20}\)

**Unexpected End of Statement**
Most valid statements in Maple must end in either a colon or a semicolon. An error is returned if you press ENTER on an input region that is incomplete.

---

\(^\text{18}\)For more information on toolbar icons, refer to `?worksheet,reference,toolbar`.

\(^\text{19}\)For more information about clearing the Maple internal memory and the `restart` command, refer to `?restart`.

\(^\text{20}\)You can use the `parse` routine for finding errors in statements, and the Maple debugger for finding errors in programs. For more information, see chapter 8, or refer to `?parse` and `?debugger`. 
If you press ENTER to enter a procedure definition on multiple lines, the following error displays.

```maple
> p := proc()
```

Error, unexpected end of statement

To prevent this error message from displaying as you enter a procedure definition, hold the SHIFT key and press the ENTER key at the end of each line, instead of pressing only the ENTER key.

```maple
> p := proc()
> "Hello World"
> end proc;
```

If you neglect to enter a trailing semicolon or colon, Maple inserts a semicolon and displays the following warning message.

```maple
> 1 + 2
```

Warning, inserted missing semicolon at end of statement,

```maple
1 + 2;
```

3

Maple also inserts a semicolon after end proc in procedure definition.

```maple
> p := proc()
> "Hello World"
> end proc
```

Warning, inserted missing semicolon at end of statement,

```maple
...ld"; end proc;
```

```maple
p := proc() "Hello World" end proc;
```

Missing Operator

The most common error of this type is omitting the multiplication operator.

```maple
> 2 a + b;
```
Error, missing operator or ‘;‘
You can avoid this error by using * to indicate multiplication.

\[ 2a + b \]

Invalid, Wrong Number or Type of Arguments
An error is returned if the argument(s) to a Maple library command are incorrect or missing.

\[ > \text{evalf}(); \]
Error, invalid input: evalf expects 1 or 2 arguments, but received 0

\[ > \text{solve}(y=3x+4, 5); \]
Error, (in solve) invalid arguments

\[ > \cos(x, y); \]
Error, (in cos) expecting 1 argument, got 2

If such an error occurs, check the appropriate online help page for the correct syntax. Enter \texttt{?topic\_name} at the Maple prompt.

Unbalanced Parentheses
In complicated expressions or nested commands, it is easy to omit a closing parenthesis.

\[ > [(1,0), (0,1)]; \]
Error, ‘]‘ unexpected

In a valid statement, each (, {, and [ requires a matching ), }, and ], respectively.

\[ > [(1,0), (0,1)]; \]
[1, 0, 0, 1]
Assignment Versus Equality

When you enter statements in a Maple session, it is important that you understand the difference between equality (using \( = \)) and assignment (using \( := \)).

The equal sign \( = \) creates an equation. An equation is commonly used to test whether two expressions (the left-hand side and the right-hand side) are equal. The test is usually performed by using the Maple \texttt{evalb} command.\(^{21}\)

\begin{verbatim}
> x = 2;

x = 2

> x;

x

> evalb(x=2);

false

> x + 5;

x + 5
\end{verbatim}

The assignment operator \( := \) assigns to the left-hand side the value of right-hand side. Once an assignment is made, the left-hand side can be used in place of the value of the right-hand side. The left-hand side must evaluate to a name (for example, the left-hand side cannot be a number).\(^{22}\)

\begin{verbatim}
> x := 2;

x := 2

> x;

x
\end{verbatim}

\(^{21}\)For more information about equations and Boolean testing, see page 84 or refer to \texttt{?evalb}.

\(^{22}\)For more information about names and assignment, see pages 26 and 111, respectively.
1.6 Exercises

1. Assign the integers 12321, 23432, and 34543 to the names a, b, and c. Use these names to find the sum and difference of each pair of numbers.

2. Write two procedures. The first requires two inputs and finds their sum. The second requires two inputs and finds their product. Use these procedures to add and multiply pairs of numbers. How could you use these procedures to add and multiply three numbers?

3. Display your procedure definitions. Are they identical to the code you entered to write them? 23

1.7 Conclusion

This chapter presented a basic overview of the Maple system and the Maple programming language. The Maple system consists of three main components: the kernel, which contains compiled built-in commands; the library which contains routines written in the Maple programming language; and, the interface, which handles the input and output of mathematical expressions and functions. You were introduced to the essential elements of writing and executing Maple procedures, along with common syntax errors related to writing procedures.

23For more information about procedure definitions, see chapter 6.
To learn more about the Maple programming language, read the remaining chapters in this guide and, when you encounter other example programs, try to write variations. Study the details, exceptions, and options in these chapters, as the need arises. References to related topics in other chapters, manuals, and online help pages that provide additional information are included where relevant.
Before programming in Maple, it is important to learn the properties and roles of the basic elements of the Maple language.

In This Chapter

- Basic elements of the Maple language: the character set and tokens
- Maple tokens: reserved words, operators, names, strings, and natural numbers; including the types of each and related functions
- Using special characters
- Maple data types related to the tokens

2.1 Character Set

The Maple character set consists of letters, digits, and special characters. The letters are the 26 lower case letters:

```
| a | b | c | d | e | f | g | h | i | j | k | l | m | n | o | p | q | r | s | t | u | v | w | x | y | z |
```

and the 26 upper case letters:

```
| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
```

The 10 decimal digits are:
Table 2.1  Special Characters

<table>
<thead>
<tr>
<th>Character</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>blank</code></td>
<td>(</td>
</tr>
<tr>
<td><code>;</code></td>
<td>)</td>
</tr>
<tr>
<td><code>:</code></td>
<td>[</td>
</tr>
<tr>
<td><code>+</code></td>
<td>]</td>
</tr>
<tr>
<td><code>-</code></td>
<td>{</td>
</tr>
<tr>
<td><code>*</code></td>
<td>}</td>
</tr>
<tr>
<td><code>/</code></td>
<td>`'</td>
</tr>
<tr>
<td><code>^</code></td>
<td>`'</td>
</tr>
<tr>
<td><code>!</code></td>
<td>&quot;</td>
</tr>
<tr>
<td><code>=</code></td>
<td></td>
</tr>
<tr>
<td><code>&lt;</code></td>
<td>&amp;</td>
</tr>
<tr>
<td><code>&gt;</code></td>
<td>_</td>
</tr>
<tr>
<td><code>@</code></td>
<td>%</td>
</tr>
<tr>
<td><code>$</code></td>
<td>\</td>
</tr>
<tr>
<td><code>.period</code></td>
<td>#</td>
</tr>
<tr>
<td><code>,comma</code></td>
<td>?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>0, 1, 2, 3, 4, 5, 6, 7, 8, 9</td>
</tr>
</tbody>
</table>

There are also 32 special characters, which are listed in Table 2.1. The use of special characters is discussed in 2.3 Using Special Characters.

2.2   Tokens

The Maple language combines characters into tokens. The set of tokens consists of reserved words (also called *keywords*), programming-language operators, names, strings, and natural integers.

Reserved Words

The Maple *keywords* are reserved words that have special meanings. Thus, you cannot change them or use them as variables in procedures. The keywords are listed in Table 2.2. You can find information about a specific keyword in the subsequent chapters of this guide or in the online help pages.¹

¹For general information about reserved words in Maple, refer to `?keywords`.
Table 2.2  Reserved Words

<table>
<thead>
<tr>
<th>Keywords</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>break, next</td>
<td>loop control</td>
</tr>
<tr>
<td>if, then, elif, else</td>
<td>if statement</td>
</tr>
<tr>
<td>for, from, in, by, to, while, do</td>
<td>for and while loops</td>
</tr>
<tr>
<td>proc, local, global, option, error, return options, description</td>
<td>procedures</td>
</tr>
<tr>
<td>export, module, use end</td>
<td>modules</td>
</tr>
<tr>
<td>assuming</td>
<td>assume facility</td>
</tr>
<tr>
<td>try, catch, finally</td>
<td>exception handling</td>
</tr>
<tr>
<td>read, save</td>
<td>read and save statements</td>
</tr>
<tr>
<td>quit, done, stop</td>
<td>ending Maple</td>
</tr>
<tr>
<td>union, minus, intersect, subset</td>
<td>set operators</td>
</tr>
<tr>
<td>and, or, not, xor</td>
<td>Boolean operators</td>
</tr>
<tr>
<td>implies</td>
<td>implication operator</td>
</tr>
<tr>
<td>mod</td>
<td>modulus operator</td>
</tr>
</tbody>
</table>

Programming-Language Operators
There are three types of Maple language operators: nullary, unary, and binary.\(^2\)

In Maple, there are three nullary operators (operators that take no arguments and return values from the environment of the Maple session). They are constructed from the Maple ditto operator. The percent sign % is the ditto operator. It is a special Maple name used to refer to previously computed non-NULL expressions. Specifically, the following nullary operators are defined as:

\[
\begin{align*}
\% & \quad \text{last expression} \\
\%\% & \quad \text{second-last expression} \\
\%\%\% & \quad \text{third-last expression}
\end{align*}
\]

The % operator *re-evaluates* the last expression computed, the %\% operator *re-evaluates* the second-last expression computed, and the %\%\% operator *re-evaluates* the third-last expression computed.

\(^2\)For more information about the order of precedence of programming-language operators, see Table 3.4 on page 89 or refer to ?precedence.
Table 2.3  Programming Binary Operators

<table>
<thead>
<tr>
<th>Operator</th>
<th>Meaning</th>
<th>Operator</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>addition</td>
<td>&lt;</td>
<td>less than</td>
</tr>
<tr>
<td>-</td>
<td>subtraction</td>
<td>&lt;=</td>
<td>less or equal</td>
</tr>
<tr>
<td>*</td>
<td>multiplication</td>
<td>&gt;</td>
<td>greater than</td>
</tr>
<tr>
<td>/</td>
<td>division</td>
<td>&gt;=</td>
<td>greater or equal</td>
</tr>
<tr>
<td>~</td>
<td>exponentiation</td>
<td>&lt;&gt;</td>
<td>not equal</td>
</tr>
<tr>
<td>$</td>
<td>sequence operator</td>
<td>-&gt;</td>
<td>arrow operator</td>
</tr>
<tr>
<td>@</td>
<td>composition</td>
<td>union</td>
<td>set union</td>
</tr>
<tr>
<td>@@</td>
<td>repeated composition</td>
<td>minus</td>
<td>set difference</td>
</tr>
<tr>
<td>&amp;string</td>
<td>neutral operator</td>
<td>intersect</td>
<td>set intersection</td>
</tr>
<tr>
<td>,</td>
<td>expression separator</td>
<td>::</td>
<td>type declaration,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>concatenation</td>
</tr>
<tr>
<td>.</td>
<td>non-commutative</td>
<td>or</td>
<td>logical or</td>
</tr>
<tr>
<td>..</td>
<td>ellipsis</td>
<td>xor</td>
<td>exclusive or</td>
</tr>
<tr>
<td>mod</td>
<td>modulo</td>
<td>implies</td>
<td>implication</td>
</tr>
<tr>
<td>:=</td>
<td>assignment</td>
<td>subset</td>
<td>subset</td>
</tr>
</tbody>
</table>

Note: The nullary operators do not reference the results of the lines located above the execution groups in which they are used. They reference the results of the most recently performed computations in the Maple session, regardless of the execution group or worksheet that contains them. Also, since the ditto operators do not recall the results of past computations, but re-evaluate the results of these computations, the use of local variables to save and recall computed expressions is preferred in procedures. For more information about local variables, see Variables on page 201.\(^3\)

The Maple binary and unary operators, and their meanings, are listed in Table 2.3 and Table 2.4, respectively. For additional information about these operators, see page 70.

Names
A name in Maple is a sequence of one or more characters that uniquely identifies a command, file, variable, or other entity. There are two distinct types of names: indexed names and symbols, which are non-indexed

\(^3\)For more information about the ditto operators, refer to ?ditto.
## Table 2.4 Programming Unary Operators

<table>
<thead>
<tr>
<th>Operator</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>unary plus (prefix)</td>
</tr>
<tr>
<td>-</td>
<td>unary minus (prefix)</td>
</tr>
<tr>
<td>!</td>
<td>factorial (postfix)</td>
</tr>
<tr>
<td>$</td>
<td>sequence operator (prefix)</td>
</tr>
<tr>
<td>not</td>
<td>logical not (prefix)</td>
</tr>
<tr>
<td>&amp;string</td>
<td>neutral operator (prefix)</td>
</tr>
<tr>
<td>.</td>
<td>decimal point (prefix or postfix)</td>
</tr>
<tr>
<td>%integer</td>
<td>label (prefix)</td>
</tr>
</tbody>
</table>

names. For more information about indexed names, see page 34.

The simplest instance of a name consists of a sequence of letters, digits, and underscores. If you require a name that includes blank spaces, use left single quotes (for more information, see page 28).

```maple
> My_Name_1;

My_Name_1
```

You can confirm that the previous statement is a valid name (and symbol) by using the `whattype` command.

```maple
> whattype(%);

symbol
```

A name cannot begin with a number, and the maximum length of a name is system dependent.

```maple
> 1myname;

on line 258, syntax error, missing operator or ';';
1myname;
```

Maple is case-sensitive, so, for example, the name `Apple` is different from the name `apple`.

Other Maple names include:

- mathematical functions such as `sin` and `cos`
- Maple commands such as `expand` or `simplify`
- type names such as `integer` or `list`
- variables that represent values, for example, `x` and `y` in the expression `x + y`
- programming variables (A name becomes a programming variable only after Maple assigns it a value. Otherwise, the name remains an unknown.)

For example, in the first statement below, `y` is a name that does not have a value. In the second statement, the variable `x` has the value `3`.

```
> 2*y - 1;

2*y - 1

> x := 3; x^2 + 1;

x := 3

10
```

Names that begin with an underscore are reserved by Maple for internal use only, and names of the form `\texttt{name}` are permitted for spreadsheet references (refer to `\texttt{cell references}` in `\texttt{spreadsheet,references}`).

In general, names can also be formed by using left single quotes or concatenation. Other categories of names in Maple include indexed names, initially-known names, environment variables, constants, and protected names. These are discussed in the following sections.

**Forming a Name By Using Left Single Quotes** You can form a name in Maple by enclosing any sequence of characters in left single quotes (also called back quotes). To form a name that includes blank spaces, use left single quotes.

```
> 'This is a name!';

This is a name!

> whattype(%);
```
symbol

Because the name is not indexed, it is of type symbol.

With the exception of keywords and names that contain blank spaces, any valid Maple name formed without using left single quotes is the same as the name formed by surrounding the name with left single quotes. For example, $x$ and $'x'$ refer to the same name $x$.

**Name Concatenation**  Names can be formed through concatenation by using the `cat` command or the superseded concatenation operator, $\mid\mid$.

**The cat Command**  You can construct a name (or string) by using the `cat` command

```
cat( sequence )
```

where `sequence` contains any number of expressions, separated by commas.

The `cat` command is commonly used to concatenate names and strings, and the result returned has the type (name or string) of the first argument to `cat`. (In the following examples, any argument that is enclosed by double quotes is a string.)

```maple
> cat(a, b);

ab
```

```maple
> cat(a, "b");

ab
```

```maple
> cat("a", b);

“ab”
```

```maple
> cat(a, 2);

a2
```

---

4You can also use the `cat` command to form strings. For more information, see page 41 or refer to ?cat.
Chapter 2: Maple Language Elements

```maple
> cat("a", 2);

"a2"
```

If the result from the `cat` command evaluates to a name, then it is a global name—a name that can be recognized anywhere in the Maple session. To illustrate, consider the following procedures.⁵

```maple
> globalassign := proc()
>   local a;
>   a := 5;
>   assign( cat('a'), 2 );
> end proc:

> a;

a

> globalassign();
> a;

2
```

In procedure `globalassign`, the assignment statement⁶ assigns 2 to the global name `a`. If the statement `assign( cat('a'), 2 );` were `assign('a', 2 );`, the local variable `a` would be assigned 2, but the global variable `a` would be unchanged.⁷

**The Concatenation Operator `||`** You can also concatenate names by using the concatenation operator `||` in one of the following formats.

```
name || name
name || naturalInteger
name || string
name || ( expression )
```

⁵For more information about procedure syntax, see chapter 6.
⁶For more information about the assignment statement, see page 111.
⁷For more information about local and global variables, see Variables on page 201.
The concatenation operator is a binary operator that requires a name (or a string) as its left operand. Since a name can appear on the left-hand side of any concatenation operator, you can perform a succession of concatenations.

\[ i := 5; \]

\[ i := 5 \]

\[ i \| 7; \]

\[ i7 \]

\[ p \| "in"; \]

\[ pin \]

\[ a \| (2*i); \]

\[ a10 \]

\[ a \| i \| b; \]

\[ a5b \]

Maple *never* fully evaluates the left-most object, but evaluates it to a name. For example, study the result of \( i \| 7 \) and \( a \| i \| b \) in the previous examples. The \( i \) is evaluated to 5 only if it is *not* the left-most object in the concatenation.

In general, Maple evaluates expressions from left to right. However, Maple evaluates concatenations from *right to left*. Maple evaluates the right-most operand, then concatenates to the left operand.

**Example 1** If the right operand evaluates to an integer, string, or name, then the result of the concatenation is a string or name (depending on the type of the left-most operand).

\[ n := 4: p \| (2*n+1); \]

\[ p9 \]
Example 2  If the right operand evaluates to another type of object, for example, a general expression, the result of the operation is an unevaluated concatenated object.

\[
p || (2m + 1)
\]

Example 3  If the right-hand expression is a sequence or a range and the operands of the range are integers or character strings, then Maple returns a sequence of names.

\[
x || (a, b, 4, 67);
\]

\[xa, xb, x4, x67\]

Example 4  If more than one range appears, the extended sequence of names is constructed.

\[
x || (1..2) || (1..3);
\]

\[x11, x12, x13, x21, x22, x23\]

The use of the concatenation operator to form sequences has been superseded by the seq function. For more information, see page 183 or refer to ?seq.

Differences Between cat and ||  Although you can use both cat and || to concatenate, there are subtle differences in the way each performs. The concatenation operator does not evaluate its first argument, whereas cat does.\(^8\)

\[
a := 2;
\]

\[a := 2\]

\[
a || 4;
\]

\[a4\]

\(^8\text{In this example, the result from cat(a,4) is the name 24 not the integer 24.}\)
Also, the result of the statement that uses the concatenation operator is evaluated; this is not the case for the `cat` command.

```maple
> a4 := 5;

a4 := 5
```

```maple
> a || 4;

5
```

```maple
> cat(a, 4);

a4
```

**Note:** In general, it is recommended that you use `cat`.

**Special Characters in Names** To make the left single quote character appear in a name, enter a backslash character (`\`) followed by a left single quote (`'`) where you want the left single quote character to appear. The backslash is the escape character in Maple. For more information, see **Escape Characters** on page 51 or refer to `?backslash`.

```maple
> 'a'b';

a'b
```

Similarly, to cause a backslash to appear as one of the characters in a name, enter two consecutive backslashes, `\`. You must escape the backslash because the backslash also acts as a line continuation character. For more information, see page 47.

```maple
> 'a\b';

a\b
```
The special backslash character is only one character, as is demonstrated by using the \texttt{length} command.

\begin{verbatim}
> length(\%);
3
\end{verbatim}

\textbf{Indexed Names} Another form of a name in Maple is the \textit{indexed} name (or subscripted name), which has the following syntax.

\begin{verbatim}
name [ sequence ]
\end{verbatim}

Since an indexed name is itself a valid name, you can add a succession of subscripts.

\begin{verbatim}
> A[1,2];
A_{1,2}

> A[i,3*j-1];
A_{i,3j-1}

> b[1][1], data[Cu,gold][1];
b_{11}, data_{Cu,gold1}
\end{verbatim}

You can assign a value to an indexed name.

\begin{verbatim}
> f[Cu] := 1.512;
f_{Cu} := 1.512

> a[1]^2; a[1] := 3; a[1]^2;
a_1^2
\end{verbatim}
### Table 2.5 Initially Known Names

<table>
<thead>
<tr>
<th>Name</th>
<th>Meaning</th>
<th>Name</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catalan</td>
<td>Catalan’s lasterror</td>
<td>lasterror</td>
<td>stores most recent error</td>
</tr>
<tr>
<td>constants</td>
<td>initially-known symbolic constants</td>
<td>libname</td>
<td>pathname of Maple library(ies)</td>
</tr>
<tr>
<td>Digits</td>
<td>number of digits in floating-point computations</td>
<td>NULL</td>
<td>empty expression sequence</td>
</tr>
<tr>
<td>FAIL</td>
<td>cannot determine value</td>
<td>Order</td>
<td>truncation order for series</td>
</tr>
<tr>
<td>false</td>
<td>Boolean evaluation</td>
<td>Pi</td>
<td>mathematical constant</td>
</tr>
<tr>
<td>gamma</td>
<td>Euler’s constant</td>
<td>printlevel</td>
<td>control display of information</td>
</tr>
<tr>
<td>I</td>
<td>complex number</td>
<td>true</td>
<td>Boolean evaluation</td>
</tr>
<tr>
<td>infinity</td>
<td>mathematical infinity</td>
<td>undefined</td>
<td>undefined</td>
</tr>
</tbody>
</table>

**Note:** The use of an indexed name such as `A[1,2]` does not automatically imply that `A` is an array, as in some languages. The statement

```plaintext
```

forms a formula in four indexed names. However, if `A` does evaluate to an array, Array, or table, `A[1,2]` refers to element `(1,2)` of the array or table.\(^9\)

**Initially Known Names** Maple ships with a collection of names that are initially known to the Maple system. These are names of global or environment variables and names of constants related to functions. Table 2.5 lists some of the initially known Maple names.\(^10\)

**Environment Variables** Environment variables influence the execution of operations in a software program. You can display a sequence of all active environment variables by entering the following command.\(^11\)

---

\(^9\)For more information about arrays, Arrays, and tables, see chapter 4 or refer to `?array`, `Array`, and `?table`.

\(^10\)For general information about initially known names, refer to `?ininames`.

\(^11\)For more information about assigned names, see pages 111–114 or refer to `?anames`. 
Chapter 2: Maple Language Elements

> anames('environment');

Testzero, UseHardwareFloats, Rounding, %, _ans, %%%, Digits, index/newtable, mod, %, Order, printlevel, Normalizer, NumericEventHandler

You can use environment variables in assignment statements. However, if an environment variable is assigned a value in a procedure body, its pre-procedure value is restored upon exit from the procedure. Also, the value of the environment variable is available to all subprocedures called from that procedure, unless locally superseded.$^{12,13,14}$

> Digits;

10

> evalf(2/13);

0.1538461538

> envtest := proc()
> Digits := 3;
> evalf(2/13);
> end proc:

> envtest();

0.154

> Digits;

10

> evalf(2/13);

$^{12}$For more information about environment variables, refer to ?envvar. For more information about procedures, see chapter 6 or refer to ?procedures.

$^{13}$The evalf command is used to evaluate its argument to a floating-point (decimal) number. For more information, refer to ?evalf.

$^{14}$For more information on variable scoping, see Variables on page 201 or refer to chapter 1 of the Maple Advanced Programming Guide.
2.2 Tokens

0.1538461538

**Constants** You can display a sequence of all the currently active symbolic constants in Maple by using the global variable `constants`.

```maple
> constants;
false, γ, ∞, true, Catalan, FAIL, π
```

Maple also has numeric constants: integers, fractions, floating-point numbers, and complex numbers. For more information, see pages 63–68.\(^\text{15}\)

**Protected Names** A protected name has a predefined meaning. You cannot directly assign a value to it. For example, the names of built-in functions such as `sin` (the sine function), utility operations such as `degree` (computes the degree of a polynomial), commands such as `diff` (differentiation), and type names such as `integer` and `list`, are protected names. If you attempt to assign to any of these names, an error is returned.

```maple
> list := [1,2];
```

Error, attempting to assign to ‘list‘ which is protected

The Maple system protects these names from re-assignment. However, even though it is not recommended, it is possible to make assignments to these names by first unprotecting them as illustrated by the following statements.\(^\text{16}\)

```maple
> unprotect(sin);
> sin := "a sin indeed";
```

```
sin := “a sin indeed”
```

As a result, components of Maple that rely on the sine function do not work properly.

---

\(^{15}\)For general information about constants in Maple, refer to \?constants.

\(^{16}\)You can usually undo assignments made to Maple system names by entering a `restart` command, or by ending the session. However, in general, it is dangerous to reassign Maple system names; using the `unprotect` command to modify Maple system names is not recommended.
Chapter 2: Maple Language Elements

> plot( 1, 0..2*Pi, coords=polar );

Plotting error, empty plot

To check whether a name is protected, use the type command.\(^\text{17}\)

> type(diff, protected);

\textit{true}

To display all the protected names, use the following command.\(^\text{18}\)

\begin{verbatim}
select( type, unames(), anames(anything), protected);
\end{verbatim}

To prevent a user from assigning a value to a name, use the protect command when writing programs.

> mysqr := x -> x^2;

\textit{mysqr := } \textit{x \rightarrow x^2}

> type(mysqr, protected);

\textit{false}

> protect( mysqr );
> mysqr := 9;

\textit{Error, attempting to assign to ‘mysqr‘ which is protected}

\textbf{Strings}

A \textit{string} is a sequence of characters that evaluates to itself. To create a string, enclose any sequence of characters in double quotes.

> "This is a string";

\textquote{This is a string}
You cannot assign to a string.

```maple
> "hello" := 5;
Error, invalid left hand side of assignment
```

Do not confuse the double quote character, which delimits a string, with the left single quote character (’), which forms a name (symbol), or right single quote (’), which delays evaluation. For more information on left and right single quotes, see pages 28 and 100, respectively.

In the following sections, strings and string operations are described. For information on the StringTools package, refer to ?StringTools.

**Length of a String**  There is no practical limit on the length of a Maple string (the maximum string length is system dependent). On most Maple implementations, this means that a string can contain more than half a million characters.

Use the `length` command to determine the length of a string.

```maple
> length("What is the length of this string?");
34
```

All characters between, but excluding, the double quotes are counted. Each blank space is counted as one character.

**The Empty String**  The empty string (or null string) is represented by two double quotation marks with no enclosed characters (not even a blank space).

```maple
> "";

```

```maple
> length(%); 
0
```

```maple
> whattype(%%); 
string
```
The null string is *not* the same as the global variable `NULL`, which is an empty expression sequence. The output for the null string consists of zero printed characters.

**Substrings** You can extract a substring of a string by using the `substring` command, or by using subscripts.

The `substring` command returns the string consisting of the characters specified by `range`.

```maple
substring( exprString, range );
```

If `range` is a nonzero integer `m`, then the `m`th character of `exprString` is returned as a one-character string. If `range` is specified as `m..n`, where `m` and `n` are nonzero integers, then the substring starting from the `m`th character and ending with the `n`th character of `exprString` is returned. Negative values in `range` indicate character positions counted from the right end of the string.

```maple
> S := "abcdef";
S := "abcdef"

> substring(S, 5);
"e"

> substring(S, 3..7);
"cdef"

> substring(S, -3..-1);
"def"
```

Alternatively, you can access a substring by using a string with a subscripted integer range (also called a *selection operation*).

---

19It is more efficient to use the selection operation than the `substring` command to access a substring of a string. Therefore, whenever possible, use a selection operation instead of the `substring` command for accessing substrings from procedures. For more information about selection, see page 114.
> S := "This is a string";

\[ S := \text{"This is a string"} \]

> S[6];

\[ \text{"i"} \]

> S[6..9];

\[ \text{"is a"} \]

> S[-6..-1];

\[ \text{"string"} \]

**String Concatenation**  Like names, strings can also be formed through concatenation by using the `cat` command or the superseded concatenation operator, `||`.\(^\text{20}\)

**The `cat` Command**  You can construct a string by using the `cat` command, where `sequence` contains any number of expressions, separated by commas.

\[ \text{cat( sequence )} \]

The `cat` command is commonly used to concatenate strings with names and integers, and the result returned has the type (name or string) of the first argument to `cat`.

> `cat("a", b);`

\[ \text{"ab"} \]

> `cat("a", 2);`

\[ \text{"a2"} \]

\(^{20}\)For more information on the differences between `cat` and `||`, see pages 29–32 or refer to `?cat`. 
Chapter 2: Maple Language Elements

> i := 5;

\[ i := 5 \]

> cat( "The value of i is ", i, "." );

“The value of i is 5.”

**The Concatenation Operator \(||\)** You can also concatenate strings by using the concatenation operator \(||\) in one of the following formats.

| string \(||\) name |
|-------------------|
| string \(||\) naturalInteger |
| string \(||\) string |
| string \(||\) ( expression ) |

The concatenation operator is a binary operator that requires a string (or a name) as its left operand. Since a string can appear on the left-hand side of any concatenation operator, Maple accepts a succession of concatenations.

> "The "|| "value of i is " || i;

“The value of i is 5”

In general, Maple evaluates expressions from left to right. However, Maple evaluates concatenations from *right to left*. Maple evaluates the right-most operand, then concatenates to the left operand. If the right operand evaluates to an integer, string, or name, the result of the concatenation is a string (or name, depending on the type of the left-most operand). If the right operand is evaluated to another type of object, for example, a general expression, the result of the operation is an *unevaluated* concatenated object.

> n := 4: "p" || (2*n+1);

“p9”

> "p" || (2*m+1);

“p”||(2 m + 1)
If the right-hand expression is a sequence or a range, and the operands of the range are integers or character strings, Maple returns a sequence of strings.

```maple
> "var" || (a, b, 4, 67);

"vara", "varb", "var4", "var67"
```

If more than one range appears, an extended sequence of strings is constructed.

```maple
> "x" || (1..2) || (1..3);

"x11", "x12", "x13", "x21", "x22", "x23"
```

The use of the concatenation operator to form sequences has been superseded by the `seq` function. For more information, see page 183 or refer to `?seq`.

For examples that illustrate the differences between `cat` and `||`, see page 32.

**Special Characters in Strings** To make the double quote character appear in a string, enter a backslash character (\) followed by a double quote (") where you want the double quote character to appear. You must do this since Maple does not know which double quote ends the string, and the backslash acts as the escape character in Maple. For more information, see page 51 or refer to `?backslash`.

```maple
> "a"\"b";

"a""b"
```

Similarly, to cause a backslash to appear as one of the characters in a string, enter two consecutive backslashes, \\ . You must escape the backslash in this manner since the backslash also acts as a line continuation character in Maple. For more information, see page 47.

```maple
> "a\\b";

"a\b"
```
The special backslash character mentioned above counts as only one character, as is demonstrated by using the \texttt{length} command.

\begin{verbatim}
> length(%);
3
\end{verbatim}

A reserved word enclosed in double quotes is a valid Maple string, distinct from its usage as a token.

\begin{verbatim}
> "while";
"while"
\end{verbatim}

\textbf{Parsing Strings} The \texttt{parse} command accepts any Maple string and parses the string as if it had been entered or read from a file.

\begin{verbatim}
parsed( exprString, option );
\end{verbatim}

The string must consist of exactly one Maple expression. The expression is parsed, and returned unevaluated.

\begin{verbatim}
> parse("a+b");
a + b
> parse("a+b;");
a + b
\end{verbatim}

If the string is syntactically incorrect, the \texttt{parse} command returns an error of the form "\texttt{incorrect syntax in parse: ... (number)}". The \texttt{number} indicates the offset in characters, counted from the beginning of the string, at which the syntax error was detected.

\begin{verbatim}
> parse("a++b");
Error, incorrect syntax in parse: ‘+‘ unexpected (4)
\end{verbatim}

Partial statements or expressions cannot be parsed. Multiple statements or expressions cannot be parsed, unless they comprise one larger statement (such as a loop, or a procedure definition).
If the option statement is specified, the string must consist of exactly one Maple statement. In this case, the statement is parsed and evaluated, and then the result is returned.\(^{21}\)

```maple
> parse("sin(Pi)");

sin(\(\pi\))
```

```maple
> parse("sin(Pi)", statement);

0
```

**Searching a String**  To perform case-sensitive and case-insensitive string searching, use the `SearchText` and `searchtext` commands, respectively.

```maple
SearchText( pattern, exprString, range );
searchtext( pattern, exprString, range );
```

The `SearchText` command searches for exact matches of `pattern` in `exprString`. The `searchtext` command performs the same search, but it is case-insensitive. If `pattern` is found, Maple returns an integer indicating the position of the first character in `pattern` in `exprString`. If the pattern is not found in `exprString`, 0 is returned.

```maple
> SearchText("my s", "This is my string.");

9
```

```maple
> searchtext("My S", "This is my string.");

9
```

The optional `range` restricts the search to the specified range. It is equivalent to performing a search on a substring, and it is useful when the pattern occurs more than once in the string.

```maple
> SearchText("is", "This is my string.", 4..-1);

3
```

\(^{21}\)For more information about the `parse` command, see page 268 or refer to ?parse.
Converting Expressions to Strings
To convert an expression to a string, use the `convert` command.

```maple
> convert(a, string);

"a"

> convert(a+b-c*d/e, string);

"a+b-c*d/e"
```

Natural Integers
A natural integer is a sequence of one or more decimal digits.

```maple
> 00003141592653589793238462643;

3141592653589793238462643
```

2.3 Using Special Characters

Token Separators
You can separate tokens by using white space characters or punctuation marks. The separator indicates the end of one token and the beginning of the next.

Blank Spaces, New Lines, Comments, and Continuation
The white space characters are space, tab, return, and line-feed. This guide uses the terminology `new line` to refer to a return or line-feed since the Maple system does not distinguish between these characters. The terminology `blank` refers to a space or tab.

The white space characters separate tokens, but are not themselves tokens. White space characters cannot normally occur `within` a token.

```maple
> a := b;
```

---

22Maple has the facility to convert a variety of objects. For more information about expressions, see 3.2 Expressions and 3.3 Using Expressions. For more information about conversions in Maple, refer to `convert`.

23For more information about integers in Maple, see page 62 or refer to `integer`. 
on line 26, syntax error, ‘=’ unexpected:  
a: = b;

However, you can use white space characters between tokens.

> a * x + x*y;

\[a x + x y\]

The only situation in which white space is part of a token is in a name or string formed by enclosing a sequence of characters in left single quotes or double quotes, respectively. For more information, see pages 28 and 38.

Except in a string, all characters that follow a pound sign “#” on a line are part of a comment.24

> a := 1 + x + x^2; #This is a comment

\[a := 1 + x + x^2\]

Since white space and new line characters are functionally identical, you can continue statements from line to line, as described in chapter 1.

> a:= 1 + x +

> x^2;

\[a := 1 + x + x^2\]

To continue numbers and strings over multiple lines, use the backslash (\) as a line continuation character. The behavior of line continuation is as follows.

If the special character backslash \ immediately precedes a new line character, the Maple parser ignores both the backslash and the new line. If a backslash occurs in the middle of a line, Maple usually ignores it. 25

---

24 For information about comments in Maple procedures, see 6.5 Documenting Your Procedures.

25 For more information about the backslash and exceptions to this rule, refer to ?backslash.
Table 2.6 Maple Punctuation Marks

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>;</td>
<td>Semicolon</td>
</tr>
<tr>
<td>:</td>
<td>Colon</td>
</tr>
<tr>
<td>'</td>
<td>Left single quote</td>
</tr>
<tr>
<td>'</td>
<td>Right single quote</td>
</tr>
<tr>
<td></td>
<td>Vertical bar</td>
</tr>
<tr>
<td>&lt;</td>
<td>Left angle bracket</td>
</tr>
<tr>
<td>&gt;</td>
<td>Right angle bracket</td>
</tr>
<tr>
<td>(</td>
<td>Left parenthesis</td>
</tr>
<tr>
<td>)</td>
<td>Right parenthesis</td>
</tr>
<tr>
<td>[</td>
<td>Left bracket</td>
</tr>
<tr>
<td>]</td>
<td>Right bracket</td>
</tr>
<tr>
<td>{</td>
<td>Left brace</td>
</tr>
<tr>
<td>}</td>
<td>Right brace</td>
</tr>
<tr>
<td>,</td>
<td>Comma</td>
</tr>
</tbody>
</table>

You can use this rule to break up a long sequence of digits into groups of smaller sequences, to enhance readability.

> "The input should be either a list of \ variables or a set of variables";

> The input should be either a list of variables or a set of variables

> G:= 0.57721566490153286060 \ 04215933593992;

> 6512090082402\43104215933593992;

\[ G := 0.57721566490153286060512090082402431\4215933593992 \]

**Punctuation Marks**
The punctuation marks that act as token separators are listed in Table 2.6.

; and : Use the semicolon and the colon to separate statements. The distinction between these marks is that, during an interactive session, a colon prevents the result of the statement from printing.

> f:=x->x^2;

\[ f := x \rightarrow x^2 \]
Enclosing an expression, or part of an expression, in a right single quotes (also called *apostrophes*) delays evaluation of the expression (subexpression) by one level. For more information, see page 100.

```
> ''sin''(Pi);

'sin'(π)

> %;

sin(π)

> %;

0
```

To form names, enclose an expression in left single quotes.

```
> limit(f(x), x=0, 'right');

0
```

The left and right parentheses group terms in an expression and group parameters in a function call.

```
> (a+b)*c; cos(Pi);

(a + b) c

-1

> proc( x, y, z )
>    x+y+z;
> end proc:
```
[] Use the left and right square brackets to form indexed (subscripted) names and to select components from aggregate objects such as arrays, tables, and lists. For more information on data structures, see chapter 4.

\[ a[1]; L:=[2,3,5,7]; L[3]; \]

\[ a_1 \]

\[ L := [2, 3, 5, 7] \]

5

[] and {} Use the left and right square brackets to form lists, and the left and right braces to form sets. For more information on sets and lists, see chapter 4.

\[ L:=[2,3,5,2]; S:={2,3,5,2}; \]

\[ L := [2, 3, 5, 2] \]

\[ S := \{2, 3, 5\} \]

<> and | The left and right angle brackets in conjunction with the vertical bar are used to construct Matrices and Vectors. For more information, refer to \texttt{?Matrix} and \texttt{?MVshortcut}.

\[ <<1,2,3> | <4,5,6>>; \]

\[
\begin{bmatrix}
1 & 4 \\
2 & 5 \\
3 & 6
\end{bmatrix}
\]

, Use the comma to form a sequence, and to separate the arguments of a function call or the elements of a list or set.

\[ \sin(\Pi), 0, \text{limit} (\cos(x)/x, x=\text{infinity}); \]

0, 0, 0
2.4 Types and Operands

Escape Characters

An escape character indicates that the following character must be handled in a special manner. The escape characters in Maple are ?, !, #, and \.

? The question mark character, if it appears as the first nonblank character on a line, invokes the Maple help facility. The words following ? on the same line determine the arguments to the help procedure. Use either “,” or “/” to separate the words in the argument. For more information, refer to ?help.

! The exclamation mark character, if it appears as the first nonblank character on a line, passes the remainder of the line as a command to the host operating system. This facility is not available on all platforms. For more information, refer to ?system and ?escape.

# The pound sign character indicates that the characters that follow it on the line are a comment. For more information, see 6.5 Documenting Your Procedures or refer to ?comment.

\ The backslash character is used for continuation of lines and grouping characters in a token. For more information, see page 47 or refer to ?backslash.

2.4 Types and Operands

In most programming languages, data is divided into different classes of information—called data types. Types are important in Maple since they are used to decide whether an expression is a valid input in procedure calls and Maple commands. By definition, a type in Maple is any expression that is recognized by the type command. The type command has the following syntax.

```
type( expression, typeName );
```

If expression is of type typeName, the type command returns true. Otherwise, false is returned.

To determine the operands and the number of operands in an expression, use the op and nops commands, respectively. These commands have the following basic syntax.
Chapter 2: Maple Language Elements

Table 2.7  Integer Subtypes

<table>
<thead>
<tr>
<th>Subtype</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>negint</td>
<td>negative integer</td>
</tr>
<tr>
<td>posint</td>
<td>positive integer</td>
</tr>
<tr>
<td>nonnegint</td>
<td>non-negative integer</td>
</tr>
<tr>
<td>nonposint</td>
<td>non-positive integer</td>
</tr>
<tr>
<td>even</td>
<td>even integer</td>
</tr>
<tr>
<td>odd</td>
<td>odd integer</td>
</tr>
</tbody>
</table>

\[
\text{op}( i, \text{expression} );
\]
\[
\text{nops}( \text{expression} );
\]

If the optional first argument \( i \) to the \text{op} command is a positive integer, the \( i \)th operand of \text{expression} is returned.

The following sections introduce some elementary data types, for example, integers, strings, and names, but the Maple software contains many others.\(^{26}\)

**Integers**

The type of an integer is \text{integer}. The \text{type} command also understands the subtypes of integers listed in Table 2.7.

An integer has only one operand, itself.

\[
> x := 23;
\]

\[
x := 23
\]

\[
> \text{type}(x, \text{prime});
\]

\[
true
\]

\[
> \text{op}(x);
\]

\[
23
\]

\[
> \text{op}(0, x);
\]

\(^{26}\)For more information about data types and operands in Maple, see chapter 3 or refer to \text{?type} and \text{?op}.\]
2.4 Types and Operands

Integer

Strings
The type of a string is string. A string also has only one operand, itself.

```plaintext
> s := "Is this a string?"

s := “Is this a string?”
```

```plaintext
> type(s, string);

true
```

```plaintext
> nops(s);

1
```

```plaintext
> op(s);

“Is this a string?”
```

Names
The type of a name is name. However, the type command also understands the type names symbol and indexed. The type name is defined as symbol or indexed.

```plaintext
> x := ‘my name’;

x := my name
```

```plaintext
> type(x, name);

true
```

```plaintext
> type(x, symbol);

true
```
The type of an indexed name is *indexed*. The zeroth operand of an indexed name is the base name. The remaining operands are the indices (subscripts).

```maple
> x := A[1][2,3];

x := A_{1,2,3}

> type(x, name);

true

> type(x, indexed);

true

> nops(x);

2

> op(x);

2, 3

> op(0,x);

A

> y := %;

y := A

> type(y, indexed);

true

> nops(y), op(0,y), op(y);

1, A, 1
2.5 Troubleshooting

This section provides you with a list of common mistakes, examples, and hints that will help you understand and avoid common errors. Use this section to study the errors that you may encounter when entering the examples from this chapter in a Maple session.

Attempt to Assign to a Protected Name
An error occurs if you attempt to assign a value to a protected name.\footnote{For more information about protected names, see page 37 or refer to \texttt{?protect}.}
\begin{verbatim}
> int := 10;
Error, attempting to assign to ‘int’ which is protected
\end{verbatim}
Invalid Left-Hand Assignment
An error occurs if you attempt to assign a value to a string.\textsuperscript{28}

\begin{verbatim}
> "my string" := 10;
Error, invalid left hand side of assignment
\end{verbatim}

Use only valid names on the left-hand side of an assignment statement.

Incorrect Syntax in Parse
The \texttt{parse} command accepts a string as its argument. An error message is returned if the string is syntactically incorrect.\textsuperscript{29}

\begin{verbatim}
> parse("a^2--b");
Error, incorrect syntax in parse: '-' unexpected (6)
\end{verbatim}

The error message indicates the character number (counted from the left double quote) where error was detected. In this case, the 6th character (the second minus sign) caused the error.

White Space Characters within a Token
An error message is normally returned if a white space character occurs in a token.

\begin{verbatim}
> evalb(2 <= 3);
on line 71, syntax error, '=' unexpected:
evalb(2 <= 3);
\end{verbatim}

The binary operator \texttt{<=} is a token in Maple. Therefore, it cannot contain a space.

\begin{verbatim}
> evalb(2 <= 3);
true
\end{verbatim}

\textsuperscript{28}For more information about strings, see page 38 or refer to \texttt{?string}.
\textsuperscript{29}For more information about parsing, see page 44 or refer to \texttt{?parse}.
Incorrect Use of Double and Single Quotes

In Maple, double quotes form a string, left single quotes form a name, and right single quotes delay evaluation of an expression. Confusing a string with a name, or a name with delayed evaluation causes errors. Study the following examples to see the different uses of these quotes.\(^{30}\)

To form a string, enclose the expression in double quotes.

```
> "2 + 3";
"2 + 3"
```

```
> whattype(%);
string
```

To form a name, enclose the expression in left single quotes.

```
> '2 + 3';
2 + 3
```

```
> whattype(%);
symbol
```

To delay the evaluation of an expression, enclose it in right single quotes. To evaluate the expression, omit these quotes.

```
> x := 2: y := 3: f := 'x + y';
f := x + y
```

```
> f := x + y;
f := 5
```

\(^{30}\)For more information about using quotes, see Punctuation Marks on page 48 or refer to ?quotes.
2.6 Exercises

1. Using the \%, \%, and \%\% operators, find the:
   a) Sum of 5434 and 6342.
   b) Product of 92 and 310.
   c) Quotient of the result from a) divided by the result from b).
   d) Quotient of the result from b) divided by the result from a).

2. Estimate \( \pi \) to 10,000 digits.

3. Concatenate the three strings "int", "(x^2," and "x)". Parse the resulting string. Evaluate the parsed string.

4. The Fibonacci numbers are a sequence of numbers. The first two numbers in the sequence are zero (0) and one (1). For \( n \) greater than two, the \( n^{th} \) number in the sequence is the sum of the two preceding numbers. Assign values to indexed names representing the first, second, and general Fibonacci numbers.

5. Determine a random integer between 40 and 100 using the command \texttt{rand(40..100)}. Concatenate this number with the string, "The student's grade is ". Extract the student's grade from the resulting string.

6. Assign the expressions \( x^2 \) and \( x\times x \) to the names \( \texttt{a} \) and \( \texttt{b} \). Find the three operands of \( \texttt{a} \) and \( \texttt{b} \). Compare the results with those returned by using the \texttt{dismantle} function, that is, \texttt{dismantle(a)} and \texttt{dismantle(b)}. The \texttt{dismantle} function displays the internal data structure used.

2.7 Conclusion

This chapter introduced the Maple language elements. In particular, tokens are the smallest meaningful elements in Maple. Tokens are used to form Maple expressions and statements. The next chapter shows how to use the information presented in this chapter to build expressions, and discusses how to form Maple statements.
3 Maple Expressions and Statements

You construct Maple statements, most importantly expressions, from tokens, which were discussed in chapter 2.

In This Chapter
- Syntax and semantics of the Maple language
- Expressions, the most important type of statement, including the use of expressions and the action of commands on expressions
- Other important types of statements

3.1 Syntax and Semantics

Syntax and semantics define a language. Syntax, or grammar, refers to the rules that are used to combine basic elements into statements. Semantics refers to the extra information or meaning that syntax cannot capture; it determines the actions Maple performs when you enter a command.

Syntax

Syntax defines valid forms of input, for example, expressions, statements, and procedures. It dictates, for example:

- Whether parentheses are required in the expression $x^{(y^z)}$
- How to enter a string that is longer than one line
- How to enter the floating-point number $2.3 \times 10^{-3}$
If the input is not syntactically correct, a syntax error is reported. Consider the following examples.

Adjacent minus signs are not valid.

```maple
> --1
```

on line 29, syntax error, ‘-‘ unexpected:

```
--1
```

You can enter floating-point numbers using many formats.

```maple
> 2.3e-3, 2.3E-03, +0.0023;
```

0.0023, 0.0023, 0.0023

However, you must place at least one digit between the decimal point and the exponent suffix.

```maple
> 2.e-3;
```

on line 42, syntax error, missing operator or ‘;’:

```
2.e-3;
```

A correct way to enter this expression is 2.0e-3.

**Semantics**

The semantics of a language specifies how expressions, statements, and programs execute—that is, the actions Maple performs with them. It controls:

- Whether \( \frac{x}{2z} \) or \( \frac{x}{2}/z \) is equal to \( \frac{x}{(2z)} \) or \( (x/2)*z \)
- The behavior when \( \sin(x)/x \) is computed with \( x=0 \)
- The return of 1, instead of an error, for \( \sin(0)/\sin(0) \)
- The value of \( i \) after executing the following loop

```maple
> for i from 1 to 5 do print(i^2) end do;
```
These are important concepts to understand before writing Maple programs.

A common mistake is to think that \(x/2*z\) is equal to \(x/(2*z)\). Following the operator precedence rules (see Table 3.4 on page 89), this is not true.

\[
> x/2*z, x/(2*z);
\]

\[
\frac{1}{2} x z, \frac{1}{2} \frac{x}{z}
\]

The remainder of this chapter focuses on the Maple programming language syntax.

### 3.2 Expressions

Expressions are the fundamental entities in the Maple language and the most important type of Maple statement. The types of expressions are the following:

- constants
- operator-based expressions
- data structures
- function calls

Procedures and modules\(^1\) are also valid expressions because you can use them wherever an expression is accepted. This is an important feature of the Maple language. Procedures are described separately in chapter 6.

Expressions are now presented in detail, beginning with the numeric constants. The presentation shows how to input the expression, gives examples of how and where to use the expression, and illustrates the action of the \texttt{type}, \texttt{nops}, \texttt{op}, and \texttt{subsop} commands on the expression.

### Constants

The Maple language contains both symbolic and numeric constants.

\(^1\)For information on modules, refer to chapter 2 of the \textit{Maple Advanced Programming Guide} or \texttt{?module}.
Maple has a general concept of *symbolic* constants. The global variable `constants` is assigned the expression sequence containing the names of the initially known constants. For more information, see page 35.

Maple also has *numeric* constants. The numeric constants in Maple are integers, rational numbers (fractions), floating-point numbers (decimals), and the special values, `infinity` and `undefined`. The complex numeric constants are the complex integers (Gaussian integers), complex rationals, and complex floating-point numbers. The full set of real and complex numeric constants is exactly what is recognized by `type(..., complex(extended_numeric))`.

Generally, a Maple expression is of type `constant` if it is:

- Of type `complex(extended_numeric)`
- One of the initially known constants
- An unevaluated function with all arguments of type `constant`
- A sum, product, or power with all operands of type `constant`.

For example, the following expressions are of type constant: 2, `sin(1)`, `f(2,3)`, `exp(gamma)`, `4+Pi`, `3+I`, `2*gamma/Pi^(1/2)`.

```maple
> type(2, constant);
true

> type(sin(1), constant);
true

> type(2*gamma/Pi^(1/2), constant);
true
```

**Integers** In Maple, an *integer* is an optionally signed, arbitrary-length sequence of one or more decimal digits. However, the integer 0 (zero) does not have a sign.\(^2\)

\(^2\)The integer zero (0) does not have a sign. However, the floating-point zeros, \(+0.0\) and \(-0.0\), are signed. For more information, see page 65.

\(^3\)The `evalb` command evaluates a Boolean expression. For more information, see page 86 or refer to ?evalb.
3.2 Expressions

> evalb(0 = +0); evalb(0 = -0); evalb(+0 = -0);

true
true
true

In Maple, the maximum length of an integer is system dependent and can be obtained by entering `kernelopts(maxdigits)`. The number of digits is approximately:

> kernelopts(maxdigits);

268435448

A natural integer is any sequence of one or more decimal digits. A signed integer is indicated by either +natural or -natural, where natural is any natural integer.

Hence, an integer is either a natural integer or a signed integer. The type of an integer is integer.

> type(-36, integer);

true

To form arithmetic expressions involving integers in Maple, use the Maple arithmetic operators (see page 70 or refer to ?arithop). In contrast with pocket calculators, when arithmetic expressions containing only integer operands are entered, Maple performs exact arithmetic—the integers are not converted to decimal numbers.4

Rational Numbers (Fractions) A rational number (fraction) is the quotient of two integers, where the denominator is always nonzero.

integer/natural

As with integers, Maple does exact arithmetic with fractions. Maple simplifies fractions so that the denominator is positive, and reduces the fraction to lowest terms by dividing the greatest common divisor from the numerator and denominator.

---

4For more information about integers in Maple, refer to ?integer.
If the denominator is 1 after simplifying the fraction, Maple converts the fraction to an integer.

```maple
> -30/12;

-5/2
```

```maple
> 25/5;

5
```

```maple
> whattype(%);

integer
```

The type of a fraction is `fraction`. The `type` command also accepts the composite type name `rational`, which is the union of `integer` and `fraction`—that is, rational numbers.

```maple
> x := 4/6;

x := 2/3
```

```maple
> type(x, fraction);

true
```

```maple
> type(x, rational);

true
```

A fraction has two operands, the numerator and denominator. It is recommended that you use the `numer` and `denom` commands to extract the numerator and denominator of a fraction.

```maple
> op(1,x), op(2,x);

2, 3
```

```maple
> numer(x), denom(x);
```
Floating-point Numbers (Decimals)  An *unsigned float* has one of the following six forms:

- \texttt{natural.natural}
- \texttt{natural.}
- \texttt{natural.}
- \texttt{natural exponent}
- \texttt{natural.natural exponent}
- \texttt{.natural exponent}

where \texttt{natural} is a natural integer, and \texttt{exponent} is the suffix containing the letter “e” or “E” followed by a signed integer with no spaces between.

A *floating-point number* is an unsigned float or a signed float (\texttt{+unsigned float} or \texttt{-unsigned float} indicates a signed float).

\begin{verbatim}
> 1.2, -2., +.2;

1.2, -2., 0.2

> 2e2, 1.2E+2, -.2e-2;

200., 120., -0.002
\end{verbatim}

Note that

\begin{verbatim}
> 1.e2;

on line 229, syntax error, missing operator or ';';
1.e2;
\end{verbatim}

is not valid, and that spaces are significant.

\begin{verbatim}
> .2e -1 <> .2e-1;

-0.8 \neq 0.02
\end{verbatim}

The type of a floating-point number is \texttt{float}. The \texttt{type} command also accepts the composite types \texttt{numeric} (which is the union of \texttt{integer},
fraction, and float—that is, the real numbers), and extended_numeric (which is the union of integer, fraction, float, infinity, and undefined).

A floating-point number has two parts, the mantissa (or significand) \( m \) and the exponent \( e \), which represent the number \( m \times 10^e \). The decimal point is placed after the right-most digit of \( m \). To access the parts of a floating-point number, use the SFloatMantissa and SFloatExponent commands.

\[
> \ x := 231.3;
\]

\[
x := 231.3
\]

\[
> \ \mathrm{SFloatMantissa}(x);
\]

\[
2313
\]

\[
> \ \mathrm{SFloatExponent}(x);
\]

\[
-1
\]

You can also use the Float command to construct floating-point numbers.

\[
\text{Float}( \ m, \ e );
\]

This constructs the floating-point number \( m \times 10^e \). Again, \( m \) is the mantissa, \( e \) is the exponent, and the decimal point is located to the right of \( m \).

\[
> \ \text{Float}(1.2, \ -3 );
\]

\[
0.0012
\]

The mantissa \( m \) is a Maple integer. Hence, it is subject to the same restrictions in terms of number of digits as any Maple integer. Since Maple integers are machine dependent, the maximum number of digits in any integer, and therefore in \( m \), is always at least 268,435,448.\(^6\)

\(^5\)For information about the full suite of numeric types and subtypes, refer to ?numeric_type.

\(^6\)For more information, see page 62 or refer to ?maxdigits.
3.2 Expressions

The exponent \( e \) is subject to a smaller restriction, which is again machine dependent, but is always at least 2,147,483,646. You can obtain the exact values of these limits by using the \texttt{Maple_floats} command. For more information, refer to \texttt{?Maple_floats}.

You can also enter a floating-point number \( m \times 10^e \) by simply forming the expression \( m \times 10^e \).

\[
> 1.2 \times 10^{-3};
\]

0.001200000000

**Arithmetic with Floating-point Numbers** For arithmetic operations and the standard mathematical functions, if one of the operands (or arguments) is a floating-point number or evaluates to a floating-point number, then floating-point arithmetic is used automatically. The global name \texttt{Digits}, which has the value 10 as its default, determines the number of digits (in the mantissa) that Maple uses for floating-point calculations.\(^7\)

\[
> x := 2.3: y := 3.7:
> 1 - x/y;
\]

0.3783783784

In general, you can use the \texttt{evalf} command to force the evaluation of a non-floating-point expression to a floating-point expression, where possible.

\[
> x := \ln(2);
\]

\[
x := \ln(2)
\]

\[
> \text{evalf}(x);
\]

0.6931471806

An optional index to the \texttt{evalf} command specifies the precision of the evaluation.\(^8\)

\(^7\)For more information, refer to \texttt{?Digits}.

\(^8\)For more information about evaluation using floating-point arithmetic, refer to \texttt{?evalf}. 
Complex Numerical Constants  By default, \( I \) denotes the complex unit \( \sqrt{-1} \) in Maple. Therefore, the following are equivalent.

\[
> \text{sqrt}(-1), I, (-1)^{(1/2)};
\]

\[
I, I, I
\]

You can enter a complex number \( a + bi \) as the sum \( a + b*I \) or by using the \texttt{Complex} command as \texttt{Complex(a, b)}.

\[
> 2 + 3*I;
\]

\[
2 + 3I
\]

\[
> \text{Complex}(2, 3);
\]

\[
2 + 3I
\]

Maple uses a special representation for complex numeric constants, such as \( 1.3 + 4.2*I \). To select the real and imaginary parts, use the \texttt{Re} and \texttt{Im} commands.\(^9\)

\[
> z := 2+3*I;
\]

\[
z := 2 + 3I
\]

\[
> \text{Re}(z), \text{Im}(z);
\]

\[
2, 3
\]

The type of a complex number is \texttt{complex(numeric)}. This means that the real and imaginary parts are of type \texttt{numeric}—that is, integers, fractions, or floating-point numbers. Other possible type names are listed in Table 3.1.

\(^9\)In an expression such as \( x + y*I \), where \( x \) and \( y \) are symbols, Maple does not assume that \( x \) is the real part and \( y \) is the imaginary part.
### Table 3.1 Types of Complex Numbers

<table>
<thead>
<tr>
<th>Complex Type Name</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>complex(integer)</td>
<td>both $a$ and $b$ are integers, possibly 0</td>
</tr>
<tr>
<td>complex(rational)</td>
<td>both $a$ and $b$ are rationals</td>
</tr>
<tr>
<td>complex(float)</td>
<td>both $a$ and $b$ are floating-point constants</td>
</tr>
<tr>
<td>complex(numeric)</td>
<td>any of the above</td>
</tr>
</tbody>
</table>

Arithmetic with complex numbers is done automatically.

```plaintext
> x := (1 + I); y := 2.0 - I;

$x := 1 + I$

$y := 2.0 - 1. I$

> x + y;

$3.0 + 0. I$
```

Maple can evaluate elementary functions and many special functions over the complex numbers. Maple evaluates the result automatically if $a$ and $b$ are numeric constants and one of $a$ or $b$ is a decimal number.

```plaintext
> exp(2+3*I), exp(2+3.0*I);

$e^{(2+3 I)}$, $-7.315110095 + 1.042743656 I$
```

If the arguments are not complex floating-point constants, you can expand the expression in some cases into the form $a+bi$, where $a$ and $b$ are real, by using the `evalc` command.

For example, the result of the following statement is not in the form $a+bi$ because $a$ is not of type `numeric`.

```plaintext
> 1/(a - I);

$\frac{1}{a-I}$

> evalc(%);
```
\[
\frac{a}{a^2 + 1} + \frac{I}{a^2 + 1}
\]

**Note:** The `evalc` command assumes that the symbol `a` is *real*.

To use another letter, say `j`, to represent the imaginary unit, use the `interface` command as follows.

```maple
> interface(imaginaryunit = j);

j
```

```maple
> solve( {z^2 = -1}, {z} );

\{z = j\}, \{z = -j\}
```

The following command reinstates `I` as the imaginary unit.

```maple
> interface(imaginaryunit = I);

I
```

```maple
> solve( {z^2 = -1}, {z} );

\{z = I\}, \{z = -I\}
```

**Operators**

A Maple operator is a symbol indicating that a mathematical operation is to be performed. This section discusses the Maple operators, and how to create expressions involving them.

**The Arithmetic Operators** The six Maple arithmetic operators are listed in Table 3.2. They can all be used as binary operators. However, you can also use the operators `+` and `-` as prefix operators representing unary plus and unary minus, respectively.

The types and operands of the arithmetic operations follow.

- The type of a sum or difference is `+`.
- The type of a product or quotient is `*`. 
3.2 Expressions

<table>
<thead>
<tr>
<th>Operator</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>addition</td>
</tr>
<tr>
<td>−</td>
<td>subtraction</td>
</tr>
<tr>
<td>*</td>
<td>multiplication</td>
</tr>
<tr>
<td>.</td>
<td>non-commutative multiplication</td>
</tr>
<tr>
<td>/</td>
<td>division</td>
</tr>
<tr>
<td>^</td>
<td>exponentiation</td>
</tr>
</tbody>
</table>

- The type of a power is ‘^’.
- The operands of the sum \(x - y\) are the terms \(x\) and \(-y\).
- The operands of the product \(xy^2/z\) are the factors \(x\), \(y^2\), and \(z^{-1}\).
- The operands of the power \(x^a\) are the base \(x\) and the exponent \(a\).

```maple
> whattype(x-y);

+                           

> whattype(x^y);

^                           
```

**Arithmetic** Maple always computes the result to the five arithmetic operations \(x + y\), \(-y\), \(x \times y\), \(x/y\), and \(x^n\), if \(n\) is an integer, and \(x\) and \(y\) are numbers. For example, you cannot prevent Maple from simplifying \(2 + 3\) to \(5\) in output.\(^\text{10}\) If the operands are floating-point numbers, Maple performs the arithmetic computation in the floating-point environment.

```maple
> 2 + 3, 6/4, 1.2/7, (2 + I)/(2 - 2*I);

5, \frac{3}{2}, 0.1714285714, \frac{1}{4} + \frac{3}{4}I
```

```maple
> 3^(1.2), I^(1.0 - I);

> 3^(1.2), I^(1.0 - I);
```

\(^{10}\)For information on displaying output that appears unsimplified, refer to the examples in section 1.3 of the *Maple Advanced Programming Guide*. 
For numerical constants, Maple reduces fractional powers of integers and fractions as follows.

- For integers \( n, m \) and fraction \( b \),
  \[(n/m)^b \rightarrow (n^b)/(m^b).
  \]
- For integers \( n, q, r, d \) and fraction \( b = q + r/d \) with \( 0 < r < d \),
  \[n^b = n^{q+r/d} \rightarrow n^q \times n^{r/d}.
  \]

For example,
\[
> 2^{(3/2)}, (-2)^{(7/3)};
2 \sqrt{2}, 4 (-2)^{(1/3)}
\]

**Automatic Simplifications** Maple automatically performs the following simplifications for any symbol \( x \) or arbitrary expression.

\[
> x - x, x + x, x + 0, x*x, x/x, x*1, x^0, x^1;
0, 2x, x, x^2, 1, x, 1, x
\]

Note the following exceptions.
\[
> \text{infinity} - \text{infinity};
\text{undefined}
\]
\[
> \text{infinity}/\text{infinity};
\text{undefined}
\]
\[
> 0/0;
\text{Error, numeric exception: division by zero}
\]
To perform additional simplifications, use the `simplify` command.\(^{11}\)

In Maple, addition and multiplication are associative and commutative. Therefore, the following simplifications are performed where \(a, b,\) and \(c\) denote numerical constants, and \(x, y,\) and \(z\) denote general symbolic expressions.

\[
ax + bx \rightarrow (a + b)x
\]

\[
x^a \times x^b \rightarrow x^{a+b}
\]

\[
a(x + y) \rightarrow ax + ay
\]

The first two simplifications mean that Maple adds like terms in polynomials automatically. The third simplification means that Maple distributes numerical constants (integers, fractions, and floating-point numbers) over sums, but does not do the same for non-numerical constants.

\[
> 2*x + 3*x, x*y*x^2, 2*(x + y), z*(x + y);
\]

\[
5x, x^3y, 2x + 2y, z(x + y)
\]

To force Maple to display an expression in expanded form, use the `expand` command.\(^{12}\)

The simplifications that are most confusing to Maple users relate to simplifying powers \(x^y\) for non-integer exponents \(y\).

**Simplification of Repeated Exponentiation**  In general, Maple does not perform the simplification \((x^y)^z \rightarrow x^{yz}\) automatically because it does not always provide an accurate answer. For example, letting \(y = 2\) and \(z = 1/2\), the simplification would imply that \(\sqrt{x^2} = x\), which is not necessarily true. Maple performs the transformation only if it is provably correct for all complex \(x\) with the possible exception of a finite number of values, such as 0 and \(\infty\). Maple simplifies \((x^a)^b \rightarrow x^{ab}\) if \(b\) is an integer, \(-1 < a \leq 1\), or \(x\) is a positive real constant.

\[
> (x^(3/5))^(1/2), (x^(5/3))^(1/2);
\]

\[
x^{(3/10)}, \sqrt{x^{(5/3)}}
\]

\(^{11}\)For more information, refer to `?simplify`.

\(^{12}\)For more information, refer to `?expand`.  
Chapter 3: Maple Expressions and Statements

\[
> (2^{(5/3)})^{(1/2)}, (x^{(-1)})^{(1/2)};
\]

\[2^{(5/6)}, \sqrt{\frac{1}{x}}\]

Similarly, Maple does not simplify \(a^b c^b \rightarrow (ac)^b\) automatically. This simplification may introduce more unique roots.

\[
> 2^{(1/2)}+3^{(1/2)}+2^{(1/2)}*3^{(1/2)};
\]

\[\sqrt{2} + \sqrt{3} + \sqrt{2} \sqrt{3}\]

Simplifying \(\sqrt{2} \sqrt{3}\) to \(\sqrt{6}\) in the previous expression would create a third unique square root. Calculating with roots is, in general, difficult and expensive. Therefore, new roots are avoided.

Use the `combine` command to combine roots.\(^\text{13}\)

**The Non-commutative Multiplication Operator**. The . (dot) operator performs (non-commutative) multiplication on its arguments. It is left associative.

\[
> A . B;
\]

If \(A\) and \(B\) are numbers (including complex and extended numerics such as `infinity` and `undefined`), then \(A . B = A*B\). If one of \(A\) and \(B\) is a Matrix or a Vector, and the other is a Matrix, Vector, or constant, their product is computed by using the `LinearAlgebra[Multiply]` command. Arguments that are not of type Matrix, Vector, or constant are ignored, and \(A . B\) remains unevaluated. There is also a dot product operator in the `VectorCalculus` package.\(^\text{14}\)

\[
> 7 . 6;
\]

\[42\]

\[
> M:=<<1,0,2>|<0,1,2>|<0,0,2>>;
\]

\(^{13}\text{For more information, refer to }\texttt{?combine}.\)

\(^{14}\text{For more information about Matrix, Vector, and the LinearAlgebra package in }\texttt{Maple}, \text{refer to }\texttt{?LAOverview}. \text{For more information about the non-commutative multiplication operator, refer to }\texttt{?dot}. \text{For more information about the VectorCalculus dot product operator, refer to }\texttt{?VectorCalculus[DotProduct]}.\)
3.2 Expressions

\[
M := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 2 \end{bmatrix}
\]

\[
V := \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}
\]

\[
M \cdot V;
\]

\[
\begin{bmatrix} 10 \\ 0 \\ 20 \end{bmatrix}
\]

\[
\lambda \cdot M \cdot V;
\]

\[
\begin{bmatrix} 10 \\ 0 \\ 20 \end{bmatrix}
\]

In Maple, . (dot) can be interpreted as a decimal point (for example, 2.3), as part of a range operator (for example, x..y), or as the (non-commutative) multiplication operator. To distinguish between these three circumstances, Maple uses the following rule.

Any dot with spaces before and/or after it that is not part of a number is interpreted as the non-commutative multiplication operator.

For example, 2.3 is a number, 2 . 3 and 2 .3 return 6, and 2 . 3 returns an error.\(^{15}\)

\[
\begin{align*}
> & \ 2.3, \ 2 . 3, \ 2 .3; \\
& 2.3, \ 6, \ 6
\end{align*}
\]

\(^{15}\)For more information about floating-point numbers, see page 65 or refer to ?Float. For more information about the range operator, see page 77 or refer to ?range.
Chapter 3: Maple Expressions and Statements

> 2. 3;

on line 739, syntax error, unexpected number:
2. 3;

The Composition Operators @ and @@

The composition operators are @ and @@.
The @ operator represents function composition. For example, f@g denotes \( f \circ g \).

\[
> (f@g)(x);
\]

\[
f(g(x))
\]

\[
> (\sin@\cos)(\Pi/2);
\]

\[
0
\]

The @@ operator represents repeated functional composition. For example, f@@n denotes \( f^{(n)} \).

\[
> (f@@2)(x);
\]

\[
(f^{(2)})(x)
\]

\[
> \text{expand}(\%);
\]

\[
f(f(x))
\]

\[
> (D@@n)(f);
\]

\[
(D^{(n)})(f)
\]

There is no single notational rule used by mathematicians. Usually \( f^n(x) \) denotes composition. For example, \( D^n \) denotes the differential operator composed \( n \) times. Also, \( \sin^{-1}(x) \) denotes the inverse of the \( \sin \) function, that is, composition to the power \(-1\). However, some mathematicians use \( f^n(x) \) to denote ordinary powering, for example, \( \sin^2(x) \) is the square of the \( \sin \) of \( x \). In Maple \( f^n(x) \) always denotes repeated composition and \( f(x)^n \) always denotes powering.
3.2 Expressions

\[ > \sin(x)^2, (\sin(2))^{(2)}(x), \frac{1}{\sin(x)}, \arcsin(x) \]

\[ > \sin(\pi)^2, (\sin(\pi))^{(2)}(\pi); \]

\[ 0, 0 \]

The Ditto Operators \%, \%%, and \%\%\%

The sequence of expressions assigned to the three ditto nullary operators is the last three non-NULL results generated in the Maple session independent of where they are located in the session.¹⁶,¹⁷

The Range Operator ..

A range expression is represented by using the binary operator .. (two consecutive periods) between two expressions.

\[ \text{expression1 .. expression2} \]

It is important to note that this only represents a range, that is, it is a notational tool. For example, the range 1..3 is not equivalent to the expression sequence 1, 2, 3. However, the \texttt{seq} command can be used to yield the expression sequence.¹⁸

\[ > 1..3; \]

\[ 1..3 \]

\[ > \text{seq}( i, i = 1..3 ); \]

\[ 1, 2, 3 \]

A range has type ‘..’ or \texttt{range}.

\[ > r:=3..7; \]

\[ r := 3..7 \]

¹⁶ For more information about the ditto operator and nullary operators, see page 25 or refer to \texttt{?ditto}.

¹⁷ The ditto operators are technically not operators.

¹⁸ For more information about the \texttt{seq} command, see page 183 or refer to \texttt{?seq}.
A range has two operands, the left-hand limit and the right-hand limit. You can access these limits by using the `op` command or the `lhs` and `rhs` commands.

```maple
> op(1,r), op(2,r);
3, 7

> lhs(r), rhs(r);
3, 7
```

You can also use the `range` construct in conjunction with the `op` command to extract a sequence of operands from an expression.

```maple
> a := [ u, v, w, x, y, z ];

a := [u, v, w, x, y, z]

> op(2..5,a);
v, w, x, y
```

**The Factorial Operator**! Maple uses the unary operator `!` as a postfix operator that denotes the factorial function of its operand `n`.

```maple
n!
```

The expression `n!` is shorthand for the command `factorial(n)`.

```maple
> 0!, 5!;

1, 120
```

For floating-point `n`, generalized factorial function values `n!` are calculated by using `GAMMA(n+1)`.
3.2 Expressions

> 2.5!;

```
3.323350970
```

> (-2)!

Error, numeric exception: division by zero

The type of an unevaluated factorial is !. Note that in Maple, n!! does not denote the double factorial function. It denotes repeated factorial, n!! = (n!)!.

> 3!!;

720

The mod Operator The mod operator evaluates an expression modulo m, for a nonzero integer m.

```
e mod m;
```

The operator syntax e mod m is equivalent to the `mod`(e,m) command.

> 5 mod 2;

```
1
```

Maple has two representations for an integer modulo m: modp and mods. You can assign the environment variable `mod` to either modp or mods.

**modp** In the positive representation, e mod m is an integer between 0 and m – 1, inclusive. The following assignment explicitly selects the positive representation.

---

19 For information on the Maple doublefactorial command, refer to ?doublefactorial.
20 Use left single quotes around mod when it is not used as an operator because it is a reserved word. For more information about reserved words, see page 24 or refer to ?keyword.
> `mod` := modp;

```
  mod := modp
```

> 9 mod 5;

```
  4
```

This is the default representation.

**mods** In the symmetric representation, \(e \mod m\) is an integer between \(-\text{floor}((\text{abs}(m)-1)/2)\) and \(\text{floor}(\text{abs}(m)/2)\). The following assignment selects the symmetric representation.\(^{21}\)

> `mod` := mods;

```
  mod := mods
```

> 9 mod 5;

```
-1
```

Alternatively, you can invoke the `modp` and `mods` commands directly.

> modp(9,5), mods(9,5);

```
4, -1
```

The `mod` operator accepts the inert operator \(^{\wedge}\) for powering. That is, \(i^{\wedge}j \mod m\) calculates \(i^j \mod m\). Instead of separately computing the integer \(i^j\), which may be too large to compute, and then reducing modulo \(m\), Maple computes the power by using binary powering with remainders.

> 2\(^{\wedge}(2^{\wedge}100)\) mod 5;

```
Error, numeric exception: overflow
```

\(^{21}\)For more information on the `floor` function, refer to ?floor.
3.2 Expressions

\[ 2^{-100} \mod 5; \]

1

The first operand of the \texttt{mod} operator can be a general expression. Maple evaluates the expression over the ring of integers modulo \( m \). For polynomials, it reduces rational coefficients modulo \( m \). The \texttt{mod} operator accepts many functions for polynomial and matrix arithmetic over finite rings and fields, for example, \texttt{Factor} for polynomial factorization and \texttt{Nullspace} for matrix null spaces.\footnote{Do not confuse the commands \texttt{factor} and \texttt{Factor} or \texttt{int} and \texttt{Int}. The former evaluate immediately; the latter are \textit{inert} commands which Maple does not evaluate until you make the call to \texttt{mod}. For more information on inert functions, refer to \texttt{?inert}.}

\[ 1/2 \mod 5; \]

3

\[ 9x^{-2} + x/2 + 13 \mod 5; \]

\[ 4x^2 + 3x + 3 \]

\[ \text{Factor}(4x^{-2} + 3x + 3) \mod 5; \]

\[ 4(x + 3)(x + 4) \]

The \texttt{mod} command can also compute over a Galois field \( GF(p^k) \), that is, the finite field with \( p^k \) elements.\footnote{For more information and a list of commands that \texttt{mod} accepts, refer to \texttt{?mod}.}

**The Neutral Operators \&name** Maple has a neutral operator (or user-defined) facility. You can form a neutral operator symbol by using the ampersand character “\&” followed by one or more characters. There are two varieties of \&-names: alphanumerics and non-alphanumeric.

**alphanumerics** The \& character followed by any Maple name not requiring left single quotes, for example, \texttt{wedge}.

**non-alphanumeric** The \& character followed by one or more non-alphanumeric characters, for example, \texttt{&+} or \texttt{&++}.
The following characters cannot appear in an &-name

\& | ( ) [ ] { } ; : ' ' # \ %

plus the new line and blank characters.

You can use neutral operators as unary prefix operators, binary infix operators, or function calls. In any of these cases, they generate function calls with the name of the function being that of the neutral operator. (In the standard pretty-printing (output format) mode, these function calls are printed in binary operator format when exactly two operands exist, and in unary operator format when exactly one operand exists. However, the internal representation is an unevaluated function.) Consider the following example.

> a &~ b &~ c;

\((a &~ b) &~ c\)

> op(%);

\(a &~ b, c\)

> op(0,%%);

\&~

Maple imposes no semantics on the neutral operators. The user can define the operator to have a meaning by assigning the name to a Maple procedure. You can define manipulations on expressions containing such operators by using the interface to user-defined procedures for standard library commands, including \texttt{simplify}, \texttt{diff}, \texttt{combine}, \texttt{series}, \texttt{evalf}, and many others. For more information on neutral operators, refer to \textbf{Neutral Operators} in section 1.5 of the \textit{Maple Advanced Programming Guide}.

\textbf{The Relational Operators} <, >, <=, >=, and <> You can form new types of expressions from ordinary algebraic expressions by using the relational operators <, >, <=, >=, and <>. The semantics of these operators depend on whether they occur in an algebraic context or in a Boolean context.

In an algebraic context, the relational operators are simply placeholders for forming equations or inequalities. Maple fully supports addition of equations or inequalities, and multiplication of an equation or inequality
by an algebraic expression. To add or subtract two equations, for example, Maple applies the addition or subtraction to each side of the equations, yielding a new equation. In the case of multiplying an equation by an expression, Maple distributes the multiplication to each side of the equation. You can perform similar operations with inequalities.

> e := x + 3*y = z;

\[ e := x + 3y = z \]

> 2*e;

\[ 2x + 6y = 2z \]

The type of an equation is = or equation. Use the \texttt{op(0,...)} or \texttt{whattype} command to return the principal type.

> op(0,e); =

> whattype(e); =

An equation has two operands, the left-hand side and the right-hand side. In addition to using the \texttt{op} command to select the operands of an equation, you can use the \texttt{lhs} and \texttt{rhs} commands.

> lhs(e);

\[ x + 3y \]

The \texttt{type} command also accepts the types \texttt{<>}, \texttt{<}, and \texttt{<=}. Maple automatically converts inequalities involving \texttt{>} or \texttt{>=} to \texttt{<} and \texttt{<=}, respectively. All the relational types have two operands.

> e := a > b;

\[ e := b < a \]
Chapter 3: Maple Expressions and Statements

> op(e);

\[ b, a \]

In a *Boolean* context, Maple evaluates expressions to the value `true` or the value `false`. A Boolean context includes the condition in an `if` statement and the condition in the `while` clause of a loop.\(^{24}\)

In the case of the operators `<`, `<=`, `>`, and `>=`, the difference of the operands, in general, evaluates to a numeric constant that Maple compares with zero.

\[
> \text{if } 2 < 3 \text{ then "less" else "not less" end if;}
\]

"less"

You can also use the relational operators to compare strings.

\[
> \text{if } "f" \leq "m" \text{ then "first half" else "second half" end if;}
\]

"first half"

In the case of the relations `=` and `<>`, the operands can be arbitrary expressions (algebraic or non-algebraic). This equality test for expressions tests object equality of the Maple representations of the expressions, which is *not* the same as mathematical equivalence.

To evaluate a relation in a Boolean context, use the `evalb` command.

\[
> \text{evalb}( x + y = y + x );
\]

`true`

\[
> \text{evalb}( x^2 - y^2 = (x - y)(x + y) );
\]

`false`

For the latter example, apply the `expand` command to show that the equation is `true`.

\(^{24}\)For more information about `if` statements and `while` clauses, see chapter 5, or refer to `?if` and `?while`. 
3.2 Expressions

> evalb( x^2 - y^2 = expand( (x - y)*(x + y) ) );

true

You can also use the is command, instead of evalb, to evaluate relations in a Boolean context.

> is( x^2 - y^2 = (x - y)*(x + y) );

true

> is( 3<Pi );

true

The Logical Operators and, or, xor, implies, and not

Generally, you can form an expression by using the logical operators and, or, xor, implies, and not. The first four are binary operators and the last is a unary (prefix) operator. An expression containing one or more logical operators is automatically evaluated in a Boolean context.

> 2>3 or not 5>1;

false

> 3 < evalf(Pi) xor evalf(exp(1)) < evalf(Pi);

false

The precedence of the logical operators and, or, and not is analogous to that of multiplication, addition, and exponentiation, respectively. Here no parentheses are necessary.\(^{25}\)

> (a and b) or ((not c) and d);

\(a \text{ and } b \text{ or } \not c \text{ and } d\)

The xor operator is of lower precedence than or. The implies operator has the lowest precedence of the logical operators.

---

\(^{25}\)For more information about precedence of Maple operators, see Table 3.4 on page 89 or refer to ?precedence.
The type names for the logical operators \texttt{and}, \texttt{or}, \texttt{xor}, \texttt{implies}, and \texttt{not} are \texttt{and}, \texttt{or}, \texttt{xor}, \texttt{implies}, and \texttt{not}, respectively. The first four have two operands, the last has one operand.

```maple
> b := x and y or z;

b := x \texttt{and} y \texttt{or} z
```

```maple
> whattype(b);

\texttt{or}
```

```maple
> op(b);

x \texttt{and} y, z
```

In expressions with operators of the same precedence, the evaluation of Boolean expressions that involve the logical operators \texttt{and} and \texttt{or} proceeds from left to right, and terminates once Maple can determine the truth of the whole expression. Consider the evaluation of the following.

```maple
a \texttt{and} b \texttt{and} c
```

If the result of evaluating \texttt{a} is \texttt{false}, the result of the entire Boolean expression is \texttt{false}, regardless of the value of \texttt{b} and \texttt{c}. Therefore, Maple stops evaluating the expression. These evaluation rules are commonly known as \textit{McCarthy} evaluation rules and they are quite crucial for programming. Consider the following statement.

```maple
if x <> 0 and f(x)/x > 1 then ... end if;
```

If Maple always evaluated both operands of the \texttt{and} clause, when \texttt{x} is 0, evaluation would result in a \textit{division by zero} error. The advantage of the above code is that Maple attempts to check the second condition only when \texttt{x} \neq 0.

**Boolean Expressions** In general, a Boolean context requires a Boolean expression. Therefore, you must use the Boolean constants \texttt{true}, \texttt{false}, and \texttt{FAIL}, the relational operators, and the logical operators to form Boolean expressions. The \texttt{type} command accepts all these for the name \texttt{boolean}. 
The evaluation of Boolean expressions in Maple uses \textit{three-valued logic}. In addition to the special names \texttt{true} and \texttt{false}, Maple also understands the special name \texttt{FAIL}. Maple sometimes uses the value \texttt{FAIL} when it is unable to completely solve a problem. You can interpret it as the value “\texttt{unknown}.”

\begin{verbatim}
> is(sin(1),positive);

true

> is(a1,positive);

false
\end{verbatim}

In the context of the Boolean clause in an \texttt{if} statement or a \texttt{while} statement, Maple determines the branching of the program by treating the value \texttt{FAIL} as if it were the value \texttt{false}. With three-valued logic, you do not need to test for \texttt{FAIL} separately when using the \texttt{is} command. Otherwise, you would need to write

\begin{verbatim}
if is( a - 1, positive) = true then ...
\end{verbatim}

The three-valued logic allows you to write

\begin{verbatim}
if is( a - 1, positive) then ...
\end{verbatim}

The evaluation of a Boolean expression yields \texttt{true}, \texttt{false}, or \texttt{FAIL} according to Table 3.3.

It is important to note that three-valued logic leads to asymmetry in the use of \texttt{if} statements and \texttt{while} statements. For example, the following two statements are not equivalent.
if condition then statseq1 else statseq2 end if;
if not condition then statseq2 else statseq1 end if;

If condition has the value FAIL, statseq2 is executed in the first structure and statseq1 is executed in the second structure.

The Set Operators union, minus, intersect, and subset  
The union, intersect, minus, and subset commands are used for the set union, intersection, difference, and subset operations. The union and intersect commands are infix n-ary operators. The minus and subset commands are binary infix operators.

> A:={1,2,3}; B:={2,3,4};

\[
A := \{1, 2, 3\} \\
B := \{2, 3, 4\}
\]

> A union B;

\[
\{1, 2, 3, 4\}
\]

> A intersect B;

\[
\{2, 3\}
\]

> A minus B;

\[
\{1\}
\]

> A subset B;

\[
false
\]

Associated with each set operator, there is a type of the same name. For more information, refer to ?type.

The assuming Operator  
The assuming operator performs a single computation under assumptions on the name(s) in the expression. For more information about the assuming command, refer to chapter 3 of the Maple User Manual or ?assuming.
### 3.2 Expressions

#### Table 3.4 Operator Precedence Order

<table>
<thead>
<tr>
<th>Operator</th>
<th>Associativity</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td>non-associative</td>
</tr>
<tr>
<td>&amp;-operators</td>
<td>left associative</td>
</tr>
<tr>
<td>!</td>
<td>left associative</td>
</tr>
<tr>
<td>^, @@</td>
<td>non-associative</td>
</tr>
<tr>
<td>., *, /, @, intersect</td>
<td>left associative</td>
</tr>
<tr>
<td>+, -, union</td>
<td>left associative</td>
</tr>
<tr>
<td>mod</td>
<td>non-associative</td>
</tr>
<tr>
<td>subset</td>
<td>non-associative</td>
</tr>
<tr>
<td>..</td>
<td>non-associative</td>
</tr>
<tr>
<td>&lt;, &lt;=, &gt;, &gt;=, =, &lt;&gt;, in</td>
<td>non-associative</td>
</tr>
<tr>
<td>$</td>
<td>non-associative</td>
</tr>
<tr>
<td>not</td>
<td>right associative</td>
</tr>
<tr>
<td>and</td>
<td>left associative</td>
</tr>
<tr>
<td>or</td>
<td>left associative</td>
</tr>
<tr>
<td>xor</td>
<td>left associative</td>
</tr>
<tr>
<td>implies</td>
<td>non-associative</td>
</tr>
<tr>
<td>assuming</td>
<td>non-associative</td>
</tr>
</tbody>
</table>

**Precedence Rules** The order of precedence of all the operators in this section is shown in Table 3.4, from highest to lowest binding strengths.\(^{26}\)

**Data Structures**

The Maple system includes many data structures. The more common ones are mentioned here in the context of expressions. For a thorough discussion on sequences, sets, lists, tables, arrays, Arrays, and others, see chapter 4.

**Sequences, Sets, and Lists** *A sequence* is a group of expressions separated by commas.

```
expression1, expression2, expression3
```

**Example** The parameter \( s \) forms the sequence containing the expressions 5, 6, and 7.

```
> s := 5, 6, 7;
```

\(^{26}\)For more information about precedence in Maple, refer to \(?precedence\).
Chapter 3: Maple Expressions and Statements

\[ s := 5, 6, 7 \]

\[ > \text{whattype}(\%); \]

\textit{exprseq}

You can also use the \texttt{seq} operator to form sequences.

\[ > \text{seq}( i, i=5..7 ); \]

\[ 5, 6, 7 \]

Sequences occur in many Maple expressions. In particular, a sequence is the basic element for forming sets, lists, and function calls.

A \textit{set} is an unordered sequence of unique expressions. A set is formed by enclosing a sequence in braces (\{\}).

\[ > \text{myset} := \{s\}; \]

\[ \text{myset} := \{5, 6, 7\} \]

\[ > \text{whattype}(\%); \]

\textit{set}

A \textit{list} is an ordered sequence of expressions. The expressions in the list need not be unique. A list is formed by enclosing a sequence in square brackets ([]).

\[ > \text{mylist} := [s]; \]

\[ \text{mylist} := [5, 6, 7] \]

\[ > \text{whattype}(\%); \]

\textit{list}
Tables, arrays, and Arrays  The table data structure in Maple is a special object for representing data in tables. You can create a table either explicitly by using the `table` command or implicitly by assignment to an indexed name. The following statements are equivalent.

\[ T := \text{table}([(\text{Na},11) = 23]) \]

\[ T[\text{Na},11] := 23; \]

They both create a table object with one component. The purpose of a table is to allow fast access to data.

\[ T[\text{Na},11]; \]

The array data structure in Maple is a specialization of the table data structure. An array is a table with specified dimensions, where each dimension is an integer range. To create an array, use the `array` command.

\[ a := \text{array}(1..2,1..2); \]

An Array is a table-like data structure with fixed dimensions and integer indices.\(^{27}\) To create an Array, use the `Array` command.

\[ A := \text{Array}([[1,2],[3,4]]); \]

For more information about tables, arrays, and Arrays, see chapter 4.

\(^{27}\)For information on the differences between arrays and Arrays, see 4.4 arrays and Arrays.
Functions
A function expression represents a function call, or in other words, an application of a function or procedure to arguments. Such an expression is said to be of type function. A typical example of an expression of type function is the expression $f(x)$, which represents the application of the expression $f$ to the argument sequence $x$. The entire expression $f(x)$ is of type function (that is, a function call or function application), while the expression $f$ is typically not itself of type function (but it is often of type procedure).

A function call in Maple takes the following form.

\[
f( \text{sequence} )
\]

Often $f$ is a procedure, or a name that evaluates to a procedure.

\[
> \sin(x);
\]

\[
\sin(x)
\]

\[
> \text{min}(2,3,1);
\]

\[
1
\]

\[
> g();
\]

\[
g()
\]

\[
> a[1](x);
\]

\[
a_1(x)
\]

Maple executes a function call as follows. First, $f$ is evaluated (typically yielding a procedure). Next, Maple evaluates the operands of sequence (the arguments) in an unspecified order. If any of the arguments evaluates to a sequence, Maple flattens the sequence of evaluated arguments into one sequence. If $f$ evaluates to a procedure, Maple invokes this procedure on the argument sequence.\(^{28}\)

\(^{28}\)In the example below, the arrow ($\rightarrow$) notation is used. For more information about creating procedures using the arrow notation, see page 207.
As mentioned previously, a function object’s type is `function`. The operands are the arguments. The zeroth operand is the name of the function.

```plaintext
> x := 1:
> f(x);

f(1)

> s := 2,3;

s := 2, 3

> f(s,x);

f(2, 3, 1)

> f := g;

f := g

> f(s,x);

g(2, 3, 1)

> g := (a,b,c) -> a+b+c;

$g := (a, b, c) \rightarrow a + b + c$

> f(s,x);

6
```
min

> op(m);

1, y, z

> type(m, function);

true

> f := n!;

f := n!

> type(f, function);

true

> op(0, f);

factorial

> op(f);

n

In general, the function name $f$ can be one of the following.

- name
- procedure definition
- integer
- float
- parenthesized algebraic expression
- function
- module selection, for example, module_name:-f(sequence)
If \( f \) is a procedure definition, you can write, for example,
> proc(t) t*(1-t) end proc (t^2);

\[ t^2 (1 - t^2) \]

instead of the following two statements.
> h := proc(t) t*(t-1) end proc;

\[ h := \text{proc}(t) t \ast (t - 1) \text{end proc} \]

> h(t^2);

\[ t^2 (t^2 - 1) \]

If \( f \) is an integer or a float, Maple treats \( f \) as a constant operator. That is, \( f(x) \) returns \( f \).
> 2(x);

\[ 2 \]

The following rules define the meaning of a parenthesized algebraic expression.
> (f + g)(x), (f - g)(x), (-f)(x), (f@g)(x);

\[ f(x) + g(x), f(x) - g(x), -f(x), f(g(x)) \]

Recall that the @ sign denotes functional composition, that is, \( f@g \) denotes \( f \circ g \). These rules together with the previous rule mean the following.
> (f@g + f^2*2g +1)(x);

\[ f(g(x)) + f(x)^2 g(x) + 1 \]

Recall that @@ denotes functional exponentiation, that is, \( f@@n \) denotes \( f^{(n)} \) which means \( f \) composed with itself \( n \) times.
> (f@@3)(x);
Chapter 3: Maple Expressions and Statements

\[(f^{(3)})(x)\]

\[\text{expand(\%);}\]

\[f(f(f(x)))\]

Finally, \( f \) can be a function, as in the following statements.\(^{29}\)

\[\text{cos(0);}\]

\[1\]

\[\text{f(g)(0);}\]

\[f(g)(0)\]

\[\text{D(cos)(0);}\]

\[0\]

### 3.3 Using Expressions

**Investigating the Parts of an Expression**

When programming with expressions, it is important to be able to:

- Determine whether an expression has a given type\(^{30}\)
- Determine the number of operands in an expression
- Select operands from an expression
- Replace operands in an expression

The commands have been introduced previously, but they are now formally presented in Table 3.5.

Consider the following formula.

---

\(^{29}\)For more information about how to define a function by using functional operators, see page 231 or refer to \texttt{?unapply}.

\(^{30}\)An expression can have multiple types.
3.3 Using Expressions

Table 3.5 Type and Operand Commands

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>type(f, t)</td>
<td>tests if f is of type t</td>
</tr>
<tr>
<td>nops(f)</td>
<td>returns the number of operands of f</td>
</tr>
<tr>
<td>op(i, f)</td>
<td>selects the ith operand of f</td>
</tr>
<tr>
<td>subsop(i=g, f)</td>
<td>replaces the ith operand of f with g</td>
</tr>
</tbody>
</table>

```plaintext
> f := sin(x) + 2*cos(x)^2*sin(x) + 3;
f := sin(x) + 2*cos(x)^2*sin(x) + 3

> type(f, '+');
true

> type(f, '*');
false

> nops(f);
3

> op(1, f);
sin(x)

> subsop(2=0, f);
sin(x) + 3
```

The `op` command has several other useful forms. The command

```plaintext
op(i..j, f)
```
returns the sequence
Chapter 3: Maple Expressions and Statements

\[ \text{op}(i, f), \text{op}(i+1, f), \ldots, \text{op}(j-1, f), \text{op}(j, f) \]

of operands of \( f \) between \( i \) and \( j \) inclusive.

The command

\[ \text{op}([i, j, k], f) \]

is an abbreviation of

\[ \text{op}(k, \text{op}(j, \text{op}(i, f))) \]

The last object in the list can also be a range.

The command

\[ \text{op}([i1, \ldots, iN, j..k], f) \]

is an abbreviation of

\[ \text{op}(j..k, \text{op}(iN, \ldots, \text{op}(i1, f)\ldots)) \]

To see the whole sequence of operands of an expression, use

\[ \text{op}(f) \]

which is equivalent to \( \text{op}(1..\text{nops}(f), f) \).

The special operand \( \text{op}(0, f) \) generally returns the primary type of an expression.\(^{31}\) However, if \( f \) is a function, it returns the name of the function.

\[
\begin{align*}
> & \ \text{op}(0, f); \\
& + \\
> & \ \text{op}(1..3, f); \\
& \sin(x), 2 \cos(x)^2 \sin(x), 3 \\
> & \ \text{op}(0, \text{op}(1,f));
\end{align*}
\]

\(^{31}\)For a complete description of \( \text{op}(0,f) \), refer to \( \text{?op} \).
Using the results of these commands, you can start to develop an expression tree of nodes, branches, and leaves for $f$.

\[ \sin(x) + 2\cos(x)^2 \sin(x) + 3 \]

The complete tree is:

![Expression Tree Diagram]

The first “node” of the expression tree labeled “+” is a sum. This indicates the expression’s type (from $\text{op}(f, 0)$).

This expression has three branches corresponding to the three terms in the sum (from $\text{op}(1..3, f)$)—that is, $\sin(x)$, $2\cos(x)^2 \sin(x)$, and 3.

The nodes of each branch indicate the type of each term in the sum. The leaves of the tree are the names and integers in this example.$^{32,33}$

---

$^{32}$The idea of an expression tree closely models how Maple interprets expressions. More precisely, Maple uses DAGs (directed acyclic graphs) to represent arbitrary expressions. While similar to this idea of an expression tree, they have the additional property that all common subexpressions are identified by Maple only once. As a result, since all like subexpressions are shared, expressions are nonmutable, and changing an expression involves a copy operation. For more information about DAGs, refer to chapter 6 of the *Maple Advanced Programming Guide*.

$^{33}$For more information about the display of Maple data structures, refer to ?dismantle.
Evaluating and Simplifying Expressions

Example 1  To understand how Maple evaluates and simplifies expressions, consider the following example.

```
> x := Pi/6:
> sin(x) + 2*cos(x)^2*sin(x) + 3;
```

```
17/4
```

Maple first reads and parses the input. As the input is parsed, Maple builds an expression tree to represent the value

\[ \sin(x) + 2 \cos(x)^2 \sin(x) + 3. \]

Maple simplifies the expression tree, and then evaluates the result. The evaluation process substitutes values for variables and invokes any functions or procedures. In this case, \( x \) evaluates to \( \pi/6 \). Hence, with these substitutions, the expression is:

\[ \sin(\pi/6) + 2 \cos(\pi/6)^2 \sin(\pi/6) + 3 \]

Invoking the \( \sin \) and \( \cos \) functions, Maple obtains a new “expression tree”,

\[ 1/2 + 2 \times (1/2\sqrt{3})^2 \times 1/2 + 3. \]

Maple simplifies this result to obtain the fraction 17/4.

Example 2  Alternatively, consider the next example: evaluation occurs, but no simplification is possible.

```
> x := 1;
> sin(x) + 2*cos(x)^2*sin(x) + 3;
```

```
x := 1
```

```
\sin(1) + 2 \cos(1)^2 \sin(1) + 3
```

Unevaluated Expressions

In general, Maple evaluates all expressions immediately. In some situations, it is necessary to delay evaluation. An expression enclosed in right single quotes is called an *unevaluated* expression.
For example, the statements
\begin{verbatim}
> a := 1; x := a + b;
\end{verbatim}

\begin{verbatim}
a := 1
x := 1 + b
\end{verbatim}

assign the value $1 + b$ to the name $x$, while the statements
\begin{verbatim}
> a := 1; x := 'a' + b;
\end{verbatim}

\begin{verbatim}
a := 1
x := a + b
\end{verbatim}

assign the value $a + b$ to the name $x$ if $b$ has no value.

Evaluating an expression enclosed in right single quotes removes one level of quotes, so in some cases nested quotes are useful.

**Note:** There is a distinction between evaluation and simplification of statements.

**Example** Consider the following
\begin{verbatim}
> x := '2 + 3';
\end{verbatim}

\begin{verbatim}
x := 5
\end{verbatim}

which assigns the value 5 to the name $x$ though the expression is enclosed in right single quotes. First ’2+3’ simplifies to ’5’. The evaluator removes the quotes, returning 5.

The result of evaluating an expression with two levels of quotes is an expression of type `uneval`. This expression has only one operand, namely the expression inside the outermost pair of quotes.

\begin{verbatim}
> op(''x - 2'');
\end{verbatim}

\begin{verbatim}
x - 2
\end{verbatim}
Chapter 3: Maple Expressions and Statements

> whattype(''x - 2'');

uneval

To reset the value of a name, assign the unevaluated name (its initial value) to the name.\textsuperscript{34}
> x := 'x';

\[ x := x \]

Now the value of \( x \) is reset to \( x \).
Another special case of unevaluation arises in the function call

\[ 'f'(sequence) \]

Suppose the arguments evaluate to the sequence \( a \). Since the result of evaluating \( 'f' \) is not a procedure, Maple returns the unevaluated function call \( f(a) \).
> ''sin''(Pi);

\[ 'sin'(\pi) \]

> %;

\[ \sin(\pi) \]

> %;

\[ 0 \]

You will find this facility useful when writing procedures that implement simplification rules—refer to the section 1.4 of the Maple Advanced Programming Guide.\textsuperscript{35}

\textsuperscript{34}For more information about unassigning names, see page 111.
\textsuperscript{35}For more information and examples regarding unevaluated expressions in Maple, refer to ?uneval.
Substituting Subexpressions

To combine the acts of substitution and evaluation, use the two-parameter version of the eval command.

The eval command has the following syntax, where $s$ is an equation, list, or set of equations.

\[ \text{eval( expr, } s \text{ )}; \]

\[ > \text{expr := } x^3 + 3x + 1; \]

\[ \text{expr := } x^3 + 3x + 1 \]

\[ > \text{eval( expr, } x=y \text{ )}; \]

\[ y^3 + 3y + 1 \]

\[ > \text{eval( expr, } x=2 \text{ )}; \]

\[ 15 \]

\[ > \text{eval( sin(x) + x}^2, x=0 \text{ );} \]

\[ 0 \]

The subs command performs syntactic substitution. It replaces subexpressions in an expression with a new value; the subexpressions must be operands in the sense of the op command.

The subs command has the following syntax where $s$ is an equation, list, or set of equations.

\[ \text{subs( } s, \text{ expr )}; \]

The subs command traverses the expression expr and compares each operand in expr with the left-hand side(s) of the equation(s) $s$. If an operand is equal to a left-hand side of an equation in $s$, subs replaces the operand with the right-hand side of the equation. If $s$ is a list or set of equations, Maple makes the substitutions indicated by the equations simultaneously.

\[ > f := x*y^2; \]
The general syntax of the \texttt{subs} command is
\begin{verbatim}
subs( s1, s2, ..., sn, expr );
\end{verbatim}

where \( s1, s2, \ldots, sn \) are equations, or sets or lists of equations, \( n > 0 \), and \( \texttt{expr} \) is an expression. This is equivalent to the following sequence of substitutions.
\begin{verbatim}
subs( sn, ..., subs( s2, subs( s1, expr ) ) ) ;
\end{verbatim}

Thus, \texttt{subs} substitutes according to the given equations from left to right. Notice the difference between the previous example and the following one.
\begin{verbatim}
> subs( y=z, x=y, z=w, f );
\end{verbatim}
\( y w^2 \)

The difference between the \texttt{eval} and \texttt{subs} commands is demonstrated in the following example.
\begin{verbatim}
> subs( x=0, cos(x) + x^2 );
\end{verbatim}
\( \cos(0) \)
\begin{verbatim}
> eval( cos(x) + x^2, x=0 );
\end{verbatim}
1

In the preceding \texttt{subs} command, Maple substitutes 0 (zero) for \( x \) and simplifies the result. Maple simplifies \( \cos(0) + 0^2 \) to \( \cos(0) \). By using the \texttt{eval} command, Maple evaluates \( \cos(0) \) to 1 (one).

Substitution compares only operands in the expression tree of \( \texttt{expr} \) with the left-hand side of an equation.
\[ \text{eval( } a\ast b\ast c, a\ast b=d \text{ );} \]

\[
\begin{array}{c}
a 
\end{array}
\begin{array}{c}
b 
\end{array}
\begin{array}{c}
c 
\end{array}
\]

The substitution does not result in \( d\ast c \) because the operands of the product \( a\ast b\ast c \) are \( a, b, c \). That is, the products \( a\ast b, b\ast c, \) and \( a\ast c \) do not appear explicitly as operands in the expression \( a\ast b\ast c \). The easiest way to make such substitutions is to solve the equation for one unknown and substitute for that unknown.

\[ \text{eval( } a\ast b\ast c, a=d/b \text{ );} \]

\[
\begin{array}{c}
d 
\end{array}
\begin{array}{c}
c 
\end{array}
\]

You cannot always do this, and you may find that it does not always produce the results you expect. The \texttt{algsubs} routine provides a more powerful substitution facility.

\[ \text{algsubs( } a\ast b=d, a\ast b\ast c \text{ );} \]

\[
\begin{array}{c}
d 
\end{array}
\begin{array}{c}
c 
\end{array}
\]

**Displaying Large Expressions: Labels**

A label in Maple has the following form.

\[
\%\text{natural}
\]

That is, it consists of the unary operator \% followed by a natural integer.\(^{36}\)

A label is only valid after Maple introduces it. The purpose of labels is to name (label) common subexpressions to decrease the size of the printed output, increasing comprehensibility. After the \texttt{prettyprinter} displays it, you can use a label like an assigned name.

\[ \text{solve( } \{x^3 - y^3 = 2, x^2 + y^2 = 1\}, \{x, y\}); \]

\(^{36}\)The percent sign performs two roles in the Maple program: a label indicator and the ditto operator, which represents the result of the last, second last, and third last result. For more information on the ditto operator, see page 25.
\{ y = \%1, x = -\frac{1}{3} \%1 (-4 \%1^3 - 3 - \%1^2 + 6 \%1 + 2 \%1^4) \}\n
\%1 := \text{RootOf}(3 \_Z^2 + 3 - 3 \_Z^4 + 2 \_Z^6 + 4 \_Z^3)\n
After Maple executes this statement, the label \%1 is an assigned name with a value of the preceding \text{RootOf} expression.

\[
> \%1;
\]

\text{RootOf}(3 \_Z^2 + 3 - 3 \_Z^4 + 2 \_Z^6 + 4 \_Z^3)\n
Two facilities are available for use with labels. The command

\texttt{interface( labelwidth=n );}\n
specifies that Maple should not display expressions less than (approximately) \( n \) characters wide as labels. The default value is 20 characters. The command

\texttt{interface( labeling=truefalse );}\n
enables the use of \%1, \%2, ... labels for output subexpressions. You can turn off this facility by using the following command.

\[
> \text{interface(labeling=false)};
\]

**Structured Types**

A simple type check may not provide sufficient information. For example, the command

\[
> \text{type( x^2, '\^' );}
\]

\texttt{true}\n
verifies that \( x^2 \) is an exponentiation, but it does not indicate whether the exponent is, say, an integer. To do so, you must use \textit{structured} types. Consider the following example.

\[
> \text{type( x^2, name\^integer );}
\]

\texttt{true}
Because \( x \) is a name and 2 is an integer, the command returns `true`. To learn more about structured types, study the following examples. The square root of \( x \) does not have the structured type `name^integer`.

\[
> \text{type}( x^{(1/2)}, \text{name}^\text{integer} );
\]

`false`

The expression \((x+1)^2\) does not have type `name^integer`, because \(x+1\) is not a name.

\[
> \text{type}( (x+1)^2, \text{name}^\text{integer} );
\]

`false`

The type `anything` matches any expression.

\[
> \text{type}( (x+1)^2, \text{anything}^\text{integer} );
\]

`true`

An expression matches a set of types if the expression matches one of the types in the set.

\[
> \text{type}( 1, \{\text{integer}, \text{name}\} );
\]

`true`

\[
> \text{type}( x, \{\text{integer}, \text{name}\} );
\]

`true`

The type `set(type)` matches a set of elements of type `type`.

\[
> \text{type}( \{1,2,3,4\}, \text{set(\text{integer})} );
\]

`true`

\[
> \text{type}( \{x,2,3,y\}, \text{set(\{\text{integer, name}\})} );
\]

`true`
Similarly, the type \texttt{list(type)} matches a \textit{list} of elements of type \texttt{type}.

\begin{verbatim}
> type([2..3, 5..7], list(range));

true
\end{verbatim}

Note that $e^2$ is not of type \texttt{anything^2}.

\begin{verbatim}
> exp(2);

e^2

> type( %, anything^2 );

false
\end{verbatim}

Because $e^2$ is the pretty-printed (output formatted) version of \texttt{exp(2)}, it does not match the type \texttt{anything^2}.

\begin{verbatim}
> type( exp(2), 'exp'(integer) );

true
\end{verbatim}

The next example illustrates the need to use right single quotes (') to delay evaluation when including Maple commands in \texttt{type} expressions.\footnote{For more information on delayed evaluation, see page 100.}

\begin{verbatim}
> type( int(f(x), x), int(anything, anything) );

Error, testing against an invalid type
\end{verbatim}

An error is returned because Maple evaluates \texttt{int(anything, anything)}.

\begin{verbatim}
> int(anything, anything);

\frac{1}{2} \textit{anything}^2
\end{verbatim}

This is not a valid type. If you enclose the \texttt{int} command in right single quotes, the type checking works as intended.
3.3 Using Expressions

> type( int(f(x), x), 'int'(anything, anything) );

true

The type specfunc(type, f) matches the function f with zero or more arguments of type type.

> type( exp(x), specfunc(name, exp) );

true

> type( f(), specfunc(name, f) );

true

The type function(type) matches any function with zero or more arguments of type type.

> type( f(1,2,3), function(integer) );

true

> type( f(1,x,Pi), function( {integer, name} ) );

true

In addition to testing the type of arguments, you can test the number of arguments. The type anyfunc(t1, ..., tn) matches any function with n arguments of the listed types in the correct order.

> type( f(1,x), anyfunc(integer, name) );

true

> type( f(x,1), anyfunc(integer, name) );

false

> type( f(x), anyfunc(integer, name) );

false
Another useful variation is to use the And, Or, and Not type constructors to create Boolean combinations of types. Note that these are different from the logical operators and, or, and not discussed on page 85.\(^{38}\)
\[
> \text{type(Pi, 'And( constant, numeric)' );}
\]

\[
false
\]

Pi is of type symbol, not of type numeric.
\[
> \text{type(Pi, 'And( constant, Not(numeric))');}
\]

\[
true
\]

### 3.4 Statements

The statement is an important component in the Maple language. There are many types of statements in Maple.

- assignment statement
- selection statements
- repetition statements
- read and save statements
- break and next statements
- error and return statements
- use statement
- quit statement
- expressions

Expressions have been discussed at length earlier in this chapter. The remainder of this chapter discusses the assignment statement in detail, and introduces the other statements listed previously.

---

\(^{38}\)For more information on structured types, refer to ?type,structured. For more information on how to define your own types, refer to ?type,definition.
The Assignment Statement

An assignment statement can take one of the following forms, where \texttt{expr} is any expression.

\begin{center}
\begin{tabular}{l}
name := expr; \\
(name1, ..., namen) := (expr1, ..., exprn); \\
\end{tabular}
\end{center}

Once an assignment statement is entered, the left-hand side of the assignment operator is evaluated to a name. Then, the right-hand side is evaluated to an expression. Finally, the assignment is performed.\textsuperscript{39}

\begin{verbatim}
> a := 0;

a := 0
\end{verbatim}

You can perform multiple assignments in one statement. If the left-hand side is a sequence of names enclosed in parentheses, the right-hand side must evaluate to a sequence of expressions (with the same number of components) enclosed in parentheses. The assignments are then performed, matching each name with the corresponding expression.

\begin{verbatim}
> (a, b) := (1,2);

a, b := 1, 2
\end{verbatim}

\begin{verbatim}
> a; b;

1

2
\end{verbatim}

A common error is to confuse an assignment statement with an equation. An assignment statement uses the assignment operator (:=) to assign an evaluated expression to a name. An equation uses an equal sign (=) to represent the equality of two operands, the right-hand side and the left-hand side.

Consider the following examples. You can use equations as arguments to the \texttt{solve} command. The first statement assigns the equation on the right-hand side of the assignment statement to the name \texttt{eq1}. The second statement solves the equation.

\textsuperscript{39}You can also use the \texttt{assign} command to make an assignment. For more information, refer to \texttt{?assign}.
Chapter 3: Maple Expressions and Statements

> eq1 := y = 3*x - 4;

\[
eq 1 := y = 3x - 4
\]

> solve(eq1, x);

\[
\frac{1}{3}y + \frac{4}{3}
\]

Testing for Assignment You can test whether a name has an assigned value by using the `assigned` command.

```maple
assigned( name );
```

The `assigned` command returns `true` if `name` has a value different from its name. Otherwise, `false` is returned.

> assigned(a);

```
false
```

To return an expression sequence of names that are currently assigned values different from their names, use the `anames` command. It has the following syntax.

```maple
anames();
```

> a := 0: b:=1:
> anames();

```
b, a
```

You can also use `anames` to return an expression sequence of user-assigned names, names assigned values of certain types, or the current environment variables in the session. For more information, refer to ?anames.

Unassigning a Name There are many ways to unassign a name (reset its value to its name).

If a name is unassigned, it acts as an `unknown`. After a name is assigned a value, it acts as a `variable`. However, it is often desirable to unassign an assigned name, so that you can use the name as an unknown again.
3.4 Statements

One method of unassigning a name is to assign the unevaluated name (its initial value) to the name. To do this, enclose the name in right single quotes to delay evaluation.

```plaintext
> a := 4;

a := 4

> a;

4

> a := 'a';

a := a

> a;

a
```

You can also unassign names by using the `unassign` command.

```
unassign( name1, name2, ... );
```

The arguments to `unassign` must be *unevaluated* names.

```plaintext
> i := 1;

i := 1

> unassign(i);

Error, (in unassign) cannot unassign '1' (argument must be assignable)

> unassign('i');

The value returned by `unassign` is `NULL`. Another way you can unassign a name is by using the `evaln` command.
The `evaln` command evaluates \texttt{name} to a \texttt{name} (as opposed to evaluating \texttt{name} to its \texttt{value} as in other calls). Therefore, you can unassign a name by using `evaln` in the following manner.

\begin{verbatim}
> a := 4;
  a := 4
> a := evaln(a);
  a := a
> a;
  a
\end{verbatim}

**Selection Statements**

Selection is an important operation in programming. To perform selection, use the \texttt{conditional} or \texttt{if} statement. It has the following syntax.

\begin{verbatim}
if Boolean expression then
  statement sequence
elif Boolean expression then
  statement sequence
else
  statement sequence
end if
\end{verbatim}

The construct \texttt{elif conditional expression then statement sequence} can be repeated any number of times.

\begin{verbatim}
> a := sin(1.8): b := ln(1.8):
> if a > b then
>   print( "a is larger" );
> else
>   print( "a is not larger" );
> end if;

"a is larger"
\end{verbatim}
3.4 Statements

For more information about selection and the if statement, see 5.1 Selection and Conditional Execution.

Repetition Statements
Looping constructs, such as the for loop and the while loop, are used to repeat similar actions a number of times.

Example 1  Without using looping constructs, the following statements are necessary to calculate the sum of the first four natural numbers.

> total := 0;

\[
\text{total} := 0
\]

> total := total + 1;

\[
\text{total} := 1
\]

> total := total + 2;

\[
\text{total} := 3
\]

> total := total + 3;

\[
\text{total} := 6
\]

> total := total + 4;

\[
\text{total} := 10
\]

Example 2  The same calculation is simpler if you use the for loop.

> total := 0;

\[
\text{total} := 0
\]

> for i from 1 to 4 do
>     total := total + i;
> end do;
Chapter 3: Maple Expressions and Statements

\[
\text{total} := 1
\]
\[
\text{total} := 3
\]
\[
\text{total} := 6
\]
\[
\text{total} := 10
\]

The initial value of \(i\) is 1. After each execution of the assignment statement (the body of the loop), the value of \(i\) is increased by 1. The value of \(i\) is compared to 4 before the body is re-executed. If \(i\) is less than or equal to 4, Maple executes the body of the loop again. After the execution of the loop finishes, the value of \text{total} is 10 and the value of \(i\) is 5.

> total;

10

**Example 3** Alternatively, you can calculate the sum of the first four natural numbers by using the \textbf{while} loop.

> total := 0; i := 0;

\[
\text{total} := 0
\]
\[
i := 0
\]

> while i <= 4 do
>    total := total + i;
>    i := i + 1;
> end do;
Before each cycle through the while loop, Maple checks whether \( i \) is greater than 4. If it is not, then Maple executes the body of the loop. When the execution of the loop finishes, the value of \( \text{total} \) is 10 and the value of \( i \) is 5.

**Note:** In the for loop, \( i \) is incremented in the for \( i \) from 1 to 4 do part of the statement. In the while loop, \( i \) is incremented by the \( i := i+1 \) statement in the loop body.

The while and the for loop are both special cases of a more general repetition statement. It has the following syntax.

\[
\text{for name from expr by expr to expr while expr do}
\text{statement sequence}
\text{end do;}
\]

For more information about repetition using loops, see 5.2 Repetition.

**The read and save Statements**

You can interact with Maple either directly by entering a command using the keyboard, or indirectly by accessing information from a file. The read and save statements read and save Maple data and procedures to and from files. For more information about these statements, see chapter 7.
The break and next Statements
The break and next statements are used for controlling how a repetition statement is executed.

The break statement causes Maple to exit from the repetition statement. Execution then proceeds with the first statement that follows the repetition statement.

```maple
> L := [9,8,7,6]:

> for i in L do
>   if i=8 then
>     break
>   end if;
>   print(i);
> end do;

9
```

The next statement causes Maple to immediately proceed to the next iteration in the repetition statement.

```maple
> for i in L do
>   if i=8 then
>     next
>   end if;
>   print(i);
> end do;

9

7

6
```

An error occurs if Maple evaluates break or next in a context outside a repetition statement. For more information about the break or next statement, see page 179.

The error and return Statements
The error and return statements control flow in procedures. The error statement raises an exception and interrupts the execution of the current statement in a procedure. For more information about the error statement, see page 219. The return statement causes an immediate return to the point where the current procedure was invoked. For more information about the return statement, see page 217.
The **use** **Statement**

The `use` statement specifies local bindings of names, module exports, and operator overloading. It has the following syntax:

```maple
use exprseq in stateseq end use;
```

where `stateseq` is a sequence of statements and `exprseq` is a sequence of expressions. The expressions can be any of the following.

- equation of the form `name = expression`
- module member selection `m:-e`, which is equivalent to the equation `e=m:-e`
- module expression `m`, which is equivalent to the equations `e=m:-e` for all exports `e` of `m`.

```maple
> use StringTools in
>   a:=Random(10);
>   Encode(a, 'encoding' = 'base64');
> end use;

a := "Φd8p!<vΥ"

"/9BkOHAhPHaEzw=="
```

The **quit** **Statement**

The `quit` statement terminates the current Command-line Maple session. It has the following syntax.

```maple
quit;
```

The keywords `done` and `stop` are equivalent to `quit`.

You can also use `quit` as a procedure to return a specific integer value to the operating system.

```maple
> 'quit'(0);
```

### 3.5 Troubleshooting

This section provides you with a list of common mistakes, examples, and hints that will help you understand and avoid common errors. Use this
section to study the errors that you may encounter when entering the examples from this chapter in a Maple session.

**Syntax Errors**
If input is not syntactically correct, Maple reports a syntax error.

```maple
> 2+-3;
```

```maple
on line 22, syntax error, ‘-‘ unexpected:
2+-3;
```

If the error occurs in a Maple expression and you cannot determine the cause, use the `parse` command to indicate the location of the error. For more information, see page 44 or refer to `?parse`.

```maple
> parse("2+-3");
```

`Error, incorrect syntax in parse: ‘-‘ unexpected (4)`

If Maple returns an error after you enter a Maple command, refer to the corresponding online help page, by using `?topic`, to determine the correct calling sequence.

```maple
> plot(x^2);
```

Plotting error, empty plot

```maple
> ?plot
```

**Reserved Word Unexpected**
An error occurs in Maple if you try to improperly use a reserved word.\(^{40}\)

```maple
> mod(4, 2);
```

```maple
on line 61, syntax error, reserved word ‘mod‘ unexpected:
mod(4, 2);
```

\(^{40}\)For more information about reserved words, see page 24 or refer to `?keyword`.\)
To avoid this error, use left single quotes when you must use reserved words as a name, for example, in function calls.

```maple
> 'mod'(4,2);
0
```

**Break or Next Not in Loop**

An error occurs if Maple evaluates `break` or `next` in a context different from a repetition statement.\(^{41}\)

```maple
> break;
Error, break or next not in loop
```

### 3.6 Exercises

1. Find the numerator and denominator of the irreducible form of \(\frac{4057114691}{4404825097799}\).

2. Construct floating-point numbers using the floating-point number constructor. Construct the number \(917.3366\) using a positive exponent, and then using a negative exponent. Construct a floating-point approximation of \(1/3\).

3. Without using the `Digits` environmental variable, find the difference between \(\pi\) estimated to 20 digits and 10 digits.

4. Calculate the negative complex root of \(-1369\), and then sum 3 and the root. Find the inverse of this complex sum. Find the inverse of \((a*b)/c+((a-d)/(b*e))*I\) in standard form, where \(a\), \(b\), \(c\), \(d\), and \(e\) are real.

5. Using the `time` command, determine the time required to multiply two ten-by-ten matrices.

6. Compute \(3^{3^{98}}\) modulo 7.

\(^{41}\)For more information about `break` and `next`, see page 179.
7. Use Maple to verify de Morgan’s laws.

8. Contrast the behavior of functions and expressions by performing the following commands.
   a) Define a function \( f \) equal to \( x^3 \). Define an expression \( g \) equal to \( x^3 \).
   b) Evaluate \( f \) and \( g \) at 2.
   c) Evaluate \( f \) and \( g \) at \( y \).
   d) Assign the value 2 to \( x \). Evaluate \( f \) and \( g \).

9. Swap the values of two variables using one statement.

10. Sum the smallest 100 prime integers.
    
    *Hint*: Use the `ithprime` or `nextprime` function.

### 3.7 Conclusion

This chapter discussed the syntax and semantics of Maple expressions and statements. There are many expression types in Maple, and by using expression trees you can understand the types and the operands in an expression. Expressions are commonly used in Maple statements. Maple has many types of statements in its language, including assignments, conditional and looping statements, and statements to read from and save to files.

With the knowledge of how to form valid Maple statements, you can proceed to the next chapter which discusses Maple data structures.
4 Basic Data Structures

Maple supports a variety of data structures such as tables, arrays (a special case of tables), Arrays, stacks, and queues. Two basic data structures commonly used in Maple are sets and lists.

In This Chapter

- Defining and manipulating sets, lists, tables, arrays, and Arrays
- Selecting elements from basic data structures
- Converting between data structures
- Stacks and queues, two additional data structures that are implemented using tables

4.1 Sets

A set is an unordered sequence of unique expressions enclosed in braces (\{\}). Recall that a sequence is a group of expressions separated by commas.

\{ sequence \}

Maple removes duplicate members and reorders them in a manner convenient for internal storage. The sequence can be empty, so \{\} represents an empty set.\(^1\)

\[ > \{x, y, y\}; \]

\(^1\)Compare the results of these examples to those in 4.2 Lists.
Manipulating a Set
There are a variety of commands that perform operations on sets.

Set Arithmetic   The most commonly used built-in set arithmetic operators are \texttt{union}, \texttt{minus}, and \texttt{intersect}. These operators perform set union, set difference, and set intersection, respectively.

\begin{verbatim}
> s := \{x, y, z\};

s := \{x, y, z\}

> t := \{w, y, z\};

t := \{y, z, w\}

> s union t;

\{x, y, z, w\}

> s minus t;

\{x\}

> s intersect t;

\{y, z\}
\end{verbatim}
**Testing Set Equality** You can force the evaluation of expressions that involve relational operators by using `evalb`. To perform equality testing with sets, use the `evalb` command in conjunction with the relational operator `=`.

```latex
> evalb(s = t);
false
```

**Testing Set Membership** To test for set membership, use the `member` command or the `in` operator.

```latex
> member(x, s);
true

> evalb(x in s);
true

> member(w, s);
false
```

**Applying a Function to the Members of a Set** To apply a function to the members of a set, use the `map` command. The simplest form of the `map` command syntax, where `f` is a function or procedure and `S` is a set, is:

```latex
map(f, S);
```

In this form, the `map` command replaces each member `Si` of `S` with `f(Si).

```latex
> map(f, {a,b,c});
{f(a), f(b), f(c)}

> S := {1,-2,3,-4};
S := \{-4, -2, 1, 3\}
```
Chapter 4: Basic Data Structures

> map(abs,S);

\{1, 2, 3, 4\}

> map(x->x^2, S);

\{1, 4, 9, 16\}

You can also apply a function to the members of a set by using the \texttt{in} command.

> for elem in S do
> elem^3;
> end do;

\[-64\]
\[-8\]
\[1\]
\[27\]

4.2 Lists

A list is an \textit{ordered} sequence of expressions enclosed in square brackets \([\textit{[]}\]).

\begin{verbatim}
[ sequence ]
\end{verbatim}

The ordering of the list is the same as the \texttt{sequence} ordering (specified by the user). Also, unlike sets, duplicate entries are retained in the list. In the case where \texttt{sequence} is empty, \[\texttt{[]}\] represents an empty list.\(^2\)

> [x, y, y];

\([x, y, y]\)

\(^2\)Compare the results of these examples to those in 4.1 Sets.
4.2 Lists

\[ [a, 1, b, 2]; \]
\[ [a, 1, b, 2] \]

\[ [y[1], x, x[1], y[1]]; \]
\[ [y_1, x, x_1, y_1] \]

The elements of a list can be any expression, even other lists.

\[ L := [[1], [2, a], [X, Y, Z]]; \]
\[ L \]

Maple gives nested lists whose inner lists have the same number of elements a special name—listlist.\(^3\)

\[ M := [[a, b], [1, 2], [3, 4]]; \]
\[ M \]

\[ \text{type}(M, \text{list}); \]
\[ \text{true} \]

\[ \text{type}(L, \text{listlist}); \]
\[ \text{false} \]

\[ \text{type}(M, \text{listlist}); \]
\[ \text{true} \]

The following command creates a list of sets.

\[ [\text{seq}(\text{seq}(i^j, j=1..3), i=-2..2)]; \]
\[ \{\{-8, -2, 4\}, \{-1, 1\}, \{0\}, \{1\}, \{2, 4, 8\}\} \]

\(^3\)For more information about nested lists, refer to ?listlist.
By changing the braces and brackets, you can create a set of lists, list of lists, or set of sets.

> \{seq( [ seq( i^j, j=1..3 ) ], i=-2..2 ) \};

\{[ -2, 4, -8 ], [ -1, 1, -1 ], [ 0, 0, 0 ], [ 1, 1, 1 ], [ 2, 4, 8 ] \}

**Manipulating a List**

There are a variety of commands that perform operations on lists.

**Testing List Membership**

To test whether an element is in a list, use the `member` command.

> L := [1,2,3];

\[ L := [1, 2, 3] \]

> member(2, L);

\textit{true}

> member(5, L);

\textit{false}

You can also determine the position of an element in a list. If `member` returns \textit{true} and a third argument, for example \textquote{p}, is included in the calling sequence, the element's position in the list is assigned to \textit{p}.

> member(2, L, \textquote{p});

\textit{true}

> p;

\textit{2}

**Concatenating Lists**

Use the \texttt{op} command to extract the expression sequence of elements from the individual lists. Concatenate two lists by creating a new list from the expression sequences.
4.2 Lists

> L1 := [a, b, c];

\[ L1 := [a, b, c] \]

> L2 := [y, z];

\[ L2 := [y, z] \]

> L3 := [op(L1), op(L2)];

\[ L3 := [a, b, c, y, z] \]

You can also concatenate parts of lists with another list by using the \texttt{op} command to select particular elements from a list.\(^4\)

> L4 := [op(2, L1), op(L2)];

\[ L4 := [b, y, z] \]

\textbf{Inserting Elements into a List} To prepend and append elements to a list, use the previous concatenation method.

> L1 := [grape];

\[ L1 := [grape] \]

> [banana, op(L1)]; \# prepend element

\[ [banana, grape] \]

> [op(L1), orange]; \# append element

\[ [grape, orange] \]

You can also insert elements in a list by using the \texttt{nops} command.\(^5\)

\(^4\)For more information about selection using the \texttt{op} command, see page 155.

\(^5\)The \texttt{nops} command determines the number of operands in an expression. For more information about \texttt{nops}, refer to \texttt{?nops}.
Replacing Elements in a List  To replace an element in a list, use a selection operation in an assignment statement.\(^6\)

\[
> L1 := [\text{banana, grape, orange}];
\]

\[
L1 := [\text{banana, grape, orange}]
\]

\[
\]

\[
L1 := [\text{banana, grape, orange}]
\]

\[
\]

\[
L1 := [\text{kiwi}]
\]

\[
> L1;
\]

\[
[\text{banana, kiwi, orange}]
\]

To replace all occurrences of an expression in a list, use the eval command.

\[
> L1 := [\text{banana, grape, orange, grape}];
\]

\[
L1 := [\text{banana, grape, orange, grape}]
\]

\[
> \text{eval(L1, grape=kiwi)};
\]

\[
[\text{banana, kiwi, orange, kiwi}]
\]

\(^6\)For more information about the selection operation, see page 152 or refer to ?selection.
4.2 Lists

Table 4.1: Sorting Options

<table>
<thead>
<tr>
<th>List Element Type</th>
<th>F</th>
<th>Resulting sort Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>numeric</td>
<td>&lt;</td>
<td>ascending</td>
</tr>
<tr>
<td>numeric</td>
<td>&gt;</td>
<td>descending</td>
</tr>
<tr>
<td>string or symbol</td>
<td>lexorder</td>
<td>lexicographic</td>
</tr>
</tbody>
</table>

Reversing the Order of Elements in a List  To reverse the order of the elements in a list, use a combination of the selection operation, and the seq and nops commands.\(^7\)

\[ L1 := \{\text{banana, grape, orange}\}; \]

\[ L1 := \{\text{banana, grape, orange}\} \]

\[ \{ \text{seq( L1[-i], i=1..nops(L1) )} \}; \]

\[ \{\text{orange, grape, banana}\} \]

Sorting a List  To sort the elements in a list, use the sort command. It has the following syntax where \( L \) is a list and \( F \) is an optional parameter that specifies the sort order.

\[ \text{sort(} \ L, \ F \ \text{);} \]

If \( F \) is not specified, the elements of the list \( L \) are sorted in ascending order. Otherwise, Table 4.1 displays the permissible values of \( F \) and the resulting sort order for lists containing elements of a particular type.\(^8\)

\[ \text{sort(}[\text{z,e,b}]); \]

\[ [b, e, z] \]

\[ \text{sort([2.5, 7/3, 10]}); \]

---

\(^7\)For more information on the seq command, see page 183 or refer to ?seq.

\(^8\)The algorithm that Maple uses for sort is a recursive implementation of merge sort with an early detection of sorted sequences. For more information about sorting lists, refer to ?sort.
Applying a Function to the Elements of a List  Two commonly used commands that apply functions to the elements of a list are the map command and the zip command.

You can apply a function to the elements of a list by using the map command. The simplest map command syntax, where \( f \) is a function or procedure and \( L \) is a list, is:

\[
\text{map}(f, L);
\]

In this form, the map command replaces each element \( L_i \) of \( L \) with \( f(L_i) \).

\[
> \text{map}(f, [a, b, c]);
\]

\[
[f(a), f(b), f(c)]
\]

\[
> L := [1, -2, 3, -4];
\]

\[
L := [1, -2, 3, -4]
\]

\[
> \text{map}(\text{abs}, L);
\]

\[
[1, 2, 3, 4]
\]

\[
> \text{map}(x \rightarrow x^2, L);
\]

\[
[1, 4, 9, 16]
\]

You can apply a binary (two parameter) function to the elements of two lists by using the zip command. It has the following syntax where \( f \) is any binary function, and \( L_1 \) and \( L_2 \) are lists. If the optional argument \( d \) is specified, it is used as the default value in \( f \) when \( L_1 \) and \( L_2 \) are not the same size.
zip( f, L1, L2, d );

If d is specified and L1 and L2 are the same size, d is ignored.

> zip((x,y)->x+y, [1,2,3], [4,5,6]);

> zip((x,y)->x+y, [1,2,3], [4,5], 10);

4.3 Tables

The **table** data structure in Maple is used for representing data in a **table**—an arrangement of rows and columns. To create a table explicitly, use the **table** command. The optional parameters F and L specify an indexing function, and a list or set that specifies the initial values of the table, respectively.

table( F, L );

If L is a **list of entries**\(^9\), the indices are set to the natural integers 1, 2, ..., \(^{10}\)

> T := table([a, b, c]);

\[
T := \text{table}([1 = a, 2 = b, 3 = c])
\]

> T0 := table([cos, sin, tan]);

\[
T0 := \text{table}([1 = \cos, 2 = \sin, 3 = \tan])
\]

\(^9\)The use of a **set** of initial values is ambiguous because there is no fixed ordering of set elements. Hence, the ordering of the entries in the result may not correspond to the ordering in which the entries are given.

\(^{10}\)You can also create a table implicitly by using an indexed name. For more information, see page 135 or refer to **table**.
Chapter 4: Basic Data Structures

If $L$ is a list of equations or a set of equations, the left-hand side of each equation is the table index (key), and the right-hand side is the table entry (value). Unlike arrays and Arrays (see 4.4 arrays and Arrays or refer to ?array or ?Array), in which indices must be integers, the indices of a table can be any value.

```plaintext
> T1 := table([0=alpha, 1=beta]);

T1 := table([0 = α, 1 = β])
```

```plaintext
> T2 := table({y1=x, y3=x^2, y2=x^3});

T2 := table([y1 = x, y3 = x^2, y2 = x^3])
```

```plaintext
> T3 := table([(shirt, S) = 12, (shirt, M) = 8, (shirt, L) = 9]);

T3 := table([(shirt, S) = 12, (shirt, L) = 9, (shirt, M) = 8])
```

If $L$ is not specified, an empty table is created.

```plaintext
> table();

table([])
```

The optional indexing function $F$ can be a procedure or a name that describes the semantics of indexing in the table. If $F$ is not specified, then ordinary indexing (integer) is assumed. The built-in indexing functions are: symmetric, antisymmetric, sparse, diagonal, and identity. For more information about indexing functions, refer to ?indexingfunctions.

The purpose of a table is to enable fast access to data. To access the value of a table entry, enter the table name followed by a table key (in square brackets).

```plaintext
> T0[3];

tan
```

---

11For more information about table entry selection, see 4.5 Selecting Elements from a Data Structure or refer to ?selection.
An Alternate Method for Generating a Table

A table can be created implicitly by making an assignment to an indexed name \( T \), where \( T \) does not have a previous value.

\[
T[\text{indexExpr}] = \text{entryValue};
\]

In this format, \( T \) is the name of the table, \( \text{indexExpr} \) is the index of the entry, and \( \text{entryValue} \) is the actual value of the entry in the table.

For example, the following statements have the same effect.

\[
> T4 := \text{table}((\text{Cu},1) = 64));
\]

\[
T4 := \text{table}((\text{Cu},1) = 64))
\]

\[
> T4[\text{Cu},1] := 64;
\]

\[
T4_{\text{Cu},1} := 64
\]

Both statements create a table object with one component.

Table Evaluation Rules

Tables have special evaluation rules. If the name \( T \) is assigned to a table, the result of evaluating \( T \) is the name \( T \), not the value of \( T \) (contents of the table).\(^{12}\)

\[
> T5 := \text{table}([1,2,3,4]);
\]

\[
T5 := \text{table}([1 = 1, 2 = 2, 3 = 3, 4 = 4])
\]

\[
> T5;
\]

\[
T5
\]

\(^{12}\)In Maple, an expression is normally evaluated by using full recursive evaluation—that is, the result of the expression is the fully evaluated expression. For more information about evaluation rules, see page 14 or refer to ?last_name_eval.
To access the value of a table object, apply the eval command.

> eval(T5);

\[ \text{table([1 = 1, 2 = 2, 3 = 3, 4 = 4])} \]

**Manipulating a Table**

There are a variety of commands that perform operations on tables.

**Inserting Entries**  To add new entries to a table, use subscript notation. Thus, \( T := \text{table([4])} \) is equivalent to \( T[1] := 4 \) (implicit creation).

> mytable[1] := apple;

\[ mytable := \text{apple} \]

If there is no entry in the table at the specified key, a new entry is created.


\[ mytable := \text{banana} \]

> eval(mytable);

\[ \text{table([1 = apple, 2 = banana])} \]

If there is already an entry value for the specified key, it is updated to the new value.


\[ mytable := \text{orange} \]

> eval(mytable);

\[ \text{table([1 = apple, 2 = orange])} \]
Example  To assign a value to a table key, use a procedure that accepts an indexed name. Consider the following procedure.

\[
\text{keyvalues} := \text{proc}(b) \rightarrow b^3 \text{ end proc:}
\]

If the first parameter to procedure \text{keyvalues} is valid, the assignment is made.

\[
\text{newtable} := \text{table(): #create an empty table} \\
\text{newtable}[1] := \text{keyvalues}(2): \ #\text{assign the value 8 to newtable[1]} \\
\text{eval(newtable);}
\]

\[
\text{table([1 = 8])}
\]

To automatically assign values to table keys, use a flow control statement.\textsuperscript{13}

\[
\text{newtable} := \text{table();} \\
\text{newtable} := \text{table([])} \\
\text{for i to 5 do newtable}[i] := \text{keyvalues}(i) \text{ end do:} \\
\text{eval(newtable);}
\]

\[
\text{table([1 = 1, 2 = 8, 3 = 27, 5 = 125, 4 = 64])}
\]

To determine whether a table component is assigned, use the \text{assigned} command.

\[
\text{assigned(newtable[1]);}
\]

\[
\text{true}
\]

Removing Entries  To remove an entry from a table, assign an unevaluated table entry to its name by using right single quotes on the right-hand side of an assignment statement, or use the \text{evaln} command. For example, \text{T[1]} := ’T[1]’ removes the first entry from \text{T}. Alternatively, you can use \text{evaln(T[1])} or \text{unassign(’T[1]’)}.

\textsuperscript{13}For more information about flow control statements, see \textbf{5.1 Selection and Conditional Execution}. 
Displaying Table Indices  To return the sequence of the table entries, use the \texttt{indices} command, where \( T \) is a table.\(^{14}\)

\begin{verbatim}
indices(T);
\end{verbatim}

\begin{verbatim}


> indices(T);

[[shirt, M]], [[shirt, L]], [[shirt, S]]
\end{verbatim}

Displaying Table Entries  To return a sequence of the table values, use the \texttt{entries} command, where \( T \) is a table.\(^{15}\)

\begin{verbatim}
entries(T);
\end{verbatim}

\begin{verbatim}
> entries(T);

[16], [36], [42]
\end{verbatim}

Alternatively, you can use the \texttt{op} command to return the operands of a table, for example, its entries and indexing function. (If no indexing function has been specified, \texttt{NULL} is returned.) Table 4.2 lists the operands

\footnotesize\(^{14}\)The order of the keys in the returned sequence does not necessarily correspond to the order of the keys in the table because tables in Maple are implemented by using hash tables. However, there is a one-to-one correspondence between the order of the indices and the order of the entries.

\footnotesize\(^{15}\)The order of the entries in the returned sequence does not necessarily correspond to the order of the entries in the table.
### Table 4.2 Table Operands

<table>
<thead>
<tr>
<th>Operand</th>
<th>op Command</th>
</tr>
</thead>
<tbody>
<tr>
<td>table structure</td>
<td>op(T)</td>
</tr>
<tr>
<td>data type</td>
<td>op(0, eval(T))</td>
</tr>
<tr>
<td>indexing function</td>
<td>op(1, eval(T))</td>
</tr>
<tr>
<td>table entries</td>
<td>op(2, eval(T))</td>
</tr>
</tbody>
</table>

and corresponding op command that you can use to access them. In the op commands shown, T is the name of the table.\(^\text{16}\)

The call op(T) returns the actual table structure.

> op(T4);  

\[
\text{table}([(Cu, 1) = 64])
\]

The object assigned to T4 is a table.

> op(0, eval(T4));  

\[
\text{table}
\]

The table T4 has no indexing function and the list of entries has only one key/value pair.

> op(1, eval(T4));  
> op(2, eval(T4));  

\[
[(Cu, 1) = 64]
\]

**Copying a Table** If two names evaluate to the same table, an assignment to a component of either affects both.

> T6 := table([w, x, y, z]);  

\[
T6 := \text{table}([1 = w, 2 = x, 3 = y, 4 = z])
\]

\(^{16}\)Tables have special evaluation rules. To access the table object, you must apply the eval command. For more information, see page 135.
Chapter 4: Basic Data Structures

\begin{verbatim}
> b := T6:
> b[1] := 7;

b_1 := 7

> eval(T6);

\text{table}(\{1 = 7, 2 = x, 3 = y, 4 = z\})

> eval(b);

\text{table}(\{1 = 7, 2 = x, 3 = y, 4 = z\})

In this example, there is only one table. The name \texttt{b} points to the name \texttt{T6} which points to the table structure.

To create a copy of a table that is independent of the original, use the \texttt{copy} command.

\texttt{copy(T6);};

\begin{verbatim}
> c := copy(T6);

\texttt{c := table(}\{1 = 7, 2 = x, 3 = y, 4 = z\}\texttt{)}

> c[1] := 10:
> eval(T6);

\text{table}(\{1 = 7, 2 = x, 3 = y, 4 = z\})

> eval(c);

\text{table}(\{1 = 10, 2 = x, 3 = y, 4 = z\})
\end{verbatim}

\textbf{Applying a Function to the Entries of a Table}  To apply a function to the entries of a table to create a new table, use the \texttt{map} command. The indices of the table remain the same.
In the following example, the first statement creates a table of polynomials, and the second statement creates a (new) table of their derivatives by using the map command.\(^\text{17}\)

\[\begin{align*}
&> T7 := \text{table}([x, x^2 + 2, x^3 - x + 1, 1/x^2]): \\
&> \text{map}(\text{diff}, T7, x); \\
&\text{table}(\{1 = 1, 2 = 2 x, 3 = 3 x^2 - 1, 4 = -\frac{2}{x^3}\})
\end{align*}\]

To modify the values in-place, that is, change the values in the original table, use the following method.

\[\begin{align*}
&> \text{for entry in op(2,eval(T6)) do} \\
&> \quad T6[lhs(entry)] := 2*rhs(entry) \\
&> \text{end do;}
\end{align*}\]

\[\begin{align*}
T6_1 &:= 14 \\
T6_2 &:= 2 x \\
T6_3 &:= 2 y \\
T6_4 &:= 2 z \\
&> \text{eval(T6);}
\end{align*}\]

\[\text{table}(\{1 = 14, 2 = 2 x, 3 = 2 y, 4 = 2 z\})\]

### 4.4 arrays and Arrays

In Maple, there are two kinds of arrays: arrays and Arrays.

An array is a specialization of the table data structure, with zero or more specified dimensions \(B\), where each dimension is an integer range (index bound). As in the case of tables (see 4.3 Tables or refer to \texttt{?table}), \(F\) is the optional indexing function, and \(L\) is the initial list of values.

\(^{17}\)For more information about the map command, refer to \texttt{?map}.\]
The number of ranges in B is called the *dimension* of the array. If no index bound is specified in B, it is deduced from the initial list of values.

The list of initial values L can be a:

- list of equations (similar to tables)
- list of values (one-dimensional)
- nested list of lists (row-by-row representation)

If L is the empty list, the array is zero-dimensional and the index bound is NULL (a sequence of zero integer ranges). If L is given as a list of n values, the array is one-dimensional and the index bound is the range 1..n. If L is a list of lists, the number of dimensions can be deduced from the level of nesting of the lists and each dimension is indexed from 1.

All parameters to the `array` command are optional and can appear in any order, but at least one bound or one list must appear. If the bounds are not specified, they are deduced from the list of initial values. If the bounds are specified without an initialization list, the array values are unassigned.

```plaintext
> a2 := array(1..3); #empty 1-d array
    a2 := array(1..3, [])

> a3 := array(1..3, [a,b,c]); #1-d array
    a3 := [a, b, c]

> a4 := array([a,b,c]); #1-d array
    a4 := [a, b, c]

> a5 := array(1..3, 1..2); #empty 3 x 2 array
    a5 := array(1..3, 1..2, [])

> a6 := array([[1,2,3],[4,5,6]]); #2-d array
```
4.4 arrays and Arrays

Similarly, an Array is a table-like structure with fixed dimensions and integer indices. The difference between arrays and Arrays is the underlying data structure. An array uses a hash table, while an Array uses a fixed size data block.

The Array command has the same form as array, but accepts more optional parameters. If the bounds are specified without an initialization list, all Array values are assigned 0 (zero). For more information, refer to ?Array.

```
Array( F, B, L );
```

> A2 := Array(1..3); #empty 1-d Array

```
A2 := [0, 0, 0]
```

> A3 := Array(1..3, [a,b,c]); #1-d Array

```
A3 := [a, b, c]
```

> A4 := Array([a,b,c]); #1-d Array

```
A4 := [a, b, c]
```

> A5 := Array(1..3, 1..2); #empty 3 x 2 Array

```
A5 :=
[ 0 0 ]
[ 0 0 ]
[ 0 0 ]
```

> A6 := Array([[1,2,3],[4,5,6]]); #2-d Array

```
A6 :=
[ 1 2 3 ]
[ 4 5 6 ]
```

As in the creation of a table, the optional indexing function F can be a procedure or a name that describes the semantics of indexing in the array or Array. If F is not specified, ordinary indexing (integer) is
implied. The built-in indexing functions are: symmetric, antisymmetric, sparse, diagonal, and identity.\(^{18}\) For more information about indexing functions, refer to ?indexingfunctions.

\[
\begin{bmatrix}
3 & a7_{1,2} \\
a7_{1,2} & a7_{2,2}
\end{bmatrix}
\]

### Evaluation Rules for arrays

There are special evaluation rules for arrays. If the name \(a\) has been assigned to an array, the result of evaluating \(a\) is the name \(a\), not the value of \(a\) (contents of the array). As with a table, to access the value of the array object, apply the eval command.\(^{19}\)

\[
\begin{bmatrix}
1, 2, 3, 4
\end{bmatrix}
\]

In contrast, it is not necessary to use eval for Arrays.

\[
\begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6
\end{bmatrix}
\]

\(^{18}\)There are additional built-in indexing function names for Arrays. For a complete list, refer to ?rtable_indexingfunctions.

\(^{19}\)For more information about evaluation rules, see page 14 or refer to ?last_name_eval.
Manipulating arrays and Arrays

There are a variety of commands that perform operations on arrays and Arrays. You manipulate arrays and Arrays in a manner that is similar to that used for tables.

Inserting Entries  To assign entries in arrays and Arrays, use subscript notation, that is, square brackets.

```plaintext
> a3[3] := d;

a3_3 := d

> eval(a3);

[a, b, d]


A3_3 := d

> A3;

[a, b, d]

> a7 := array(symmetric,1..2,1..2);

a7 := array(symmetric, 1..2, 1..2, [])

> a7[1,1] := 3;

a7_{1,1} := 3

> a7[1,2] := 4;

a7_{1,2} := 4

> eval(a7);

3 4
4 ?_{2,2}
```
Chapter 4: Basic Data Structures

Removing Entries  To remove entries from an array, assign an unevaluated array entry to its name by using right single quotes on the right-hand side of an assignment statement, or use the evaln command. For example, \(a[1] := 'a[1]'\) removes the first entry from \(a\). Similarly, you can use evaln(a[1]).

\[
\text{a3[3] := 'a3[3]'};
\]

\[
a3_3 := a3_3
\]

\[> \text{eval(a3)};
\]

\[
[a, b, ?_3]
\]

Entries cannot be removed from Arrays. Unless you specify storage=sparse in its definition, each entry must have an assigned value.

Displaying array Indices  To return a sequence of the indices to the array, use the indices command, where \(a\) is an array (not an Array).

\[
\text{indices( a )};
\]

The order of the indices in the returned sequence does not necessarily correspond to the order of the indices in the array.

\[> \text{indices(a3)};
\]

\[
[1], [2]
\]

Displaying array Entries  To return a sequence of the array entries, use the entries command, where \(a\) is an array (not an Array).

\[
\text{entries( a )};
\]

The order of the entries in the returned sequence does not necessarily correspond to the order of the entries in the array.

\[> \text{entries(a3)};
\]

\[\text{For a list of Array storage formats, refer to ?Array.}\]
Alternatively, you can use the `op` command to return the operands of an array, that is, its entries, data type, indexing function, and bounds. (If no indexing function has been specified, nothing is returned.) Table 4.3 lists these operands and the corresponding `op` command that you can enter to access them. In the `op` commands, `a` is the name of the array.\(^{21}\)

The object assigned to `a7` is the actual array structure.

\[
[a], [b]
\]

As with tables, the first operand is the indexing function (if any).

\[
> \text{op}(1, \text{eval}(a7));
\]

\textit{symmetric}

The second operand is the sequence of ranges that indicate the bounds.

\(^{21}\text{Recall that arrays (like tables) have special evaluation rules. To access the array object, apply the \texttt{eval} command.}\)
Table 4.4 Array Operands

<table>
<thead>
<tr>
<th>Operand</th>
<th>$op$ Command</th>
</tr>
</thead>
<tbody>
<tr>
<td>all operands</td>
<td>$op(A)$</td>
</tr>
<tr>
<td>data type</td>
<td>$op(0, A)$</td>
</tr>
<tr>
<td>indexing function</td>
<td>$op(1, A)$</td>
</tr>
<tr>
<td>Array bounds</td>
<td>$op(2, A)$</td>
</tr>
<tr>
<td>Array entries</td>
<td>$op(3, A)$</td>
</tr>
<tr>
<td>other operands</td>
<td>$op(4, A)$</td>
</tr>
</tbody>
</table>

$\gg op(2, \text{eval}(a7));$

1..2, 1..2

The third operand is a list of the entries in the array.
$\gg op(3, \text{eval}(a7));$

$[(1, 1) = 3, (1, 2) = 4]$

The example above displays only two entries in array $a7$ because the values of the entries located at $(2, 1)$ and $(2, 2)$ are implicitly specified by the symmetric indexing function.

Similarly, you can use the $op$ command to return the operands of an Array, that is, the data type, indexing function, bounds, entries, entry data type, storage type, and order. Table 4.4 lists the operands and the corresponding $op$ commands.

$\gg op(A6);$

1..2, 1..3, $(2, 3) = 6$, $(2, 1) = 4$, $(2, 2) = 5$, $(1, 1) = 1$, $(1, 3) = 3$, $(1, 2) = 2$, $\text{datatype} = \text{anything}$, $\text{storage} = \text{rectangular}$, $\text{order} = \text{Fortran\_order}$

Copying arrays and Arrays If two names evaluate to the same array or Array, an assignment to a component of either affects both.

$\gg a8 := \text{array}([w, x, y, z]);$

$a8 := [w, x, y, z]$
> b := a8;

\[ b := a8 \]

> b[1] := 7;

\[ b_1 := 7 \]

> eval(a8);

\[ [7, x, y, z] \]

> eval(b);

\[ [7, x, y, z] \]

> A8 := Array([1,2,4,8]);

\[ A8 := [1, 2, 4, 8] \]

> B := A8;

\[ B := [1, 2, 4, 8] \]

> B[1] := 0;

\[ B_1 := 0 \]

> A8;

\[ [0, 2, 4, 8] \]

> B;

\[ [0, 2, 4, 8] \]

In the preceding examples, there is only one array (or Array). The name \( b \) (\( B \)) points to the name \( a8 \) (\( A8 \)) that points to the array (or Array) structure.
To create a copy of an array that is independent of the original, use the `copy` command.

```plaintext
> c := copy(a);

c := [7, x, y, z]

> c[1] := 10;

c[1] := 10

> eval(a);

[7, x, y, z]

> eval(c);

[10, x, y, z]
```

To create a copy of an Array that is independent of the original, use the `Array` command.

```plaintext
> C := Array(A);

C := [0, 2, 4, 8]

> C[1] := Pi;

C[1] := \pi

> A;

[0, 2, 4, 8]

> C;

[\pi, 2, 4, 8]
```
Applying a Function to the Entries of an array or Array  To apply a function to the entries of an array or Array to create a new array or Array, use the \texttt{map} command.

In the following example, the first statement creates an array of polynomials, and the second statement creates an array of the factored polynomials by using the \texttt{map} command.\footnote{For more information about the \texttt{map} command, refer to \texttt{?map}.}

\begin{verbatim}
> a8 := array([2*x^4 - x^2, x^2 + 2*x, x^3 - x + 2*x]);
a8 := [2 x^4 - x^2, x^2 + 2 x, x^3 + x]

> map(factor, a8);
[x^2 (2 x^2 - 1), x (x + 2), x (x^2 + 1)]
\end{verbatim}

The \texttt{Map} command in the \texttt{LinearAlgebra} package applies a function to the entries in an \texttt{Array}, returning a new \texttt{Array}.

\begin{verbatim}
> A8 := Array([x,23,y-2]);
A8 := [x, 23, y - 2]

> LinearAlgebra[Map](x->x*y,A8);
[x y, 23 y, (y - 2) y]
\end{verbatim}

To modify the values in-place, that is, change the values in the original array or Array, use the following method.\footnote{This method (with \texttt{op(2,\ldots)}) is used to modify tables in-place.}

\begin{verbatim}
> for entry in op(3,A8) do
>   A8[lhs(entry)] := rhs(entry) + 1
> end do;

A8_1 := x + 1
A8_2 := 24
A8_3 := y - 1
\end{verbatim}
4.5 Selecting Elements from a Data Structure

Selecting elements from a data structure is most commonly achieved by performing a selection operation. However, there are a number of other methods by which you can choose an element.

The Selection Operation []

The selection operation, [], selects components from aggregate objects, for example, sequences, sets, lists, tables, arrays, and Arrays. The selection operation uses the following syntax, where name represents the object and sequence indicates the selected entry (or entries).

\[
\text{name[ sequence ]}
\]

Sequences, Sets, and Lists

If name evaluates to a sequence, set, or list, and sequence evaluates to an integer, a range, or NULL, Maple performs a selection operation.\(^\text{24}\)

If sequence evaluates to an integer \(i\), then Maple returns the \(i^{th}\) operand of the set, list, or sequence. Negative integers cause elements to be counted from the end of the sequence, set, or list. For example, \(-1\) indicates the last element.

\[
\begin{align*}
> \text{s := x, y, z;} \\
> \text{L := [s, s];} \\
\text{L := [x, y, z, x, y, z]} \\
> \text{s := {s, s};} \\
\text{S := \{x, y, z\}}
\end{align*}
\]

\(^{24}\)For more information about selecting elements, refer to ?selection.

\(^{25}\)Although you can use the selection operation to select elements of a set, the order of elements in a set is session dependent. Do not make any assumptions about this order.
If sequence evaluates to a range, the lower bound must be less than or equal to one more than the upper bound. Specifically, \( A[3..3] \) selects a single element, \( A[3..2] \) produces an empty selection, and \( A[3..1] \) is not permitted.

\[
> L[2..3];
\]

\[ [y, z] \]

\[
> L[3..3];
\]

\[ [z] \]

\[
> L[3..2];
\]

\[ [] \]

\[
> L[3..1];
\]

Error, invalid subscript selector

If sequence evaluates to NULL, Maple returns a sequence containing all operands of the aggregate object.

\[
> S[];
\]

\[ x, y, z \]
Tables, arrays, and Arrays  If name evaluates to a table, Maple returns the value in the table entry, if there is one. If there is not an entry with sequence as its key, the value returned is an indexed name. However, if sequence evaluates to a range, the selection remains unevaluated.

```maple
T := table([(1,1)=Bob, (1,2)=Mary, (2,1)=Sally, (2,2)=Joe]);
T[2,2];

Joe

T[3,1];

T_{3,1}
```

Because arrays are implemented as tables, the array behavior is the same.

```maple
a := array([w,x,y,z]);
a[3];
a[1..3];
```

In this case, Arrays behave like lists. The range selection evaluates to a subArray.

```maple
B := Array([[1,2,3],[4,5,6],[7,8,9]]);
```
4.5 Selecting Elements from a Data Structure

\[ B := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \]

\[ B[1..2,1..2]; \]

\[ \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \]

The \texttt{op} Command

You can use the \texttt{op} command to extract operands from sets, lists, tables, and arrays.\textsuperscript{26} To use \texttt{op} in this manner, use the following syntax, where \( i \) is an integer or integer range that represents the operand(s) to extract and \( e \) is the data structure.\textsuperscript{27}

\[ \text{op}(i, e); \]

Sets and Lists

Notice that if negative integers are used to indicate the operands in a set or list, the elements are counted from the end: using \(-1\) indicates the last element, using \(-2\) indicates the second last, and so on.

\[ S := \{10,11,12,13\}; \]

\[ S := \{10, 11, 12, 13\} \]

\[ \text{op}(1, S); \]

10

\[ \text{op}(2..3, S); \]

11, 12

\textsuperscript{26}In general, the use of \texttt{op} to extract operands from a list or set is less efficient than using the selection operation because the \textit{entire} list or set is evaluated. Therefore, if possible, use the selection operation.

\textsuperscript{27}You cannot use the \texttt{op} command on a sequence because Maple interprets the elements of the sequence as individual arguments to \texttt{op}. Instead, you must access elements of a sequence by using the selection operation \[\]. For more information on sequence selection, see page 152 or refer to \texttt{?sequence}. 
Chapter 4: Basic Data Structures

L := [a, b, c, d, e];

L := [a, b, c, d, e]

> op(3..-1, L);

c, d, e

Tables, arrays, and Arrays

Unlike sets, lists, and Arrays, tables and arrays have special evaluation rules. For more information on using \texttt{op} with tables, see 4.3 Tables. For more information on using \texttt{op} with arrays and Arrays, see 4.4 arrays and Arrays.

The select, remove, and selectremove Commands

You can also use the \texttt{select}, \texttt{remove}, and \texttt{selectremove} commands to select the elements of a list or set that satisfy a criterion.

The simplest forms are:

\begin{verbatim}
select( f, x )
remove( f, x )
selectremove( f, x )
\end{verbatim}

where \( f \) is a Boolean-valued function and \( x \) is an expression which must be a sum, product, list, set, function, or indexed name.

The \texttt{select} command returns a new object of the same type as \( x \) that contains only the operands of \( x \) for which the Boolean-valued function \( f \) returns \texttt{true}.

\begin{verbatim}
> X := [seq(i,i=1..10)];

X := [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

> select(isprime,X);

[2, 3, 5, 7]
\end{verbatim}

The \texttt{remove} command is (almost) the complement of \texttt{select}. It returns a new object of the same type as \( x \) that contains only the operands of \( x \) for which the Boolean-valued function \( f \) returns \texttt{false}. 

\begin{verbatim}
> remove(isprime,X);

[1, 4, 6, 8, 9, 10]
\end{verbatim}
4.5 Selecting Elements from a Data Structure  •  157

\[ > \text{remove(isprime,X)}; \]
\[ \{1, 4, 6, 8, 9, 10\} \]

The selectremove function returns two new objects of the same type as \( x \), the first containing the operands for which the Boolean-valued function \( f \) returns \texttt{true} and the second containing the operands for which \( f \) returns \texttt{false}. The result is computed more efficiently than using both \texttt{select} and \texttt{remove} because it evaluates \( f(x_i) \) once for each operand.

\[ > \text{selectremove(isprime,X)}; \]
\[ \{2, 3, 5, 7\}, \{1, 4, 6, 8, 9, 10\} \]

**Note:** If \( f(x_i) \) returns \texttt{FAIL}, none of \texttt{select}, \texttt{remove}, and \texttt{selectremove} returns an object containing \( x_i \).

The general forms of these commands are

<table>
<thead>
<tr>
<th>Function</th>
<th>Syntax</th>
</tr>
</thead>
<tbody>
<tr>
<td>\texttt{select}</td>
<td>\texttt{select(f, x, y1, \ldots, yn)}</td>
</tr>
<tr>
<td>\texttt{remove}</td>
<td>\texttt{remove(f, x, y1, \ldots, yn)}</td>
</tr>
<tr>
<td>\texttt{selectremove}</td>
<td>\texttt{selectremove(f, x, y1, \ldots, yn)}</td>
</tr>
</tbody>
</table>

where \( f \) is a function, \( x \) is a sum, product, list, set, function or indexed name, and \( y1, \ldots, yn \) are expressions. The expressions \( y1, \ldots, yn \) are passed to the function \( f \). That is, for each operand of \( x \), the Boolean evaluation \( f(x_i, y1, \ldots, yn) \) is calculated.

\[ > X := \{2, \sin(1), \exp(2x), x^{1/2}\}; \]
\[ X := \{2, \sin(1), e^{(2x)}, \sqrt{x}\} \]

\[ > \text{select(type, X, 'function');} \]
\[ \{\sin(1), e^{(2x)}\} \]

\[ > \text{remove(type, X, 'constant');} \]
\[ \{e^{(2x)}, \sqrt{x}\} \]
Chapter 4: Basic Data Structures

\[ \text{integers} := [10..15] \]

\[ \text{integers} := [10, 11, 12, 13, 14, 15] \]

\[ \text{select(isprime, integers)}; \]

\[ [11, 13] \]

\[ \text{remove(isprime, integers)}; \]

\[ [10, 12, 14, 15] \]

\[ \text{selectremove(isprime, integers)}; \]

\[ [11, 13], [10, 12, 14, 15] \]

4.6 Converting Between Data Structures

You can convert the structure of the result by using a variety of methods. This is useful because some results are in the form of a data structure that is not accepted by other Maple commands.

Converting a Sequence to a Set or a List

Many Maple commands return results in the form of a sequence. To convert a sequence into a set or list, enclose the result in braces or square brackets, respectively.

For example, the `solve` command returns a sequence of values if it finds multiple solutions.

\[ s := \text{solve}(x^4 - 2x^3 - x^2 + 4x - 2, x); \]

\[ s := 1, 1, \sqrt{2}, -\sqrt{2} \]

The result is a sequence of values, not an equation. Enclosing the solutions in square brackets creates a list.

\[ [s]; \]

\[ [1, 1, \sqrt{2}, -\sqrt{2}] \]
\[ [1, 1, \sqrt{2}, -\sqrt{2}] \]

Alternatively, you can convert the result to a set by enclosing \( s \) in braces. Note that duplicate solutions do not appear in the set.

\[
> \{s\};
\]

\[
\{1, \sqrt{2}, -\sqrt{2}\}
\]

**Converting Other Data Structures**

In general, to convert the structure of one result to another, use the `convert` command.

**Example**  Convert a table to a list.

\[
> T := \text{table}([x, x^2, x^3, x^4]);
\]

\[
T := \text{table}([1 = x, 2 = x^2, 3 = x^3, 4 = x^4])
\]

\[
> \text{convert}(T, \text{`list`});
\]

\[
[x, x^2, x^3, x^4]
\]

For a complete list of conversions between data structures, refer to `?convert`.

### 4.7 Other Maple Data Structures

In addition to the data structures already discussed in this chapter, Maple has a wide variety of other data structures. In particular, two data structures commonly used in programming are stacks and queues.

**Stacks**

A *stack* is a data structure in which the removal order of the elements is the reverse of the entry order. You can only insert (or *push*) and delete (or *pop*) elements from the *top* of the stack. This behavior is referred to
as LIFO (last in, first out). Stacks are implemented as tables in Maple. They are created by using the \texttt{stack} commands.\footnote{For more information about stacks in Maple, refer to \texttt{?stack}.}

Begin by creating a new (empty) stack.

\begin{verbatim}
> s := stack[new]();

s := table([0 = 0])
\end{verbatim}

Push the letters \texttt{h}, \texttt{a}, \texttt{t} onto the stack.

\begin{verbatim}
> stack[push](h, s);

h

> stack[push](a, s);

a

> stack[push](t, s);

t
\end{verbatim}

Check the \texttt{depth}, the value of the \texttt{top} element, and the contents of the stack.

\begin{verbatim}
> stack[depth](s);

3

> stack[top](s);

t

> eval('s');

\texttt{table([0 = 3, 1 = h, 2 = a, 3 = t])}
\end{verbatim}
Pop the letters off the stack. The original order of the letters is reversed.\(^29\)

\[
> \text{while not stack[empty](s) do stack[pop](s); end do;}
\]

\[
t
a
h
\]

**Queues**

A *queue* is a data structure in which the removal order of elements is the same as the entry order. This behavior is referred to as **FIFO** (first in, first out). Queues are implemented as tables in Maple and are created by using the `queue` commands.\(^30\)

Create a new (empty) queue.

\[
> q := \text{queue[new]();}
\]

\[
q := \text{table([0 = 0])}
\]

Place the letters h, a, t in the queue.

\[
> \text{queue[enqueue]}(q, h);
\]

\[
h
\]

\[
> \text{queue[enqueue]}(q, a);
\]

\[
a
\]

\[
> \text{queue[enqueue]}(q, t);
\]

\[
t
\]

Check the `length`, the value of the `front` element, and the contents of the queue.

\(^29\)The order of this result is the reverse of that in the example in the following subsection *Queues.*

\(^30\)For more information about queues in Maple, refer to ?queue.
> queue[length](q);

3

> queue[front](q);

h

> eval(‘q’);

table([0 = 3, 1 = h, 2 = a, 3 = t])

Remove the letters from the queue. The original order of the letters is maintained.31

> while not queue[empty](q) do queue[dequeue](q); end do;

h

a

t

Other important data structures are the connected graph and adjacency matrix. For more information on these data structures, see 6.10 Using Data Structures to Solve Problems.

There are many other data structures in Maple. For example, the matrix32 and vector33 constructors create special cases of arrays, and the Matrix34 and Vector35 constructors create special cases of Arrays.

4.8 Troubleshooting

This section provides you with a list of common mistakes, examples, and hints that will help you understand and avoid common errors. Use this

31 The order of this result is the reverse of that in the example in the preceding subsection Stacks.
32 For information on the matrix constructor, refer to ?matrix.
33 For information on the vector constructor, refer to ?vector.
34 For information on the Matrix constructor, refer to ?Matrix.
35 For information on the Vector constructor, refer to ?Vector.
section to study the errors that you may encounter when entering the examples from this chapter in a Maple session.

**Wrong Number of Parameters in Function**
Maple generates an error if a sequence is used as a parameter to `op` or `nops`.\(^{36}\)

\[
> s := a, b, c;
\]

\[
s := a, b, c
\]

\[
> \text{op}(2, s);
\]

```
Error, invalid input: op expects 1 or 2 arguments, but received 4
```

To avoid this error, convert `s` to a list (by enclosing `s` in square brackets) in the call to `op` or `nops`.

\[
> \text{op}(2, [s]);
\]

\[
b
\]

**Invalid Subscript Selector**
Maple generates an error if, during a selection operation, an invalid range is specified. An invalid range occurs if the following rule is broken: *the lower bound must be less than or equal to one more than the upper bound.* In the case that the range is an integer in a selection operation from a list, you cannot access the zeroth operand because lists are indexed by natural integers 1, 2, ...\(^{37}\)

\[
> L := [a, b, c, d];
\]

\[
L := [a, b, c, d]
\]

\[
> L[3..1];
\]

```
Error, invalid subscript selector
```

\(^{36}\) For more information, refer to `?op`.

\(^{37}\) For more information about selection and ranges, refer to `?selection`.
Chapter 4: Basic Data Structures

> L[0];
Error, invalid subscript selector

To avoid this error, use a range with a lower bound that is less than the upper bound.

> L[1..3];

\[a, b, c\]

Requires Range or Initialization List for Building arrays
Maple generates an error if no parameters are specified in a call to `array`.

> a := array();

Error, needs ranges or initialization list for building arrays

Unlike a table (for which an empty parameter list is permissible), in a call to `array` you must specify either the bound or an initialization list.

> a := array(1..3);

\[a := array(1..3, [])\]

Error in array Bound or Array Index out of Range
Maple generates an error if you try to assign a value to an array or Array index that is outside the array bound.

> a := array(1..3);

\[a := array(1..3, [])\]

> a[4] := p;

Error, 1st index, 4, larger than upper array bound 3
4.9 Exercises

1. Define a set with elements that are the powers of 13 modulo 100 for exponents ranging from 1 to 1000. Is 5 a member of the set? Why is it beneficial to use a set instead of a list?

   Hint: You can determine the set by using one statement if you use the seq command.

2. Generate the sums of 4 and the first 100 multiples of 3. Determine the sums that are square-free composite numbers.

   Hint: The numtheory package has a function that you need to use.
3. Find floating-point approximations for the sum of the square root and cubic root of each of the first 15 powers of 2.

*Hint:* Use `map`, `seq`, and `zip`.

4. Write a procedure that implements the sieve of Eratosthenes: Count the number of integers (less than or equal to a given integer) that are prime.

### 4.10 Conclusion

When you write a procedure, you generally have freedom to choose the data structure used for the data. The choice of data structure can have a great impact on the complexity and efficiency of the procedure. This chapter presented a variety of data structures that you can use to write procedures in Maple.

Procedures were introduced in chapter 1. Detailed information about procedures is provided in chapter 6.

The next chapter formally discusses the statements and other constructs that control the number of times and order in which operations are executed. These include the `for...while...do` looping construct, and the `if...then...else` selection statement.
5 Flow Control

In the Maple programming language, there are control structures that direct the flow of execution in a procedure. In particular, you can organize the Maple statements in a procedure by sequence, selection, or repetition. You have primarily used sequential flow in the preceding chapters. This chapter describes the Maple control structures for selection and repetition.¹

In This Chapter
- Selection and conditional execution control structures, the if statement and the ‘if’ operator
- Repetition control (looping) structures, the for and while statements
- Looping commands: map; select, remove, and selectremove; zip; seq, add, and mul; and $, sum, and product

5.1 Selection and Conditional Execution

A selection (or conditional execution) control structure selects one statement to execute from many listed. In Maple, to control the selection of a statement, use the if statement, or the ‘if’ operator.

The if Statement
The most general form of the if statement has the following syntax.

¹Procedure calls and exception handlers (try...catch...finally statements) are also forms of flow control in Maple. For more information about procedure calls, see chapter 6. For more information about handling exceptions, see chapter 8.
Chapter 5: Flow Control

The `elif` conditional expression then construct can appear zero, one, or many times. The `else` construct can be excluded.

This section describes various forms of this general if statement.

**Simple Forms of the if Statement** The following is the syntax of two simpler forms of the general if statement.

```markdown
if expr then
  statseq
end if

if expr then
  statseq1
else
  statseq2
end if
```

Maple executes these selection statements as follows.

1. The conditional expression (\textit{expr}) in the if clause is evaluated. The conditional expression can be any Boolean expression, which evaluates to \texttt{true}, \texttt{false}, or \texttt{FAIL}, formed by using:
   - relational operators: $<$, $<=$, $>$, $>=$, $=$, and $<>$
   - logical operators: \texttt{and}, \texttt{or}, and \texttt{not}
   - logical names: \texttt{true}, \texttt{false}, and \texttt{FAIL}

   Otherwise, the if statement returns an error.

   ```maple
   > x := -2:
   > if x then
   >   0
   > else
   >   1
   > end if;
   
   Error, invalid boolean expression
   ```
2. If the result of the if clause is the Boolean value true, Maple executes the statement sequence in the then clause. If the result of the if clause is the Boolean value false or FAIL, Maple executes the statement sequence in the else clause (if there is one).²

```maple
if x<0 then
  0
else
  1
end if;
```

0

You can omit the else clause if you do not want to specify an action if the condition is false.

```maple
if x>0 then
  x := x-1
end if;
```

**Nested Selection Statements** A selection statement can be nested—that is, the statement sequence in the then clause or else clause can be any statement (or sequence of statements), including an if statement.

```maple
if x>0 then
  print("Positive")
else
  if x=0 then
    print("Zero")
  else
    print("Negative")
  end if
end if;
```

“Negative”

The following example demonstrates the behavior of if statements in the case that the conditional expression evaluates to FAIL.

```maple
r := FAIL:
```

²For more information about Boolean expressions in Maple, see page 86 or refer to ?boolean.
Chapter 5: Flow Control

```markdown
> if r then
  >   print(1)
> else
  >   if not r then
  >     print(0)
  >   else
  >     print(-1)
  > end if
> end if;
```

General Forms of the if Statement  For more complicated scenarios, use one of the following two forms of the if statement.

```
if expr then
  statseq
elif expr then
  statseq
end if
```

```
if expr then
  statseq
elif expr then
  statseq
else
  statseq
end if
```

The `elif expr then statseq` construct can appear more than once. The following example implements the mathematical sign function by using an `elif` clause.

```markdown
> x := -2;

x := -2
```

```markdown
> if x<0 then
  > -1
> elseif x=0 then
  > 0
> else
  > 1
> end if;
```
Maple does not have a formal case statement. However, you can use the if statement as a case statement with the optional else clause as the default case. For example, to write a program that accepts a parameter n with four possible values (0, 1, 2, and 3), use the following code.\footnote{A case statement is one that directs a program to choose one action from a list of alternatives, depending on the value of a given variable. Case statements (or similar constructs) are common to most programming languages.} \footnote{For more information about the error statement, see page 219 or refer to ?error.}

```maple
> n := 5;

n := 5

> if n=0 then
> 0
> elif n=1 then
> 1/2
> elif n=2 then
> sqrt(2)/2
> elif n=3 then
> sqrt(3)/2
> else error "bad argument: \%1", n;
> end if;

Error, bad argument: 5
```

The ‘if‘ Operator
The operator form ‘if‘ requires three arguments and it returns the evaluation of the second or third argument, depending on the truth value of the first.

\[
\text{‘if‘}(\text{conditional expr}, \text{true expr}, \text{false expr})
\]

The first argument must evaluate to true, false, or FAIL. If the first argument evaluates to true, the second argument is evaluated and returned. If the first argument evaluates to false or FAIL, the third argument is evaluated and returned. When the operator form is used, the
name of this function must be enclosed in right single quotes (') because if is a Maple reserved word.\textsuperscript{5}

\begin{verbatim}
> b := 4;

b := 4

> 'if'(b>5, 10, 11);

11
\end{verbatim}

This 'if' operator statement is equivalent to the following if statement.

\begin{verbatim}
> if b>5 then
> 10
> else
> 11
> end if;
\end{verbatim}

11

### 5.2 Repetition

A loop structure (repetition statement) executes a section of code multiple times. Maple has two general forms of repetition statements that you can use to perform looping in procedures. They have the following syntax.

```
for name from expr by expr to expr while expr do
    statement sequence
end do
```

\textsuperscript{5}For more information about reserved words in Maple, see page 24 or refer to ?reserved.
5.2 Repetition

for name in expr while expr do
    statement sequence
end do

Many of the clauses in these general repetition statements are optional. As a result, you can extract two special cases of loop-control statements—the for and the while statements. These special cases provide the ability to execute a statement sequence repeatedly, either for a counted number of times (by using the for clause) or until a condition is satisfied (by using the while clause).6

The for Loop

The for loop is used to repeatedly execute a sequence of statements for a counted number of times. The for loop has two forms: the for...from loop and the for...in loop.

The for...from Loop  A typical for...from loop has the following syntax.

for name from expr by expr to expr
    while expr do
        statseq
    end do

The following clauses are optional.

• for name
• from expr
• by expr
• to expr
• while expr

You can also omit the sequence of statements statseq. Excluding the for clause, which must appear first, the clauses can appear in any order. If you omit a clause, it has the default value shown in Table 5.1.

A typical for...from loop is used to generate a sequence of results.

---

6You can replace many of the loops that use for or while statements with more efficient and concise special forms. For more information, see 5.3 Looping Commands.
Table 5.1 for Clauses and Their Default Values

<table>
<thead>
<tr>
<th>Clause</th>
<th>Default Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>for</code></td>
<td><code>dummy_variable</code></td>
</tr>
<tr>
<td><code>from</code></td>
<td>1</td>
</tr>
<tr>
<td><code>by</code></td>
<td>1</td>
</tr>
<tr>
<td><code>to</code></td>
<td>infinity</td>
</tr>
<tr>
<td><code>while</code></td>
<td>true</td>
</tr>
</tbody>
</table>

> for i from 2 to 5 do
>     i^2
> end do;

4

9

16

25

This sequence of results is generated as follows.

- Maple assigns `i` the value 2.
- Because 2 is less than 5, Maple executes the statement between the `do` and the `end do`.
- Then `i` is incremented by 1 to 3, and tested again.
- The loop executes until `i` is strictly larger than 5. In this case, the final value of `i` is 6.

> i;

6

You can also write the previous example by using the following statement.

> for i from 2 by 1 to 5
>     while true do
>     i^2
> end do:
When the by clause is negative, the for loop counts down.

```maple
> for i from 5 to 2 by -1 do
>   i^2
> end do;
```

```
25
16
9
4
```

**Example** To find the first prime number greater than $10^7$, you could write:

```maple
> for i from 10^7
>   while not isprime(i) do
>   end do;
```

After this statement is executed, \( i \) is the first prime larger than $10^7$.

```maple
> i;
```

```
10000019
```

Notice that the body of the loop is empty. Maple permits the empty statement. Improve the program by considering only odd numbers.

```maple
> for i from 10^7+1 by 2
>   while not isprime(i) do
>   end do;
```

```maple
> i;
```

```
10000019
```

The following code demonstrates how to repeat an action \( n \) times, in this case, throwing a die five times.

```maple
> die := rand(1..6):
> to 5 do
die();
end do;

If all of the clauses in the `for` statement are omitted, an infinite loop is produced.

```plaintext
do statseq end do
```

This has the same effect as the following code, but the `do` loop does not assign a value to a dummy variable each iteration.

```plaintext
for dummy_variable from 1 by 1 to infinity
while true do
  statseq
end do
```

Such a loop statement repeats indefinitely unless Maple encounters a `break` statement (see page 179), `return` statement (see page 219), `quit` statement (see page 119), or error.

**The `for...in` Loop** The `for...in` loop has the following syntax.

```plaintext
for name in expr
while expr do
  statseq
end do
```

The loop index `name` is assigned the operands of the first `expr`. You can test the value of the index in the optional `while` clause, and, of course, the value of the index is available when you execute the `statseq`. If the object in `expr` contains at least one operand, then the value of the index variable `name` remains assigned at the end of the loop.

For example, given a list `L` of integers, to find the integers that are less than or equal to 7, use the following `for` loop.
5.2 Repetition

\[ L := [7, 2, 5, 8, 7, 9]; \]

\[ L := [7, 2, 5, 8, 7, 9] \]

\[ \text{for } i \text{ in } L \text{ do} \]
\[ \quad \text{if } i \leq 7 \text{ then} \]
\[ \quad \quad \text{print}(i) \]
\[ \quad \quad \text{end if;} \]
\[ \quad \text{end do;} \]

7
2
5
7

This code cycles through the operands of object L, in this case, a list. The object can be, for example, a set, sum of terms, product of factors, or string of characters.

**The while Loop**

The for loop is used to repeatedly execute a sequence of statements until a condition is satisfied. The while loop is a for loop with all of its clauses omitted except the while clause.

```
while expr do
  stmtseq
end do
```

The expression `expr` is called the *while condition*. It must be a Boolean-valued expression. That is, it must evaluate to `true`, `false`, or `FAIL`.

\[ x := 256; \]

\[ x := 256 \]

\[ \text{while } x > 1 \text{ do} \]
\[ \quad x := x/4 \]
\[ \quad \text{end do;} \]
The **while** loop behaves as follows.

- Maple evaluates the **while condition**. An error occurs if the while condition does not evaluate to **true**, **false**, or **FAIL**.

- If the condition evaluates to **true**, Maple executes the body of the loop.

- The loop repeats until the while condition evaluates to **false** or **FAIL**.\(^7\)

\[
\begin{align*}
x := 64 \\
x := 16 \\
x := 4 \\
x := 1
\end{align*}
\]

\[
\begin{align*}
> x := 1/2 :

> \text{while } x>1 \text{ do} \\
> \quad x := x/2 \\
> \text{end do;}

> x;

\frac{1}{2}
\end{align*}
\]

\[
\begin{align*}
> \text{while } x \text{ do} \\
> \quad x := x/2 \\
> \text{end do;}

\text{Error, invalid boolean expression}
\end{align*}
\]

**Control within Loops**

Within the Maple language reside two additional loop control constructs: **break** and **next**.

\(^7\)Maple evaluates the while condition before it executes the body of the loop.
The break Statement  When Maple executes a break statement, the result is to exit from the repetition statement in which it occurs. Execution then proceeds with the first statement following this repetition statement.\(^8\)

\[
L := [2, 5, 7, 8, 9]
\]

\[
L := [2, 5, 7, 8, 9]
\]

```
> L := [2, 5, 7, 8, 9];

> for i in L do
>     print(i);
>     if i=7 then
>         break
>     end if;
> end do;

2
5
7
```

The next Statement  When Maple executes a next statement, it proceeds immediately to the next iteration. For example, to skip the elements in a list that are equal to 7, use the following for loop.

\[
L := [7, 2, 5, 8, 7, 9]
\]

\[
L := [7, 2, 5, 8, 7, 9]
\]

```
> L := [7,2,5,8,7,9];

> for i in L do
>     if i=7 then
>         next
>     end if;
>     print(i);
> end do;

2
5
8
9
```

\(^8\)For more information about the break statement, refer to \texttt{break}.
An error occurs if Maple encounters the names `break` or `next` in a context different from a repetition statement.

> next;
Error, break or next not in loop

### 5.3 Looping Commands

The previous section described the `for` and `while` loops. Some processes that involve loops are used so often that Maple provides special-purpose commands for them. These commands help to make writing programs simpler and more efficient. You can group these eight loop-based Maple commands into three categories.\(^9\)

1. `map`, `select`, `remove`, `selectremove`
2. `zip`
3. `seq`, `add`, `mul`

**The `map` Command**

The `map` command applies a function to every element of an aggregate object. The simplest form of the `map` command is

\[
\text{map}( f, x )
\]

where `f` is a function and `x` is an expression. The `map` command replaces each operand \(x_i\) of the expression \(x\) with \(f(x_i)\).\(^{10}\)

> `map( f, [a,b,c] );`

---

\(^9\) When possible, use these Maple commands instead of a generic `for` or `while` loop since the code for these commands is built into the Maple kernel. Therefore, it is usually more efficient to perform computations by using them. However, there are circumstances in which it is not desirable to use these special looping commands. For more information, see page 185.

\(^{10}\) The exceptions are for a table, array, Array, or rtable. Maple applies the function to the entries of the table, array, or Array not the operands or indices. For an rtable, the function is applied to each element of the rtable, and then returns a new rtable of the mapped result.
Example  Given a list of integers, you can create a list of their absolute values and of their squares by using the `map` command.

> L := [ -1, 2, -3, -4, 5 ];

$L := [-1, 2, -3, -4, 5]$

> q:=map(abs, L);

$q := [1, 2, 3, 4, 5]$

> map(x->x^2, L);

$[1, 4, 9, 16, 25]$

The general syntax of the `map` command is

```
map( f, x, y1, ..., yn )
```

where `f` is a function, `x` is any expression, and `y1, ..., yn` are expressions. The action of `map` is to replace each operand $x_i$ of `x` by $f(x_i, y1, ..., yn)$.

> map( f, [a,b,c], x, y );

$[f(a, x, y), f(b, x, y), f(c, x, y)]$

> map( (x,y) -> x^2+y, L, 1);

$[2, 5, 10, 17, 26]$

The `select`, `remove`, and `selectremove` Commands

The `select` command returns the operands for which the specified Boolean-valued function returns `true`. The `remove` command returns the operands for which the specified Boolean-valued function returns `false`. The `selectremove` command returns two objects: the operands for which the specified Boolean-valued function returns `true` and the operands for
which the specified Boolean-valued function returns false. The select, remove, and selectremove commands have the same syntax as the map command.

\[
X := 2x^2 - 3y^4z + 3zw + 2y^3 - z^2wy
\]

> select(has, X, z);

\[-3y^4z + 3zw - z^2wy\]

> remove( x -> degree(x)>3, X );

\[2x^2 + 3zw + 2y^3\]

For more information about these commands, see page 156 or refer to ?select.

The \texttt{zip} Command

The \texttt{zip} command merges two lists or vectors, and then applies a binary function. The \texttt{zip} command has two forms

\[
\text{zip}(f, u, v) \\
\text{zip}(f, u, v, d)
\]

where \(f\) is a binary function, \(u\) and \(v\) are both lists or vectors, and \(d\) is any value. The \texttt{zip} command takes each pair of operands \(u_i, v_i\), and creates a new list or vector from \(f(u_i, v_i)\).\(^{11}\)

> zip( (x,y) -> x || y, [a,b,c,d,e,f], [1,2,3,4,5,6] );

\[[a1, b2, c3, d4, e5, f6]\]

If the lists or vectors are not the same length, the length of the result depends on whether you provide the argument \(d\).

If you do not specify \(d\), the length of the result is the same as the length of the smaller list or vector.

\(^{11}\)There is a similar command for rtable-based Matrices and Vectors. For more information, refer to ?Zip.
5.3 Looping Commands

> zip( (x,y) -> x+y, [a,b,c,d,e,f], [1,2,3] );

\[ [a + 1, b + 2, c + 3] \]

If \( d \) is specified, the length of the result of the \texttt{zip} command is the same as the length of the longer list or vector. Maple replaces the missing value(s) with \( d \).

> zip( (x,y) -> x+y, [a,b,c,d,e,f], [1,2,3], xi );

\[ [a + 1, b + 2, c + 3, d + \xi, e + \xi, f + \xi] \]

The \texttt{seq}, \texttt{add}, and \texttt{mul} Commands

The \texttt{seq}, \texttt{add}, and \texttt{mul} commands form sequences, sums, and products, respectively. They have the following syntax.

\[
\begin{align*}
\text{seq} & (f, i = a..b) \\
\text{add} & (f, i = a..b) \\
\text{mul} & (f, i = a..b)
\end{align*}
\]

where \( f \), \( a \), and \( b \) are expressions, and \( i \) is a name. The expressions \( a \) and \( b \) must evaluate to numerical constants (except in \texttt{seq}, for which they can be single character strings).

The index name \( i \) is successively assigned the values \( a, a+1, \ldots, b \) (or up to the last value not exceeding \( b \)). The result returned by \texttt{seq} is the sequence that Maple produces by evaluating \( f \) at each value of \( i \).

> seq(i, i = 4.123 .. 6.1);

\[ 4.123, 5.123 \]

> seq(i^2, i=1..4);

\[ 1, 4, 9, 16 \]

The result returned by \texttt{add} is the sum of the sequence. The result of \texttt{mul} is the product of the sequence.

> add(i^2, i=1..4);
> mul(i^2, i=1..4);

576

> add(x[i], i=1..4);

\[x_1 + x_2 + x_3 + x_4\]

In the case that \(a\) is greater than \(b\), the result returned by \texttt{seq}, \texttt{add}, and \texttt{mul} is the NULL sequence, 0, and 1, respectively.

> mul(i^2, i = 4..1);

1

You can also use the \texttt{seq}, \texttt{add}, and \texttt{mul} commands with the following syntax.

\begin{verbatim}
seq(f, i = X)
add(f, i = X)
mul(f, i = X)
\end{verbatim}

where \(f\) and \(X\) are expressions, and \(i\) is a name.

Using this form, the index name \(i\) is successively assigned the operands of the expression \(X\) (or the characters of string \(X\)). The result returned by \texttt{seq} is the sequence that Maple produces by evaluating \(f\) at each value of \(i\).

> a := x^3 + 3*x^2 + 3*x + 1;

\[a := x^3 + 3x^2 + 3x + 1\]

> seq(degree(i,x), i=a);

3, 2, 1, 0

> seq(i, i="square");

“s”, “q”, “u”, “a”, “r”, “e”
The result of \texttt{add} is the sum of the sequence. The result of \texttt{mul} is the product of the sequence.

\begin{verbatim}
> add(degree(i,x), i=a);

\end{verbatim}

\begin{verbatim}
> a := [23,-42,11,-3];

\end{verbatim}

\begin{verbatim}
\quad a := [23, -42, 11, -3]

> add(i^2, i=a);

\end{verbatim}

\begin{verbatim}
\quad 2423

> mul(abs(i), i=a);

\end{verbatim}

\begin{verbatim}
\quad 31878
\end{verbatim}

This form of the \texttt{seq} function can be used to generate a sequence of indexed names.

\begin{verbatim}
> seq(x[i], i=1..5);

\end{verbatim}

\begin{verbatim}
\quad x_1, x_2, x_3, x_4, x_5
\end{verbatim}

\section*{Using Specialized Looping Commands}

Using the specialized looping commands described in this section can make writing Maple programs simpler and more efficient. However, there are conditions in which it is preferable to use the selection and repetition constructs instead of these specialized commands, and others in which the use of one specialized looping construct is recommended over another. This section illustrates some of these situations.

\textbf{if versus map} The use of \texttt{map} or \texttt{zip} to simplify procedure code is not always recommended. For example, it is common for a procedure to move to the next iteration once a result is determined without performing all the operations. In these cases, use an \texttt{if} statement instead of the \texttt{map} or \texttt{zip} command.
Example  Consider the following procedure that uses an \texttt{if} statement.

\begin{verbatim}
> IsStringList := proc( e ) local i;
>     if type( e, 'list' ) then
>         for i from 1 to nops( e ) do
>             if not type( e[ i ], 'string' ) then
>                 return false
>             end if
>         end do;
>         true
>     else
>         false
>     end if
> end proc:
\end{verbatim}

This procedure can be written in a simpler form by using \texttt{map}.

\begin{verbatim}
> IsStringList := proc( e )
>     type( e, 'list' ) and member( false, map( type, e, 'string' ) )
> end proc:
\end{verbatim}

However, this “simpler” form allocates storage for a new set that is otherwise unrelated to the problem and tests \textit{every} element of the list. Procedure \texttt{IsStringList} could also be written by using the following code.

\begin{verbatim}
> IsStringList := proc( e )
>     type( e, 'list' ) and evalb( remove( type, e, 'string' ) = [] )
> end proc:
\end{verbatim}

Unfortunately, this version also tests \textit{every} element of \texttt{e} to perform the test.

For example, if you call the latter two versions of \texttt{IsStringList} on a 1,000,000 element list whose third member is \textit{not} a string, it performs 1,000,000 type tests, while the first version of \texttt{IsStringList} (that uses an \texttt{if} statement) stops after the third element.

This example illustrates that if a procedure can return early, it is better to implement it by using an \texttt{if} statement.

\texttt{seq}, \texttt{add}, and \texttt{mul} \textit{versus} \$\texttt{, sum, and product}  The sequence operator, \$\texttt{, and the \texttt{sum} and \texttt{product} commands are very similar to the \texttt{seq, add, and mul} commands. However, they differ in an important way—the index variable \texttt{i} and the end points \texttt{a} and \texttt{b} do not have to be integers.

\begin{verbatim}
> x[k] $ k=1..n;
\end{verbatim}
5.4 Troubleshooting

\[ x_k \sum_{k=1}^n (k = 1..n) \]

\[
\frac{1}{3} (n + 1)^3 - \frac{1}{2} (n + 1)^2 + \frac{1}{6} n + \frac{1}{6}
\]

These commands are designed for symbolic sequences, sums, and products. The index variable \( k \) is a global variable to which you must not assign a value. If a previous value was assigned to \( k \), an error message is returned.

\[
> k := 10;
\]

\[ k := 10 \]

\[
> \text{sum}(k^2, k=0..n);
\]

Error, (in sum) summation variable previously assigned, second argument evaluates to 10 = 0 .. n

Using $, sum, and product To produce a symbolic sequence, sum, or product, you must use $, sum, or product. For example, if the end points are unknown, use $, sum, or product. If you are producing an explicit finite sequence, sum, or product—that is, when the range points \( a \) and \( b \) are integers—use seq, add, or mul.\textsuperscript{12}

For more information about the $ operator, refer to $?$. For more information about the sum and product commands, refer to ?sum and ?product, respectively.

5.4 Troubleshooting

This section provides you with a list of common mistakes, examples, and hints that will help you understand and avoid common errors. Use this

\textsuperscript{12}The sum and product commands are not built into the kernel. Therefore, in general, they are not as efficient as add and mul. For more information about efficiency, see page 330 or refer to ?efficient.
section to study the errors that you may encounter when entering the examples from this chapter in a Maple session.\textsuperscript{13}

**Cannot Evaluate Boolean in if Statement**

The conditional expression in an if clause must be a Boolean expression formed by using relational operators, logical operators, and logical names. Maple generates an error if the Boolean expression cannot be evaluated to \texttt{true}, \texttt{false}, or \texttt{FAIL}.\textsuperscript{14}

\begin{verbatim}
> p := proc( x )
>   if x < 0 then
>     -x
>   else
>     x
>   end if
> end proc;

p := proc(x) if x < 0 then -x else x end if end proc

> p(a);
Error, (in p) cannot determine if this expression is true or false: a < 0

Instead, you must use a parameter that can be compared to 0 in procedure p.

> p(-2);
2

To avoid this error message, use type-checking in the formal parameter list of the procedure definition. For more information, see page 197.

**Value in Loop Must Be Numeric or Character**

Maple generates an error if the expression in the from or to part of a for statement does not result in a value of type numeric or a character (string of length 1).\textsuperscript{15}

\textsuperscript{13}You can also use the Maple debugger to find errors in programs. For more information, see chapter 8 or refer to \texttt{debugger}.

\textsuperscript{14}For more information about Boolean expressions, see page 86 or refer to \texttt{boolean}.

\textsuperscript{15}For more information about types, see 2.4 Types and Operands or refer to \texttt{type}.
> for i from a to d do print(i) end do;

Error, initial value in for loop must be numeric or character

> for i from 1 to "z" do print(i) end do;

Error, final value in for loop must have same type as initial

Study the following examples to see how to fix these errors.

> for i from "a" to "d" do print(i) end do;

“a”

“b”

“c”

“d”

> for i from 1 to 2 do print(i) end do;

1

2

> for i from evalf(tan(Pi/8)) to evalf(sqrt(17)) by 1.5
>    do print(i) end do;

0.4142135625

1.914213562

3.414213562

Variable Previously Assigned

Maple generates an error if the index variable in a sum or product command has a previously assigned value. This occurs because the index variable in these commands is the global variable.\textsuperscript{16}

\textsuperscript{16}For more information about the global variables, see \textbf{Variables} on page 201.
> k := 100;

\[ k := 100 \]

> sum(k^2, k=0..4);

Error, (in sum) summation variable previously assigned, second argument evaluates to 100 = 0 .. 4

It is recommended (and often necessary) that both the expression and index arguments be enclosed in right single quotes to delay evaluation, so that the name (not value) is used.\(^{17}\)

> sum('k^2', 'k'=0..4);

30

Wrong Parameters in Function $$

Maple generates an error if the index variable in a statement containing the $\$ operator has an assigned value. This occurs because the index variable, which is the same as the global variable, is evaluated to its value, and hence cannot be used as a counting variable.\(^{18}\)

> i := 100;

\[ i := 100 \]

> a[i] $ i = 1..3;

Error, invalid input: $ expects its 2nd argument, range, to be of type name, but received 100 = 1 .. 3

It is recommended that \(i\) be enclosed in right single quotes to delay evaluation.\(^{19}\)

> 'a[i]' $ 'i' = 1..3;

\(^{17}\)For more information on delaying evaluation, see Unevaluated Expressions on page 100.

\(^{18}\)For more information about the global variables, see Variables on page 201.

\(^{19}\)For more information on delaying evaluation, see Unevaluated Expressions on page 100.
5.5 Exercises

1. Find the product of the square root of all prime numbers less than 100.
   
   **Hint:** The function `isprime` determines the primality of an integer.

2. Find the sum of all odd composite numbers less than 150.

3. Find the sum of the first 30 powers of 2.

4. Write a looping structure that finds the four substrings (of a string assigned to the name `MyString`) containing only lower case letters, upper case letters, decimal digits, and special characters.
   
   **Hint:** You can use relational operators to compare characters.

5. Write a procedure, `SPLIT`, that, upon input of a product `f` and a variable `x`, returns a list of two values. The first item in the list should be the product of the factors in `f` that are independent of `x`, and the second item should be the product of the factors that contain an `x`.
   
   **Hint:** Use the `has`, `select`, `remove`, and `selectremove` commands.

5.6 Conclusion

This chapter discussed flow control in the Maple programming language. Normally, the statements in a procedure body are executed sequentially. However, you can control the order in which operations are executed by using the `if...then...else` selection statement, or the `for...while...do` looping construct. For certain procedures, it is more efficient to use the specialized looping commands: `map`, `select`, `remove`, and `selectremove`; `zip`, `seq`, `add`, and `mul`; and `$`, `sum`, and `product`. 
6 Maple Procedures

A Maple procedure definition is a prearranged group of statements entered within a proc()...end proc construct. In this construct, you must declare the parameters and variables that the procedure uses, and specify the statements that form the body of the procedure. You can also define simple one-line procedures using functional operators.

In chapter 1, a brief introduction to procedures was presented. This chapter describes the syntax and semantics of a Maple procedure in detail, and discusses how to create mathematical functions using functional operators.

In This Chapter
- Structure of procedures
- Using procedures
- Automatic simplification of procedures
- Procedure return values
- Adding comments, such as copyright statements, and help pages for procedures
- Alternate methods for defining procedures
- The procedure object
- Implementing data structures, using procedures, to solve problems

6.1 Defining a Procedure

A Maple procedure definition has the following general syntax.
The letter P represents the formal parameter names, which may be NULL (an empty expression). The local variables, global variables, options, description, and procedure body statements are optional.

Example   The following is a simple Maple procedure definition. It contains two formal parameters, x and y, and one statement in the body of the procedure, but no local variables, global variables, options, or description.

\[
\begin{align*}
&> \text{proc}(x,y) \\
&> \quad x^2 + y^2 \\
&> \quad \text{end proc;}
\end{align*}
\]

\[\text{proc}(x, y) x^2 + y^2 \text{ end proc}\]

Naming a Procedure
You can name a procedure by using an assignment statement in the same manner as for other Maple objects. In general, a procedure must be assigned to a name so that you can invoke it with procedure calls. For information about unnamed procedures, see page 232.¹

\[
\begin{align*}
&> f := \text{proc}(x,y) \\
&> \quad x^2 + y^2 \\
&> \quad \text{end proc;}
\end{align*}
\]

\[f := \text{proc}(x, y) x^2 + y^2 \text{ end proc}\]

Executing a Procedure
You can execute (or invoke) a procedure assigned to a name by using a procedure call.

¹You can assign a procedure to any number of names. For example, both f and g are assigned the same procedure by using the following commands. \(f := \text{proc}(x) x^2 \text{ end proc; \quad g := \text{eval}(f); \quad \text{map(addressof@eval, [f, g])};\)
procedureName( A );

The procedureName is the name to which the procedure definition is assigned, and the letter A represents the expression sequence of actual parameters used in the procedure call.

When Maple executes the statements in the body of a procedure, it substitutes the formal parameters P in the procedure definition with the actual parameters A in the procedure call. Normally, the result a procedure returns is the value of the last executed statement in the body of the procedure.\(^2\)

**Note:** Maple evaluates the actual parameters before they are substituted for the formal parameters. These parameters are *not* evaluated again during execution of the procedure. Prior to substitution, each actual parameter is also checked against its specified type, if any (see page 197). If the type checking fails, the procedure returns an error. The order in which Maple tests the types of the actual parameters is unspecified. Type checking actual parameters is optional.

In the following procedure call, Maple executes the statements in the body of procedure F and replaces the formal parameters, x and y, with the actual parameters, 2 and 3. The result of the last, and in this case only, statement in the procedure is the returned value of procedure F.

\[ > \text{f}(2,3); \]

The number of actual parameters need not be the same as the number of specified formal parameters. If too few actual parameters are specified, an error occurs if (and only if) a missing parameter is used during the execution of the procedure body. Maple does not process extra parameters.

\[ > \text{f} := \text{proc}(x,y,z) \]
\[ > \quad \text{if } x>y \text{ then} \]
\[ > \quad \quad x \]
\[ > \quad \quad \text{else} \]
\[ > \quad \quad z \]
\[ > \quad \quad \text{end if} \]
\[ > \quad \text{end proc;} \]
\[ > \text{f}(1,2,3,4); \]

\[^2\text{For more information about return values, see 6.4 Procedure Return Values.}\]
6.2  Procedure Components

The following sections describe the components of a procedure definition in detail. These are the formal parameters, local variables, global variables, options, description, and body statements.

**Formal Parameters**

```maple
proc( P )
    ...
end proc
```

The letter P represents the formal parameter names in a procedure definition. It can be NULL (an empty expression), a sequence of names, or a sequence of names and their type definitions.

**Procedures without Formal Parameters**  If no input is required to run a procedure, that is, no formal parameters are required, enter empty parentheses in the heading of the procedure definition and in procedure calls.

```maple
> F := proc()
> "Hello World"
> end proc;

    F := proc() "Hello World" end proc

> F();
```
Specifying Formal Parameters  Procedures that do not require formal parameters usually perform very simple tasks. In general, procedure execution requires input. In these cases, you must enter formal parameters as a sequence of names between the parentheses in the heading of the procedure definition.

\[
F := \text{proc}(x, y) \text{gcd}(x, y) \text{end proc}
\]

You must specify actual parameters between parentheses in the calling sequence.

\[
F(4, 6);
\]

Type-checking Formal Parameters  To restrict the type of parameters, specify the parameter types in the formal parameter list of a procedure definition. To include the type declaration that parameter \( p \) must be of type \( t \), the parameter must be specified by using the following syntax.

\[
p :: t
\]

Although this is optional, it is recommended because it makes the procedure more robust. At invocation, each argument is checked against the type specified for the parameter. If any fail the type check, an error message is generated.\(^3\)

\[
F := \text{proc}(x :: \text{integer}, y :: \text{integer}) \text{gcd}(x, y) \text{end proc}
\]

\(^3\)Maple has many expression types. For more information, refer to \texttt{?type}. For more information about parameters and type checking, refer to \texttt{?paramtype}.
> F(4, 6.3);

Error, invalid input: F expects its 2nd argument, y, to be of type integer, but received 6.3

> F(4, 6);

2

If you do not declare the type of a parameter, it can have any type. For example, proc(x) is equivalent to proc(x::anything). In that case it is recommended that you use the latter form to inform other users that the procedure works for any input.

The closing bracket of the formal parameter list can optionally be followed by ::type_name; where type_name specifies a Maple type. Unlike the case of formal parameters, this is not a type declaration, but rather an assertion—a statement about the procedure that you assert to be true. This optional assertion facility checks the return value type if you set kernelopts(assertlevel=2). If the type violates the assertion, an exception is raised.

> F := proc(x :: numeric ) :: integer;
> x/2;
> end proc:
> F(3);

3
2

> kernelopts(assertlevel=2):
> F(3);

Error, (in F) assertion failed: F expects its return value to be of type integer, but computed 3/2

In this case, executing procedure F causes an exception because its definition asserted that the return value must be of type integer.

\footnote{For more information about assertions and exceptions, see 8.3 Detecting Errors.}
Evaluation Rules  Parameters play a special role in procedures. Maple replaces them with arguments (actual parameters) when you invoke the procedure.

Examine the following procedure that squares its first argument and assigns the answer to the second argument, which must be a name.

\begin{verbatim}
Sqr1 := proc(x::anything, y::name)
    y := x^2;
end proc;
\end{verbatim}

The procedure squares the value of \( d \) and assigns the result to the name \( \text{ans} \).

To demonstrate parameter evaluation, first assign the name \( a \) the value \( b \). Then assign the name \( b \) the value \( c \).

\begin{verbatim}
a := b;
b := c;
\end{verbatim}

Use \( a \) as the first argument. Reset \( \text{ans} \) to a name so that the procedure type check does not fail.

\begin{verbatim}
ans := 'ans';
\end{verbatim}

\begin{verbatim}
Sqr1(a, ans);
\end{verbatim}
c^2

> ans;

> c^2

From the answer, it is clear that the value c is assigned to the parameter x.

Maple evaluates the arguments before invoking the procedure.

When you call a procedure, Maple evaluates the arguments as determined by the context of the call. For example, if you call Sqr1 from inside a procedure, Maple evaluates the local variable a to one level. For more information about procedure variable evaluation rules, see page 203. Thus, in the procedure g below, Maple evaluates a to b not c.

```maple
> g := proc()
> local a,b,ans;
> a := b;
> b := c;
> Sqr1(a,ans);
> end proc;

> g();

b^2
```

Whether you call a procedure from the interactive level or from inside a procedure, Maple evaluates the arguments to the level specified by the context before invoking the procedure. Once Maple evaluates the arguments, it replaces all occurrences of the corresponding formal parameters with the actual arguments. Then Maple invokes the procedure.

Because Maple only evaluates parameters once, you cannot use them as local variables. This is demonstrated by the following procedure.

```maple
> Cube := proc(x::anything, y::name)
> y := x^3;
> y;
> end proc;
> ans := 'ans';
```
6.2 Procedure Components

\begin{verbatim}
ans := ans

> Cube(2, ans);

ans

> ans;

8

Maple replaces each y with ans, but does not evaluate these occurrences of ans again. Thus, the final line of Cube returns the name ans, not the value assigned to ans.

Use parameters only to pass information into the procedure. Parameters are objects that are evaluated to zero levels.

Variables

A variable represents a data item, such as a numerical value or a character string, that can change its value during the execution of a program. This section describes local variables, global variables, and their use in Maple procedures.

A local variable is a variable that has meaning only within a particular procedure. If the name of a local variable is used outside the procedure, it (usually) refers to a different instantiation of the name. Global variables are recognized inside and outside the procedure.

Local and Global Variable Declarations

Variables that occur inside a procedure can be global or local to the procedure. Variables that occur outside a procedure are always global. Local variables in different procedure invocations are distinct variables even if they have the same name. Thus, one procedure can change the value of a local variable without affecting variables of the same name in other procedures, or a global variable of the same name. You should always declare variables as local or global by using the following declaration statement in a procedure definition.

\begin{verbatim}
local L1, L2, ..., Ln;
global G1, G2, ..., Gm;
\end{verbatim}

In procedure Max, i and m are local variables.
Chapter 6: Maple Procedures

```maple
Max := proc()
    local i, m;
    if nargs = 0 then
        return -infinity
    end if;
    m := args[1];
    for i from 2 to nargs do
        if args[i] > m then
            m := args[i]
        end if;
    end do;
    m;
end proc:
```

Like procedure return values, you can assert the type of each local variable in the local variable declaration statement by using the following syntax.

```
local L1::type1, var2::type2, ...
```

This is not a type declaration—it is an assertion. If you use this optional assertion facility for checking the local variable types in a procedure and set `kernelopts(assertlevel=2)`, any assignment to a variable with a type assertion is checked before the assignment is performed. If the assignment violates the assertion, an exception is raised.

To illustrate how this facility works, consider the revised procedure `Max` that includes an incorrect type assertion for local variable `m`.

```maple
kernelopts(assertlevel=2):
Max := proc()
    local i, m :: string;
    if nargs = 0 then
        return -infinity
    end if;
    m := args[1];
    for i from 2 to nargs do
        if args[i] > m then
            m := args[i]
        end if;
    end do;
    m;
end proc:
Max(1,2,3);
```

Error, (in Max) assertion failed in assignment, expected string, got 1

In the case of nested procedures, where one procedure is defined within the body of another, variables can also acquire local or global
declaration from procedures which enclose them. For more information about nested procedures, refer to chapter 1 in the *Maple Advanced Programming Guide* or ?examples,lexical.

**Evaluation Rules for Procedure Variables**  Maple fully evaluates global variables, even inside a procedure. However, local variables are evaluated in a special way. During the execution of a procedure body, a local variable is evaluated only one level. The following examples clarify this concept.

Consider a sequence of Maple statements.

\[
> f := x + y;
\]
\[
f := x + y
\]

\[
> x := z^2/y;
\]
\[
x := \frac{z^2}{y}
\]

\[
> z := y^3 + 3;
\]
\[
z := y^3 + 3
\]

Since these statements undergo normal full recursive evaluation, the following result is returned.

\[
> f;
\]
\[
\frac{(y^3 + 3)^2}{y} + y
\]

The actual level of evaluation is controlled by using the `eval` command. You can use the following commands to evaluate to one, two, or three levels, respectively.

\[
> \text{eval}(f,1);
\]
\[
x + y
\]

\[
> \text{eval}(f,2);
\]
Unlike the full evaluation of the sequence of statements in the previous example, in procedures, local variables are only evaluated to one level.

```
> f := proc()
> local x, y, z;
> x := y^2; y := z; z := 3;
> x;
> end proc:
> f();
```

\[ y^2 \]

The concept of one-level evaluation is important for efficiency. It has very little effect on the behavior of programs because most programs have a sequential structure. In the case that a procedure body requires a full (recursive) evaluation of a local variable, use the `eval` command.\(^5\)

```
> f := proc()
> local x, y, z;
> x := y^2; y := z; z := 3;
> eval(x);
> end proc:
> f();
```

\[ y \]

Without the call to `eval`, the value of \( x \) is \( y^2 \).

In the same manner as global variables, you can use local variables as unknowns. This use is recommended when it is not necessary to access the variables. For example, in the `RootsOfUnity` procedure, the local variable \( x \) does not have an assigned value. The procedure uses it as the variable in the polynomial \( x^n - 1 \).

\(^5\)The concept of one-level evaluation does not occur in traditional programming languages. However, in Maple, you can assign to a variable a formula involving other variables (to which you can assign values, and so on).
6.2 Procedure Components

> RootsOfUnity := proc(n)
> local x;
> [solve( x^n - 1=0, x )];
> end proc:
> RootsOfUnity(5);

\[ [1, -\frac{1}{4} + \frac{1}{4}\sqrt{5} + \frac{1}{4}I\sqrt{2}\sqrt{5 + \sqrt{5}},
\]

\[ -\frac{1}{4} - \frac{1}{4}\sqrt{5} + \frac{1}{4}I\sqrt{2}\sqrt{5 - \sqrt{5}},
\]

\[ -\frac{1}{4} + \frac{1}{4}\sqrt{5} - \frac{1}{4}I\sqrt{2}\sqrt{5 + \sqrt{5}}] \]

Undeclared Variables

If a variable is not declared local or global in the one (or more) procedures in which it is enclosed, Maple determines its scope. A variable is automatically made local in the following two cases.

- It appears on the left-hand side of an assignment statement. For example, A in A := y or A[1] := y.
- It appears as the index variable in a for loop, or seq, add, or mul command.

If neither of these two rules applies, the variable is a global variable. To illustrate how Maple reacts to undeclared variables, consider the revised Max procedure.

> Max := proc()
> if nargs = 0 then
> return -infinity
> end if;
> m := args[1];
> for i from 2 to nargs do
> if args[i] > m then
> m := args[i]
> end if;
> end do;
> m;
> end proc:

Warning, ‘m’ is implicitly declared local to procedure ‘Max’
Warning, ‘i’ is implicitly declared local to procedure ‘Max’

Do not rely on this facility to declare local variables. Declare all your local variables explicitly. Use Maple warning messages to identify variables that you have misspelled or neglected to declare.
The `NewName` procedure creates the next unused name in the sequence \( C_1, C_2, \ldots \). The name that `NewName` creates is a global variable since neither of the two previous rules apply to `cat(C,N)`.

```maple
> NewName := proc()
>     global N;
>     N := N+1;
>     while assigned(cat(C,N)) do
>         N := N+1;
>     end do;
>     cat(C,N);
> end proc:
> N := 0;

N := 0

> NewName() * sin(x) + NewName() * cos(x);

\[ C_1 \sin(x) + C_2 \cos(x) \]

It is recommended that you do not assign a value to a global variable in a procedure. A change in the value of a global variable updates all instances of the variable, including those of which you are unaware. Therefore, use this technique judiciously.\(^6\)

### Procedure Options

```maple
proc( ... )
    variable declarations
    options O;
    description
    procedure body
end proc
```

The options of a procedure definition must appear immediately after the local and global variable declarations, if any. A procedure can have one or more options. These options are specified by using the `options` clause in a procedure definition.

\(^6\)As is the case with any global side effect, assigning to a global variable in a procedure invalidates that procedure for `option remember`. For more information about `option remember`, see page 210.
6.2 Procedure Components

options $O_1, O_2, \ldots, O_m$;

You can use any symbol to specify an option name. However, the following terms have special meanings.\(^7\)

**The arrow Option** The arrow option has meaning only when specified in conjunction with option operator. The arrow option causes the Maple prettyprinter to print the procedure using arrow notation. For more information, see page 209.

```maple
> f := proc(x, y)
> option operator;
> x^2 + y^2
> end proc;
```

\[ f := \text{proc}(x, y) \text{option operator}; x^2 + y^2 \text{end proc} \]

```maple
> f := proc(x, y)
> option operator, arrow;
> x^2 + y^2
> end proc;
```

\[ f := (x, y) \rightarrow x^2 + y^2 \]

**The builtin Option** Maple has two classes of procedures: kernel procedures written in the C programming language, and library procedures written in the Maple programming language. Because the built-in kernel functions are compiled, you cannot view their procedure definitions. The builtin option is used to identify a kernel procedure.

You see this option when you evaluate a built-in procedure. Also, instead of displaying the source code, a number is displayed. This number uniquely identifies each built-in procedure. You cannot create built-in procedures.\(^8\)

For example, the add function is in the kernel.\(^9\)

```maple
> eval(add);
```

---

\(^7\)For more information about symbols in Maple, see Names on page 26 or refer to ?symbol. For more information about procedure options, refer to ?options.

\(^8\)For information on using external compiled code, see the call_external option in this section.

\(^9\)For more information about the Maple system and built-in functions, refer to ?builtin.
You can also use type-checking to determine if a Maple procedure is built-in. An expression is of type `builtin` if it is a procedure with `option builtin`. This type identifies procedures implemented in the Maple kernel, rather than the library.

```
> type(add, 'builtin');
true

> type(int, 'builtin');
false
```

**The call_external Option** To create a link to and call functions that are external to the Maple program, use the `define_external` and `call_external` functions, respectively. The `define_external` function returns a `call_external` procedure. That is, you do not need to construct a procedure with `option call_external`. For more information about defining procedures that call external functions, refer to chapter 6 of the *Maple Advanced Programming Guide* or `?define_external`.

**The Copyright Option** To add a copyright notice to a procedure, use `option Copyright`. Maple considers any option that begins with the word `Copyright` to be a copyright option. The body of a procedure with a copyright option is not printed unless the `interface` variable `verboseproc` is at least 2—this is accomplished with the command `interface(verboseproc=2)`. All Maple library routines have the copyright option.

```
f := proc(expr::anything, x::name)
  option 'Copyright (c) 1684 by G. W. Leibniz';
  Diff(expr, x);
end proc;
```

---

10 For more information about using the `interface` variable, see page 258 or refer to `?interface`. 
The inline Option  To create an inline Maple procedure, include `option inline` in the options field of the procedure definition. An inline Maple procedure avoids the overhead of a procedure call by executing the Maple instructions directly, as if it were written inline, rather than in a separate procedure. By avoiding the overhead of the procedure call, small performance improvements can be achieved. However, not all Maple procedures can be inline. Only procedures whose body consists of a single expression or an expression sequence can be inline—the body cannot consist of a statement or statement sequence. Other restrictions also apply. For more information about procedure inlining, refer to `?inline`.

The operator Option  Functional operators (or arrow operators) provide an alternate method for representing a mathematical function (or mapping) in Maple—they are a special form of a procedure. An arrow operator consists of a sequence of arguments (generally enclosed in parentheses) followed by an arrow (\(\rightarrow\)), and then the expression that defines the function.

\[
> (x, y) \rightarrow x^2 + y^2;
\]

\[(x, y) \rightarrow x^2 + y^2\]

If `option operator` is specified in a procedure definition, it identifies the procedure as a functional operator. If option `arrow` is used in conjunction with option `operator` in the procedure definition, the Maple `prettyprinter` prints the procedure using arrow notation. See also page 207.\(^\text{11}\)

\(^{11}\)For more information about functional operators, see page 231 or refer to `?operators,functional`.
Chapter 6: Maple Procedures

> f := proc(x, y)
>     option operator;
>     x^2 + y^2
> end proc;

\[ f := \text{proc}(x, y) \text{option operator}; x^2 + y^2 \text{end proc} \]

> f := proc(x, y)
>     option operator, arrow;
>     x^2 + y^2
> end proc;

\[ f := (x, y) \rightarrow x^2 + y^2 \]

The remember Option To associate a remember table with a procedure, include option remember in the list of procedure options. If a remember table is not explicitly associated with a procedure, the remember table for the procedure is the NULL expression. A remember table that is associated with a procedure can be manipulated as other Maple tables.\(^\text{12}\)

If you specify the remember option in the options field of a Maple procedure, after each invocation of the procedure, an entry that records the result for the specified arguments is made in the procedure’s remember table. For subsequent invocations of the procedure, Maple checks whether you have called the procedure with the same parameters. If so, Maple retrieves the previously calculated result from the remember table, instead of executing the procedure.

The use of option remember and remember tables can drastically improve the efficiency of recursively-defined procedures. For example, consider the following procedure that calculates the Fibonacci numbers.\(^\text{13}\)

\[ F := \text{proc}( \text{n :: nonnegint} ) \]
\[ \text{option remember; \quad \text{if n < 2 then \quad n \quad \text{else \quad Fib(n-1) + Fib(n-2) \quad end if;}} \]

\(^\text{12}\)You can also explicitly associate a remember table with a procedure by direct assignment to the table. For example, if f is the name of the procedure definition, the commands \( f(0):=0; f(2):=1; \) creates the remember table \( \text{table}([0=0, 2=1]) \) for procedure f. For more information about tables and remember tables in Maple, refer to \texttt{?table} or \texttt{?remember}.

\(^\text{13}\)For a comparison of run-time efficiency with option remember, see Profiling a Procedure on page 332.
6.2 Procedure Components

> end proc;

\[ Fib := \text{proc}(n::\text{nonnegint}) \]
\[ \text{option remember; if } n < 2 \text{ then } n \text{ else Fib}(n - 1) + \text{Fib}(n - 2) \text{ end if} \]
\[ \text{end proc} \]

> Fib(4);

3

If you enter \texttt{interface(verboseproc=3)}, and then view the procedure definition by using the \texttt{eval} command, the contents of its remember table are printed below its procedure definition.\(^{14}\)

> interface(verboseproc=3);

1

> eval(Fib);

\[ \text{proc}(n::\text{nonnegint}) \]
\[ \text{option remember; if } n < 2 \text{ then } n \text{ else Fib}(n - 1) + \text{Fib}(n - 2) \text{ end if} \]
\[ \text{end proc} \]

# (0) = 0
# (1) = 1
# (2) = 1
# (3) = 2
# (4) = 3

Since \texttt{Fib} has the \texttt{remember} option, if you invoke the procedure again with an argument greater than 4:

- The results for all calls to \texttt{Fib} with arguments less than or equal to 4 are read from the \texttt{Fib} remember table, and

\(^{14}\)An alternate way to view the remember table of a procedure that has option \texttt{remember} is to use \texttt{op(4, eval(f))}, where \texttt{f} is the name of the procedure definition. For more information, see \texttt{Procedure Operands} on page 237, or refer to \texttt{?procedure} or \texttt{?op}. 
All calls to Fib with arguments greater than 4 are computed and appended to the Fib remember table.

> Fib(7);

> eval(Fib);

```
proc(n::nonnegint)
option remember;
    if n < 2 then n else Fib(n - 1) + Fib(n - 2) end if
end proc
# (0) = 0
# (1) = 1
# (2) = 1
# (3) = 2
# (5) = 5
# (4) = 3
# (7) = 13
# (6) = 8
```

**The system Option** If option system is specified in conjunction with option remember in the options field of a Maple procedure, entries in the procedure’s remember table are removed during garbage collection, freeing the space occupied by the remember table and its entries.\(^\text{15}\)

```
> Fib := proc( n :: nonnegint )
    option remember, system;
    if n<2 then
    n
    else
    Fib(n-1) + Fib(n-2)
    end if;
> end proc;
```

\(^{15}\)For more information about garbage collection in Maple, see page 335 or refer to ?gc.
\[ Fib := \text{proc}(n::\text{nonnegint}) \]
\[
\text{option remember, system;} \\
\quad \text{if } n < 2 \text{ then } n \text{ end if} \\
\quad \text{else } Fib(n - 1) + Fib(n - 2) \text{ end if} \\
\text{end proc} \]

**The trace Option**  If the trace option is included in a procedure definition, Maple shows the entry and exit calls, and all the internal statements when the procedure is executed. This effect is independent of the value of the printlevel variable.\(^{16}\)

\[
> \text{Fib} := \text{proc}(n::\text{nonnegint}) \\
> \quad \text{option remember, system, trace;} \\
> \quad \text{if } n < 2 \text{ then} \\
> \quad \quad n \\
> \quad \text{else} \\
> \quad \quad \text{Fib}(n - 1) + \text{Fib}(n - 2) \\
> \quad \text{end if;} \\
> \text{end proc}: \\
> \text{Fib}(3); \\
\]

{\text{--> enter Fib, args = 3}}
\{\text{--> enter Fib, args = 2}}
\{\text{--> enter Fib, args = 1}}

\[ 1 \]

{\text{ <-- exit Fib (now in Fib) = 1}}
\{\text{--> enter Fib, args = 0}}

\[ 0 \]

{\text{ <-- exit Fib (now in Fib) = 0}}
\{\text{ <-- exit Fib (now in Fib) = 1}}
\text{value remembered (in Fib): Fib(1) -> 1}

\(^{16}\)For more information about tracing a procedure and printlevel, see chapter 8 or refer to ?trace and ?printlevel.
Procedure Description

```maple
proc( ... )
variable and option declarations
description D;
procedure body
end proc
```

The last part of the procedure header is the description field. It is used to specify lines of text that describe a procedure. Its use is optional. However, if used, it must be entered after local variables, global variables, and options, and before the body of the procedure. Descriptions are specified by using the following format, where D is any symbol or string.

```
description string ;
```

The description field has no effect on the execution of a procedure. It is used only for documentation purposes.

```maple
> f := proc(x :: integer, y :: integer)
> local a;
> description "compute the average of two integers";
> a := (x + y)/2;
> end proc:
> eval(f);
```

```maple
proc(x::integer, y::integer)
local a;
description "compute the average of two integers";
a := 1/2 * x + 1/2 * y
end proc
```

Maple prints the description field even if it does not print the body of a procedure because there is a copyright option.
> f := proc(x)
>     option 'Copyrighted ?';
>     description "computes the square of x";
>     x^2; # compute x^2
> end proc:
> eval(f);

proc(x)
description "computes the square of x"
end proc

You can include more than one line of text in the description field by splitting the description string into smaller strings.

> f := proc(x)
>     description "This example shows "
>     "how to include "
>     "a multi-line description.";
>     x^2;
> end proc:
> eval(f);

proc(x)
description "This example shows how to include a \ 
multi-line description."
end proc

Procedure Body Statements
The procedure body can contain any number of valid Maple statements. The only condition on the procedure body is that it must terminate with end proc.\(^\text{17}\)

\(^{17}\)For more information about statements to perform numeric, symbolic, and graphical computations, refer to the Maple Advanced Programming Guide.
if possible. This simplification is identical to that which Maple performs in interactive mode.

Consider the following procedure.

\[
\begin{align*}
> f := &\text{proc}(x) \\
> &\text{local } t; \\
> &t := x + 0*2; \\
> &\text{if true then} \\
> &\sqrt{t} \\
> &\text{else} \\
> &t^2 \\
> &\text{end if}; \\
> &\text{end proc;}
\end{align*}
\]

\[f := \text{proc}(x) \text{ local } t; t := x; \sqrt{t} \text{ end proc}\]

The multiplication by zero is omitted and the if statement simplifies to \sqrt{t}.

Because Maple tries to simplify the body of a procedure, you should use caution if you try to hard-code a floating-point constant in a procedure body. (This also applies when specifying floating-point constants in interactive mode.) Consider procedures \(F\) and \(G\).

\[
\begin{align*}
> F := &\text{proc}(x) x*(1/100000000000000000001) \text{ end proc;} \\
\end{align*}
\]

\[F := \text{proc}(x) 1/100000000000000000001 \ast x \text{ end proc}\]

\[
\begin{align*}
> G := &\text{proc}(x) x*(1/100000000000000000000001.0) \text{ end proc;} \\
\end{align*}
\]

\[G := \text{proc}(x) \\
\begin{align*}
&x \ast 1/0.100000000000000000000000010 \ast 10^21 \\
&\text{end proc}
\end{align*}\]

If these procedures are executed with Pi as the parameter and then evaluated to a floating-point result, procedure \(F\) works correctly, but procedure \(G\) does not because the reciprocal has already been computed at 10 Digits of precision. Notice the different results.\(^{18}\)

\[
\begin{align*}
> \text{evalf[200]}( F(\text{Pi}) );
\end{align*}
\]

\(^{18}\)For more information about floating-point numbers and Digits, see page 65 and refer to ?Digits.
6.4 Procedure Return Values

When a procedure is invoked, the value that Maple returns is normally the value of the last statement in the statement sequence in the body of the procedure. There are three other types of returns.

- explicit return
- error return
- return through a parameter\(^{19}\)

In addition to these types of returns, it is also possible for a procedure to return unevaluated.

Explicit Returns

An explicit return occurs when a return statement is executed inside a procedure. The return statement causes an immediate return to the point where the current procedure was invoked. It has the following syntax, where sequence is a sequence, set, or list of zero, one, or more expressions.

\[^{19}\text{It is possible to return a value through a parameter in a Maple procedure, but it is not recommended. Parameters in Maple procedures should be treated as input parameters—used for input, but not changed in the procedure. For more information, see page 220.}\]
Chapter 6: Maple Procedures

If the procedure executes the `return` statement, the return value is sequence.

**Example**  The following procedure computes the first position \( i \) of a value \( x \) in a list of values \( L \). If \( x \) is not in the list \( L \), the procedure returns 0.

```maple
> Position := proc(x::anything, L::list) local i;
>     for i to nops(L) do
>         if x=L[i] then
>             return i
>         end if;
>     end do;
>     0;
> end proc:
```

The following `GCD` procedure computes the greatest common divisor \( g \) of two integers, \( a \) and \( b \). It returns the sequence \( g, a/g, b/g \). Procedure `GCD` treats the case \( a = b = 0 \) separately because, in that case, \( g \) is zero.

```maple
> GCD := proc(a::integer, b::integer) local g;
>     if a=0 and b=0 then
>         return 0,0,0
>     end if;
>     g := igcd(a,b);
>     g, iquo(a,g), iquo(b,g);
> end proc:
> GCD(0,0);  
0, 0, 0
> GCD(12,8);
4, 3, 2
```

You can use the Boolean constants `true`, `false`, and `FAIL` in a `return` statement. In particular, `FAIL` is often used in a `return` statement to indicate that the computation failed or was abandoned.\(^{20}\)

\(^{20}\)For more information about `return` statements, refer to `?return` and `?FAIL`. 
> Division := proc(x, y)
> if y=0 then
> return FAIL
> else
> x/y
> end if;
> end proc:
> Division(2,0);
>
FAIL

Error Returns

An error return occurs when an error statement is executed inside a procedure. The error statement indicates that an exception has occurred, and the execution of the current statement sequence in the procedure is interrupted. Then, control is passed to the next applicable catch clause in the enclosing try...catch statement (if one exists) to test whether the error can be handled. If control is not passed to a catch clause at this level, or if the exception cannot be handled by a catch clause at this level, the exception is re-raised at the next higher execution handler level (in other words, at the next enclosing try...catch statement). This process continues until either the exception is handled or execution returns to the top level (in which case the exception becomes an error). The error statement has the following syntax, where msgString is the string Maple displays when the error occurs and msgParams are the parameters substituted into msgString.\(^{21}\)

\[
\text{error msgString} \\
\text{error msgString, msgParams}
\]

If an exception becomes an error (because the exception was not caught), msgString from the error statement associated with the exception is returned.\(^{22}\)

> MyDivision := proc(x, y)
> if y=0 then
> error "trying to divide by zero"
> else
> x/y
> end if;
>
\(^{21}\)For more information, refer to ?error.

\(^{22}\)For more information about error returns and handling exceptions, see page 323 or refer to ?error.
> end proc:
> MyDivision(2,0);

Error, (in MyDivision) trying to divide by zero

Returning Values through Parameters

You can write a Maple procedure that returns a value through a parameter. Though creating side effects such as this is possible in Maple, it is not recommended.

Consider the following Boolean procedure, Member, which determines whether a list L contains an expression x. Moreover, if you call Member with a third argument, p, the position of x in L is assigned to p.

> Member := proc(x::anything, L::list, p::evaln) local i;
    > for i to nops(L) do
    >      if x=L[i] then
    >         if nargs>2 then
    >            p := i
    >         end if;
    >         return true
    >      end if;
    >    end do;
    >    false
    > end proc:

If Member is called with two arguments, that is, nargs is 2, the part of the body of Member that is executed does not refer to the formal parameter, p. Therefore, Maple does not return an error.

> Member( x, [a,b,c,d] );

false

If Member is called with three arguments, the type declaration, p::evaln, ensures that Maple evaluates the third actual parameter to a name rather than by using full evaluation.\(^\text{23}\)

> q := 78;

q := 78

\(^{23}\)If the third parameter has not been declared as evaln, enclose the name q in single right quotes ('q') to ensure that the name, and not the value, of q is passed to the procedure.
6.4 Procedure Return Values

Maple evaluates the actual parameters only once—prior to substituting them for the formal parameters in a procedure call. This means that you cannot use formal parameters in the same manner as local variables in a procedure body. Once you have assigned a value to a formal parameter, you should not refer to that parameter again.\textsuperscript{24}

The \texttt{Count} procedure is an example of this behavior. Procedure \texttt{Count} determines whether a product of factors, \texttt{p}, contains an expression, \texttt{x}. If \texttt{p} contains \texttt{x}, \texttt{Count} returns the number of factors that contain \texttt{x} in the third parameter, \texttt{n}.

```
> Count := proc(p::'*', x::name, n::evaln) local f;
>     n := 0;
>     for f in p do
>         if has(f,x) then
>             n := n+1
>         end if;
>     end do;
>     evalb( n>0 );
> end proc:
```

This version of the \texttt{Count} procedure does not work as intended.

```
> Count(2*x^2*exp(x)*y, x, m);
0 < m
```

The value of the formal parameter, \texttt{n}, inside the procedure is always \texttt{m}, the actual parameter that Maple determines when you invoke the procedure. Thus, when execution reaches the \texttt{evalb} statement, the value of \texttt{n} is the name \texttt{m}, and not the value of \texttt{m}. Even worse, if you evaluate \texttt{m} one level, you can see that the \texttt{n:=n+1} statement assigns to \texttt{m} the name \texttt{m+1}.

```
> eval(m, 1);
```

\textsuperscript{24} For a simple example, see page 200.
The $m$ in the previous result also has the value $m+1$.

```maple
> eval(m, 2);
```

$m + 2$

Therefore, if $m$ is fully evaluated, Maple enters an infinite loop and returns an error message.

```maple
> eval(m);
```

_Error, too many levels of recursion_

This example shows that, in general, it is _not_ recommended that you use parameters to return values in Maple procedures.

**Returning Unevaluated**

If a procedure cannot perform a computation, the unevaluated function invocation may be returned. For example, procedure `Max` calculates the maximum of two numbers, $x$ and $y$.

```maple
> Max := proc(x,y)
> 
> if x>y then
> x
> else
> y
> end if
> end proc:
```

This version of `Max` requires that its arguments are numerical values.

```maple
> Max(3.2, 2);
```

3.2

```maple
> Max(x, 2*y);
```

_Error, (in Max) cannot determine if this expression is true or false: 2*y < x_
Furthermore, the absence of symbolic capabilities in \texttt{Max} causes problems if you try to plot expressions involving \texttt{Max}.

\begin{verbatim}
> plot( Max(x, 1/x), x=1/2..2 );
Error, (in Max) cannot determine if this expression is true or false: 1/x < x
\end{verbatim}

The error occurs because Maple evaluates \texttt{Max(x, 1/x)} before the \texttt{plot} command is invoked. The solution is to make \texttt{Max} return unevaluated when its parameters, \texttt{x} and \texttt{y}, are not numeric. That is, in such cases \texttt{Max} should return \texttt{'Max'(x,y)}.

\begin{verbatim}
> Max := proc(x, y)
> if type(x, numeric) and type(y, numeric) then
>     if x>y then
>         x
>     else
>         y
>     end if;
> else
>     'Max'(x,y);
> end if;
> end proc:
\end{verbatim}

The new version of \texttt{Max} handles both numeric and non-numeric input.

\begin{verbatim}
> Max(3.2, 2);
3.2
> Max(x, 2*y);
Max(x, 2*y)
> plot( Max(x, 1/x), x=1/2..2 );
\end{verbatim}
You can improve \texttt{Max} so that it can find the maximum of any number of arguments. Inside the revised procedure, \texttt{args} is the sequence of actual parameters, \texttt{nargs} is the number of actual parameters, and \texttt{procname} is the name of the procedure.

\begin{verbatim}
> Max := proc() local m, i;
> m := -infinity;
> for i in \texttt{(args)} do
> if not type(i, numeric) then
> return \texttt{'}\texttt{procname'}\texttt{(args)}\texttt{'};
> end if;
> if i>m then
> m := i
> end if;
> end do;
> m;
> end proc:
> Max(3,1,4);

4

> Max(3,x,1,4);

\texttt{Max(3, x, 1, 4)}
\end{verbatim}

The \texttt{sin} function and \texttt{int} command (integration) follow the same model as the \texttt{Max} procedure. If Maple can compute the result, it is returned. Otherwise, \texttt{sin} and \texttt{int} return unevaluated.
6.5 Documenting Your Procedures

To minimize the time required to fix code, it is recommended that you annotate code with comments. To improve the readability of the code, include indentation. Other procedure documentation methods include providing a copyright notice, indicating the procedure’s purpose, and writing an online help page.

Indicating Copyright

The Copyright option is used to indicate a copyright notice for a procedure. For more information, see page 208 or refer to ?options.

Formatting Code for Readability

In chapter 1, it was shown how you can enter a procedure in a Maple session by entering its definition on one line.

\[
> f := \text{proc}(\text{x :: integer, y :: integer}) \text{ local a;} \\
> \text{description "compute the average of two integers";} \text{ a:=} \\
> (\text{x + y})/2; \text{end proc:}
\]

However, for readability, it is recommended that you enter the procedure on multiple lines.

Entering Multi-line Procedure Definitions

To enter a procedure definition on more than one line, hold the SHIFT key and press ENTER at the end of each line. You can indent any of the lines in the procedure by using the spacebar. After you enter the last line, which contains end proc, press the ENTER key.

\[
> f := \text{proc}(\text{x :: integer, y :: integer}) \text{ local a;} \\
> \text{description "compute the average of two integers";} \\
> a:= (\text{x + y})/2; \\
> \text{end proc:}
\]

Using Line Continuation

Occasionally, it is not possible to fit a statement on one line. In these cases, use the Maple line continuation character, the backslash (\). For more information, see page 47.

Including Indentation

Formatting code increases readability. However, there are no standard rules for when and how to indent procedure statements. The following are some general guidelines.

---

\[25\text{As described in chapter 1, you can enter Maple commands over multiple lines in interactive mode.}\]
• Enter every statement on a distinct line.

• Use indentation to indicate that the following line or block of code is a subblock of an if or loop statement. Use two spaces for each level of indentation. For more information about if statements and loops, see chapter 5 or refer to ?if and ?do.

```maple
> f := proc(x)
>     local a;
>     description "odd or even integer";
>     a := x/2;
>     if type(a, integer) then
>       "even"
>     else
>       "odd"
>     end if;
> end proc:
```

• Enter long comments on a separate line.

• Use continuation lines to ensure that no line exceeds 80 characters in length, or to break a long sequence of digits into groups of smaller sequences to enhance readability.

### Adding Comments

The pound sign (#) marks the characters following it on a line as a comment. In other words, Maple does not process anything on the line after the #. If you use the eval command to view the contents of a procedure definition, any comments in the definition are not displayed.

```maple
> f := proc(x)
>     x^2; # compute x^2
> end proc;
> eval(f);
```

```maple
proc(x) x^2 end proc
```

### Including a Procedure Description

The description field is used to indicate the purpose of a procedure. Unlike a comment, which Maple omits when displaying the code of a procedure, the description field provides a way to attach a one-line comment to a procedure when its code is viewed. For more information, see page 214 or refer to ?procedure.
6.6 Saving and Retrieving Procedures

As you develop a new procedure, you can save your work by saving the worksheet. When you have finished, save (only) the procedure in a file by using the save command.26

26For more detailed information about Maple I/O features, see chapter 7, or refer to ?read and ?save.
Chapter 6: Maple Procedures

The read command reads files that contain Maple statements back into a Maple session. The statements in the file are read and executed as if they were being entered directly into a Maple session.27

```maple
> read "myfile";

proc(x::complex(numeric), y::complex(numeric))
    if abs(y) < abs(x) then x else y end if
end proc
```

You can also use the read command to read Maple procedures written in your favorite text editor.

6.7 Viewing Maple Library Source Code

An important feature of Maple is its open architecture. You can access the source code of most Maple library routines (only the code for the built-in kernel routines cannot be viewed). This accessibility is helpful for learning the Maple programming language, and it is essential for enhancing existing routines to meet your specific needs. Prior to learning the commands to display the source code, it is recommended that you learn the evaluation rules for procedures.28

**Special Evaluation Rules for Procedures**

For most named objects in Maple, you can obtain the actual value of the name simply by referring to it.

```maple
> a := 3;

a := 3

> a;

3
```

---

27 When using the read command, the statements are not echoed to the display unless interface(echo) is set to 2 or higher. For more information, refer to ?interface.

28 For more information about the built-in kernel routines, see page 207 or refer to ?builtin.
In a chain of assignments, each name is fully evaluated to the last assigned expression.

```
> c := b:
> b := a:
> a := 1:
> c;
```

1

This is called *full evaluation*.

For a procedure (like a table), Maple displays only the name, not its value (the procedure definition). For example, in the previous section, \( f \) was defined as a procedure. If you try to view the body of procedure \( f \) by referring to it by name, the procedure definition is *not* displayed.

```
> f := proc( x ) x^2 end proc:
> f;
```

\( f \)

This model of evaluation that hides the procedure details is called *last name evaluation*. This approach is used because procedures can contain many subobjects. To obtain the true value of the name \( f \), use the `eval` command, which forces full evaluation:\(^{29}\)

```
> eval(f);
```

\( \text{proc}(x) \ x^2 \ 	ext{end proc} \)

### Displaying Maple Library Source Code

The `interface` routine is the mechanism by which the Maple computational engine communicates with the user interface. In particular, `interface` is used to set and query all options that affect the format of the output, but do not affect the computation. To view the source code for Maple library procedures, set the `interface` option `verboseproc` to 2.

```
> interface(verboseproc=2);
```

\(^{29}\)Last name evaluation applies to procedures, tables, and modules in Maple. For more information, refer to `?last_name_eval`. 
Then, to view the code for any Maple library routine that is not built-in or local to a module, use the name of the procedure as the argument to the `print` command. For example, use the following command to view the source code for the `blackscholes` routine in the `finance` package.

```maple
> print(finance[blackscholes]);
```

```maple
proc(Amount, Exercise, Rate, Nperiods, Sdev, Hedge)
local d1, d2, Nd1, Nd2;
option 'Copyright (c) 1995 by Waterloo Maple Inc.
All rights reserved.';

  d1 := (ln(Amount/Exercise) + (Rate + 1/2 * Sdev^2) * Nperiods)/(Sdev * Nperiods^(1/2));
  d2 := d1 - Sdev * Nperiods^(1/2);
  Nd1 := 1/2 * erf(1/2 * d1 * 2^(1/2)) + 1/2;
  Nd2 := 1/2 * erf(1/2 * d2 * 2^(1/2)) + 1/2;
  if nargs = 6 then Hedge := Nd1 end if;
  Amount * Nd1 - Exercise * exp(-Rate * Nperiods) * Nd2
end proc
```

Once you have viewed the code for a procedure, reset the interface variable to the default output level.

```maple
> interface(verboseproc=1):
```

To view the local routines of a module, set `kernelopts(opaquemodules=false)`. For more information, refer to `?kernelopts`.

---

Footnotes:

30 Viewing the source code in this manner does not display any procedure documentation.

31 You must set the `interface` option `verboseproc` because Maple library routines use the `copyright` option, which suppresses the display of the procedure body. User-defined procedures generally do not use this option. Hence, setting the `interface` option is not necessary to view user-defined procedures. To view the code of a user-defined procedure, use the `eval` command.
6.8 Alternate Methods for Defining a Procedure

Encapsulating statements in a **proc...end proc** construct is not the only way to create a procedure in Maple. Alternatively, you can use functional operators or the `unapply` command.

Functional Operators: Mapping Notation

Using a functional operator (arrow operator) is another method by which you can create a special form of a procedure in Maple, which represents a mathematical function (or mapping).

\[ (P) \rightarrow B \]

The sequence, \( P \), of formal parameters can be empty and the body, \( B \), of the procedure must be a *single* expression or if statement.

\[
> F := (x,y) \rightarrow x^2 + y^2; \\
F := (x, y) \rightarrow x^2 + y^2
\]

If the procedure requires only one parameter, you can omit the parentheses around the formal parameter.

\[
> G := n \rightarrow \text{if } n<0 \text{ then } 0 \text{ else } 1 \text{ end if}; \\
G := n \rightarrow \text{if } n < 0 \text{ then } 0 \text{ else } 1 \text{ end if}
\]

Invoking a function definition is similar to invoking a procedure.

\[
> F(1, 2); \\
5
\]

\[
> G(-1); \\
0
\]

You can also use declared parameters with the functional operator.

\[
> H := (n::even) \rightarrow n! * (n/2)!; \\
H := n::even \rightarrow n!(\frac{1}{2} n)!
\]
Chapter 6: Maple Procedures

> H(6);

4320

> H(5);

Error, invalid input: H expects its 1st argument, n, to be of type even, but received 5

The arrow notation is designed for simple one-line function definitions. It does not provide a mechanism for specifying local or global variables, or options. If these are required, use the `proc...end proc` construct.\(^\text{32}\)

**The unapply Command**

Another way to create a function in Maple is by using the `unapply` command.

\[
\text{unapply}(B, P);
\]

The sequence, \(P\), represents the formal parameters, and \(B\) represents the body of the function.\(^\text{33}\)

\[
> B := x^2 + y^2:
> F := \text{unapply}(B, x, y);
F := (x, y) \mapsto x^2 + y^2
\]

> F(1, 2);

5

**Unnamed Procedures and the map Command**

Procedures are valid Maple expressions. Therefore, you can create, manipulate, and invoke any procedure definition without assigning it to a name.\(^\text{34}\)

Consider the following mapping.

\(^{32}\)For more information about functional operators, see page 209 or refer to \texttt{?functional}.

\(^{33}\)For more information about the \texttt{unapply} command, refer to \texttt{?unapply}.

\(^{34}\)For information about named procedures, see page 194.
6.8 Alternate Methods for Defining a Procedure

> x -> x^2;

\( x \rightarrow x^2 \)

You can invoke this unnamed procedure in the following manner.

> (x -> x^2)(t);

\( t^2 \)

You can use the same method to invoke an unnamed procedure created by using the `proc...end proc` construct.

> proc(x,y) x^2 + y^2 end proc(u,v);

\( u^2 + v^2 \)

When using the `map` command, you can use unnamed procedures.

> map( x -> x^2, [1,2,3,4] );

\([1, 4, 9, 16]\)

You can add procedures, or, if appropriate, process them by using commands such as the differential operator, D.

> D( x -> x^2 );

\( x \rightarrow 2x \)

> F := D(exp + 2*ln);

\( F := \exp + 2(z \rightarrow \frac{1}{z}) \)

You can apply the result, F, directly to arguments.

> F(x);

---

35 For more information on the `map` command, see page 180.

36 For more information about the differential operator, refer to `?D`. 
Building a List of Arguments

When you enter a procedure definition, it is not always necessary to supply names for the formal parameters (see page 196). You can access the sequence of arguments (actual parameters) in the procedure by using the special name \texttt{args}.

For example, the following procedure builds a list of its arguments.

\begin{verbatim}
> f := proc()
> [args]
> end proc;
\end{verbatim}

\begin{verbatim}
f := proc() [args] end proc
\end{verbatim}

\begin{verbatim}
> f(a,b,c);
[a, b, c]
\end{verbatim}

\begin{verbatim}
> f(c);
[c]
\end{verbatim}

\begin{verbatim}
> f();
[]
\end{verbatim}

The $i$th argument is \texttt{args[i]}. Therefore, the following two procedures are equivalent, provided they are called with at least two actual parameters of type numeric.

\begin{verbatim}
> Max := proc(x::numeric,y::numeric)
> if x > y then
> x
> else
> y
> end if;
> end proc;
\end{verbatim}
6.8 Alternate Methods for Defining a Procedure

\[Max := \text{proc}(x::\text{numeric}, y::\text{numeric})\]
\[\text{if } y < x \text{ then } x \text{ else } y \text{ end if}\]
\end proc

\> Max := proc()
\> \quad \text{if } \text{args}[1] > \text{args}[2] \text{ then}
\> \quad \quad \text{args}[1]
\> \quad \quad \text{else}
\> \quad \quad \text{args}[2]
\> \quad \text{end if;}
\> \quad \text{end proc;}

\[Max := \text{proc}()\]
\[\text{if } \text{args}_2 < \text{args}_1 \text{ then } \text{args}_1 \text{ else } \text{args}_2 \text{ end if}\]
\end proc

The special name \text{nargs} is assigned the total number of actual parameters passed to a procedure. Using \text{nargs}, it is possible to write a procedure, \text{Max}, that finds the maximum of any number of arguments, but does not have any formal parameters declared in the procedure header.

\> Max := proc()
\> \quad \text{local } i, m;
\> \quad \text{if } \text{nargs} = 0 \text{ then}
\> \quad \quad \text{return } \text{-infinity}
\> \quad \text{end if;}
\> \quad m := \text{args}[1];
\> \quad \text{for } i \text{ from 2 to } \text{nargs} \text{ do}
\> \quad \quad \text{if } \text{args}[i] > m \text{ then}
\> \quad \quad \quad m := \text{args}[i]
\> \quad \quad \text{end if;}
\> \quad \text{end do;}
\> \quad m;
\> \text{end proc:}

Find the maximum of the three values 2/3, 1/2, and 4/7.

\> Max(2/3, 1/2, 4/7);
\[
\begin{array}{c}
\frac{2}{3}
\end{array}
\]

Find the maximum of the four values 1.3, 4/3, 7/5, and 9/7.

\> Max(1.3, 4/3, 7/5, 9/7);
The use of `nargs` is a convenient way of creating a procedure without specifying the number of formal parameters in the procedure definition.

### 6.9 The Procedure Object

The first part of this chapter described the syntax and semantics of a Maple procedure. This section describes the procedure object, its type and operands.

**The procedure Type**

Maple recognizes all procedures (including those created by using the mapping notation and any names to which you assign procedures) as being of type `procedure`. To verify whether a name or a statement is a procedure, use the `type` command.

```maple
> F := proc(x) x^2 end proc:
> type(F,name);

true

> type(F,procedure);

true

> type(F,name(procedure));

true

> type(eval(F),procedure);

true
```

As a consequence, you can use the following test to ensure that `F` is the name of a procedure.\(^\text{37}\)

---

\(^{37}\)For more information about `if` statements, see page 167 or refer to `?if`. 

The procedure type is \textit{structured}. Therefore, you can also perform the following types of tests:\footnote{For more information about structured types in Maple, see page 106 or refer to \texttt{?structured}.}

\begin{verbatim}
> G := proc( n::integer, s::string ) print(s); 2*n end proc:
> type( G, 'procedure(integer, string)' );
true
\end{verbatim}

**Procedure Operands**

Every Maple procedure has seven operands (the value for any of the operands can be \texttt{NULL}). Table 6.1 lists these operands and the corresponding \texttt{op} command that you can use to access them. In the \texttt{op} commands, \texttt{procName} is the name of the procedure.

**Example** Consider the following procedure.

\begin{verbatim}
> f := proc(x::name, n::posint)
    local i;
    global y;
    option Copyright;
    description "a summation"
    sum( x[i] + y[i], i=1..n );
> end proc:
\end{verbatim}
Chapter 6: Maple Procedures

The following statements indicate how to access the various parts of a procedure definition.\textsuperscript{39,40}

\begin{verbatim}
> f;  #name of the procedure

f

> eval(f);  #procedure

proc(x::name, n::posint)

description "a summation"
...
end proc

> op(1, eval(f));  #formal parameters

x::name, n::posint

> op(2, eval(f));  #local variables

i

> op(3, eval(f));  #options

Copyright

> f(t,3) := 12;  #place an entry in the remember table

f(t, 3) := 12

> op(4, eval(f));  #remember table

table([[(t, 3) = 12]])

> op(5, eval(f));  #description

"a summation"
\end{verbatim}

\textsuperscript{39}For more information about remember tables in procedures, see page 210.

\textsuperscript{40}The lexical table of a procedure stores information about lexically-scoped variables. For more information, refer to ?examples,lexical.
6.10 Using Data Structures to Solve Problems

When writing procedures you must decide how to represent the data. Sometimes the choice is straightforward, but often it requires considerable thought and planning. The appropriate choice of data structure can make your procedures more efficient, and easier to write and debug. In chapter 4, you were introduced to the basic data structures, such as sequences, lists, and sets, and shown how to use them in various Maple commands. This section illustrates, by means of examples, how such data structures are useful in writing procedures and solving problems.

**Computing an Average**

A common problem is to write a procedure that computes the average of $n > 0$ data values $x_1, x_2, \ldots, x_n$ according to the following equation.

$$
\mu = \frac{1}{n} \sum_{i=1}^{n} x_i.
$$

Before the procedure is written, think about which data structure and Maple commands to use. You can represent the data for this problem as

\[ \text{op(6, eval(f)); #global variables} \]

\[ \text{y} \]

\[ \text{op(7, eval(f)); #lexical table is NULL} \]

Alternatively, you can list all of the operands of a procedure with one command.

\[ \text{op(eval(f));} \]

\[ x::\text{name}, n::\text{posint}, i, Copyright, \text{table}([(t, 3) = 12]), \]

\[ \text{“a summation”, y} \]

**Note:** The body of a procedure is not one of its operands, so you cannot use the op command to access it.

\[ \text{op(6, eval(f));} \]

\[ y \]

\[ \text{op(7, eval(f)); #lexical table is NULL} \]

**Note:** The body of a procedure is not one of its operands, so you cannot use the op command to access it.
a list. The \texttt{nops} command returns the total number of entries in a list \texttt{X}, while the \texttt{i}th entry of the list is found by using \texttt{X[i]}.

\begin{verbatim}
> X := [1.3, 5.3, 11.2, 2.1, 2.1];

X := [1.3, 5.3, 11.2, 2.1, 2.1]

> nops(X);

5

> X[2];

5.3
\end{verbatim}

You can add the numbers in a list by using the \texttt{add} command.

\begin{verbatim}
> add( i, i=X );

22.0
\end{verbatim}

Using these ideas, write the procedure \texttt{Average} which computes the average of the entries in a list. It handles empty lists as a special case.

\begin{verbatim}
> Average := proc(X::list) local n, i, total;
>     n := nops(X);
>     if n=0 then
>         error "empty list"
>     end if;
>     total := add(i, i=X);
>     total / n;
> end proc:
\end{verbatim}

Using this procedure you can find the average of list \texttt{X} defined above.

\begin{verbatim}
> Average(X);

4.400000000
\end{verbatim}

The procedure also works if the list contains symbolic entries.

\begin{verbatim}
> Average( [ a, b, c ] );

\frac{1}{3}a + \frac{1}{3}b + \frac{1}{3}c
\end{verbatim}
**Testing for Membership**

You can write a procedure that determines whether a certain object is an element of a list or a set. Procedure `Member` accomplishes this task by using the `return` statement discussed on page 217.\(^{42}\)

\[
> \text{Member} := \text{proc}(\ a::\text{anything},\ L::\{\text{list, set}\}\ )\ \text{local}\ i; \\
> \ \ \ \ \text{for}\ i\ \text{from}\ 1\ \text{to}\ \text{nops}(L)\ \text{do} \\
> \ \ \ \ \ \ \ \ \text{if}\ a=L[i]\ \text{then} \\
> \ \ \ \ \ \ \ \ \ \ \ \ \ \ \text{return}\ \text{true} \\
> \ \ \ \ \ \ \ \ \ \ \end\text{if}; \\
> \ \ \ \ \ \ \ \ \ \ \end\text{do}; \\
> \ \ \ \ \ \ \ \ \ \ \text{false}; \\
> \ \ \end\text{proc};
\]

Test procedure `Member` on a list.

\[
> \text{MyList} := [1,2,3,4,5,6];
\]

\[
\text{MyList} := [1, 2, 3, 4, 5, 6]
\]

Use `Member` to show that 3 is a member of `MyList`.

\[
> \text{Member}(\ 3,\ \text{MyList}\ );
\]

\[
\text{true}
\]

The type of loop that `Member` uses occurs so frequently that Maple has a special version of the `for` loop for it. Compare the previous version of `Member` to the one below.\(^{43}\)

\[
> \text{Member} := \text{proc}(\ a::\text{anything},\ L::\{\text{list, set}\}\ )\ \text{local}\ i; \\
> \ \ \ \ \text{for}\ i\ \text{in}\ L\ \text{do} \\
> \ \ \ \ \ \ \ \ \text{if}\ a=i\ \text{then} \\
> \ \ \ \ \ \ \ \ \ \ \ \ \ \ \text{return}\ \text{true} \\
> \ \ \ \ \ \ \ \ \ \ \end\text{if}; \\
> \ \ \ \ \ \ \ \ \ \ \end\text{do}; \\
> \ \ \ \ \ \ \ \ \ \ \text{false}; \\
> \ \ \end\text{proc};
\]

Test the new `for` loop version of the procedure `Member` on a set.

\[
> \text{myset} := \{1,2,3,4\};
\]

---

\(^{42}\) Instead of using procedure `Member`, you can use the built-in `member` command.

\(^{43}\) For more information about loops, see 5.2 Repetition or refer to `?for`. 
Performing a Binary Search

One of the most basic and well-studied computing problems is that of searching. A typical problem involves searching a list of words (a dictionary, for example) for a specific word \( w \). There are many possible methods. One approach is to search the list by comparing each word in the dictionary with \( w \) until Maple either finds \( w \) or it reaches the end of the list.

Study the code for procedure \texttt{Search} (the first attempt at solving this problem).

\begin{verbatim}
> Search := proc(Dictionary::list(string), w::string) local x;
    > for x in Dictionary do
    > if x=w then
    >   return true
    > end if
    > end do;
    > false
> end proc:
\end{verbatim}

Unfortunately, if \texttt{Dictionary} is large (say 50,000 entries), this approach can take a long time.

You can reduce the execution time required by sorting the dictionary before you search it. If you sort the dictionary into ascending order, then you can stop searching as soon as you encounter a word greater than \( w \). On average, it is necessary to search half the dictionary.

Binary searching provides an even better approach. Check the word in the middle of the sorted dictionary. Since the dictionary is already sorted, you can determine whether \( w \) is in the first or the second half. Repeat the process with the appropriate half of the dictionary until \( w \) is found, or it is determined not to be in the dictionary.

The procedure \texttt{BinarySearch} searches the dictionary \( D \) for the word \( w \) from position \( s \) to position \( f \) in \( D \). It also uses the built-in \texttt{lexorder} command to determine the lexicographical ordering of two strings.

\begin{verbatim}
> BinarySearch := proc(D::list(string), w::string, s::integer,
    >    f::integer) local m;
    > if s>f then
    >   return false
    > end if; # entry was not found
\end{verbatim}
6.10 Using Data Structures to Solve Problems

\[ m := \text{iquo}(s+f+1, 2); \ # \text{midpoint of D} \]
\[ \text{if } w = D[m] \text{ then} \]
\[ \quad \text{true;} \]
\[ \text{elif lexorder}(w, D[m]) \text{ then} \]
\[ \quad \text{BinarySearch}(D, w, s, m-1); \]
\[ \text{else} \]
\[ \quad \text{BinarySearch}(D, w, m+1, f); \]
\[ \text{end if}; \]
\[ \text{end proc}; \]

Test \texttt{BinarySearch} on a short dictionary.

\[ \text{Dictionary} := [ \text{"induna"}, \text{"ion"}, \text{"logarithm"}, \text{"meld"} ]; \]

\[ \text{Dictionary} := [\text{"induna"}, \text{"ion"}, \text{"logarithm"}, \text{"meld"}] \]

Try searching the dictionary.

\[ \text{BinarySearch( Dictionary, } \text{"hedgehogs"}, 1, \text{nops(Dictionary)} ); \]

\[ \text{false} \]

\[ \text{BinarySearch( Dictionary, } \text{"logarithm"}, 1, \text{nops(Dictionary)} ); \]

\[ \text{true} \]

\[ \text{BinarySearch( Dictionary, } \text{"melody"}, 1, \text{nops(Dictionary)} ); \]

\[ \text{false} \]

**Plotting the Roots of a Polynomial**

You can construct lists of any type of object, including other lists. A list that contains two numbers often represents a point in the plane. The Maple \texttt{plot} command uses this structure to generate plots of points and lines.\footnote{For more information about plotting in Maple, refer to the \textit{Maple User Manual} or \texttt{?plot}. For information about graphics programming, refer to chapter 5 of the \textit{Maple Advanced Programming Guide}.}
You can use this approach to write a procedure that plots the complex roots of a polynomial.

**Example**  Consider the polynomial $x^3 - 1$.

\[
x^3 - 1\]

First, find the roots of this polynomial. You can find the numeric solutions to this polynomial by using `fsolve`.

\[
R := \{ fsolve(y=0, x, complex) \};
\]

\[
R := \{ -0.5000000000 - 0.8660254038 I, \\
-0.5000000000 + 0.8660254038 I, 1. \}
\]

Next, change this list of complex numbers into a list of points in the plane. The `Re` and `Im` commands return the real and imaginary parts of a complex number, respectively.

\[
points := map( z -> [Re(z), Im(z)], R );
\]

\[
points := \{ [-0.5000000000, -0.8660254038], \\
[-0.5000000000, 0.8660254038], [1., 0.] \}
\]

Plot the points.
Using the sequence of steps in the previous example, you can write a procedure to automate the process. The input must be a polynomial in \( x \) with constant coefficients.

```maple
RootPlot := proc( p::polynom(constant, x) ) local R, points;
    R := [ fsolve(p, x, complex) ];
    points := map( z -> [Re(z), Im(z)], R );
    plot( points, style=point, symbol=point, color=black );
end proc:
```

Test procedure `RootPlot` by plotting the roots of the polynomial \( x^6 + 3x^5 + 5x + 10 \).

```maple
> RootPlot( x^6+3*x^5+5*x+10 );
```

Generate a random polynomial by using the `randpoly` command, and then test procedure `RootPlot` again.

```maple
> y := randpoly(x, degree=100);

\[ y := -56x^{95} - 62x^{42} + 97x^8 - 73x^5 - 4x^3 \]
Connected Graphs
Suppose that there is a number of cities, some with roads between them. You want to determine whether travel by road between any two of the cities is possible.

You can express this problem in terms of graph theory by using the Maple `networks` package. You do not need to understand graph theory or the `networks` package to benefit from the examples in this section. The `networks` package is used primarily as a shortcut for drawing graphs.

Example  First, load the `networks` package.

```maple
> with(networks):
```

Create a new graph `G` using the `new` command and add a few cities (or vertices, in the terminology of graph theory).

```maple
> new(G):
> cities := {Zurich, Rome, Paris, Berlin, Vienna};
```

```maple
cities := {Rome, Zurich, Paris, Berlin, Vienna}
```

```maple
> addvertex(cities, G);
```

`Rome, Zurich, Paris, Berlin, Vienna`

To add roads between Zurich and each of Paris, Berlin, and Vienna, use the `connect` command. The roads are automatically named `e1`, `e2`, and `e3`.

```maple
> connect( {Zurich}, {Paris, Berlin, Vienna}, G );
```
Similarly, add roads between Rome and Zurich, and between Berlin and both Paris and Vienna.

\[
\begin{align*}
\text{connect( \{Rome\}, \{Zurich\}, G);} \\
\text{connect( \{Berlin\}, \{Vienna, Paris\}, G);} \\
\end{align*}
\]

Draw the graph of G.

\[
\text{draw(G)};
\]

From the drawing, it is obvious that you can travel by road between any two cities. You can also use the \texttt{connectivity} command to determine this fact.

\[
\text{evalb( connectivity(G) > 0 );}
\]

true

The data structures that the \texttt{networks} package uses are complicated because the package supports more general structures than this example requires. You must decide what data structure to use to determine how the cities and roads can best be represented in this example.
Representing the Data as a Set  Since cities have distinct names and
the order of the cities is irrelevant, represent the cities as a set of names.

\begin{verbatim}
> vertices(G);
\end{verbatim}

\begin{verbatim}
{Rome, Zurich, Paris, Berlin, Vienna}
\end{verbatim}

The networks package assigns distinct names to the roads, so it can
also represent these roads as a set of names.

\begin{verbatim}
> edges(G);
\end{verbatim}

\begin{verbatim}
{e2, e1, e3, e4, e6, e5}
\end{verbatim}

You can also represent a road as the set consisting of the two cities
that the road connects.

\begin{verbatim}
> ends(e2, G);
\end{verbatim}

\begin{verbatim}
{Zurich, Berlin}
\end{verbatim}

Thus, you can represent the roads as a set of sets.

\begin{verbatim}
> roads := map( ends, edges(G), G);
\end{verbatim}

\begin{verbatim}
roads := {{Rome, Zurich}, {Zurich, Paris},
          {Zurich, Berlin}, {Zurich, Vienna}, {Paris, Berlin},
          {Berlin, Vienna}}
\end{verbatim}

However, to determine which cities are directly connected to a city,
you must search the whole set of roads. Therefore, representing the data
as a set of cities and a set of roads is computationally inefficient for
determining whether you can travel between any two cities.

Representing the Data as an Adjacency Matrix  Instead of using a set,
try representing the data as an adjacency matrix—a square matrix with
a row for each city. The \((i, j)\)th entry in the matrix is 1 if the \(i\)th and
the \(j\)th city have a road between them, and 0 otherwise. The following is
the adjacency matrix for the graph \(G\).

\begin{verbatim}
> adjacency(G);
\end{verbatim}
The adjacency matrix is an inefficient representation if few roads exist relative to the number of cities. In that case, the matrix contains many zeros, representing an overall lack of roads. Also, though each row in the matrix corresponds to a city, you cannot tell which row corresponds to which city.

Representing the Data as a Table  Another way to represent the cities and roads is to consider neighboring cities: Paris has roads between it and both Zurich and Berlin. Thus, Berlin and Zurich are the neighbors of Paris.

> neighbors(Paris, G);

\{Zurich, Berlin\}

You can represent the data as a table of neighbors; there is one entry in the table for each city.

> T := table( map( v -> (v)=neighbors(v,G), cities ) );

\[
T := \begin{array}{c}
\text{Zurich} = \{\text{Rome, Paris, Berlin, Vienna}\}, \\
\text{Rome} = \{\text{Zurich}\}, \\
\text{Vienna} = \{\text{Zurich, Berlin}\}, \\
\text{Berlin} = \{\text{Zurich, Paris, Vienna}\}, \\
\text{Paris} = \{\text{Zurich, Berlin}\}
\end{array}
\]

The representation of a system of cities and roads as a table of neighbors is ideally suited to answering the question of whether it is possible to travel between any two cities. Start at one city and then use the table to efficiently find the cities to which you can travel. Similarly, you can find the neighbors of the neighbors. Thus, you can quickly determine how far you can travel.

You can write a procedure that determines whether you can travel between any two cities. The procedure can use the indices command to extract the set of cities from the table.
Since the \texttt{indices} command returns a sequence of lists, you must use the \texttt{op} and \texttt{map} command to generate a set.
\begin{verbatim}
> map( op, {%} );
\end{verbatim}

\{\texttt{Rome, Zurich, Paris, Berlin, Vienna}\}

Write a procedure \texttt{Connected} that determines whether you can travel between any two cities and uses the \texttt{indices} command.

The \texttt{Connected} procedure initially visits the first city, \(v\). It adds \(v\) to the set of cities that it has visited and the neighbors of \(v\) to the set of cities to which it can travel. It visits the neighbors of \(v\) and repeats the process. When \texttt{Connected} has no more new cities to which it can travel, it determines whether it has visited all the cities.

\begin{verbatim}
> Connected := proc( T::table ) local canvisit, seen, v, V;
> V := map( op, {indices(T)} );
> seen := {};
> canvisit := { V[1] };
> while canvisit <> {} do
> v := canvisit[1];
> seen := seen union {v};
> canvisit := ( canvisit union T[v] ) minus seen;
> end do;
> evalb( seen = V );
> end proc:

> Connected(T);
true
\end{verbatim}

You can add the cities Montreal, Toronto, and Waterloo, and the highways between them.
\begin{verbatim}
> T[Waterloo] := {Toronto};

\end{verbatim}

\begin{verbatim}
T_{Waterloo} := \{\texttt{Toronto}\}
\end{verbatim}

\begin{verbatim}
> T[Toronto] := \{Waterloo, Montreal\};
\end{verbatim}
You can no longer travel by road between any two cities. For example, you cannot travel from Paris to Waterloo.

```
> Connected(T);
false
```

**Choosing the Best Data Structure**  The importance of this example is not to teach you about networks, but to emphasize how the choice of data structures suited to the problem enables you to create an efficient and concise version of the procedure `Connected`. In this case, sets and tables were the best choices. The best choice for the next problem you solve may be very different. Before writing code to perform your task, consider which structures best suit your needs. The first step in good program design is to choose appropriate structures and methods for the data and task.

### 6.11 Troubleshooting

This section provides you with a list of common mistakes, examples, and hints that will help you understand and avoid common errors. Use this section to study the errors that you may encounter when entering the examples from this chapter in a Maple session.

**Missing Argument**

Maple generates an error if an argument in a procedure call is missing, but only when it is first required.

---

45 You can also use the Maple debugger for finding errors in programs. For more information, see chapter 8 or refer to `?debugger`.

46 For more information about the if statement, see 5.1 Selection and Conditional Execution or refer to `?if`.
> G := proc(x, y)
>     if x > 2 then
>     x
>     else
>     y
>     end if
> end proc:
> G(3);

3

> G(1);

Error, (in G) G uses a 2nd argument, y, which is missing

> G(1,12);

12

Incorrect Argument Type
When a procedure is invoked, Maple tests the types of the actual parameters, before executing the body of the procedure. Any of these tests can generate an error message. If no type error occurs, the procedure executes.

> Max := proc(x::numeric, y::numeric)
>     if x>y then
>     x
>     else
>     y
>     end if
> end proc:
> Max(Pi, 3);

Error, invalid input: Max expects its 1st argument, x, to be of type numeric, but received Pi

Implicitly Declared Local Variable
If a variable is used in a procedure definition, but not declared local or global, Maple determines the type of variable and issues a warning. To avoid these warnings, always declare variables as either local or global in the procedure definition.
> F := proc(x)
> y := x + 1;
> end proc;

Warning, ‘y’ is implicitly declared local to procedure ‘F’

\[
F := \text{proc}(x) \text{local} \ y; \ y := x + 1 \ \text{end proc}
\]

Understanding Names, Functions, and Remember Tables
A common mistake is to confuse names, functions, and remember tables. If no value has been assigned to a name, its value is itself.

> phi;

\[\phi\]

However, you can assign any expression to a name. Study the following examples to see the difference between assigning a value, function, and remember table to a name.

Assign the value \(t^2\) to the name \(\phi\).

> phi := t^2;

\[\phi := t^2\]

> phi;

\[t^2\]

> print(phi);

\[t^2\]

Assign a function of \(t\) to the name \(\phi\).\[47\]

> phi := t -> t^2;

---

\[47\] Functions can be defined by using the arrow notation \(\rightarrow\). For more information, see Functional Operators: Mapping Notation on page 231.
\[ \phi := t \rightarrow t^2 \]

\[ > \phi; \]
\[ \phi \]

\[ > \phi(3); \]
\[ 9 \]

\[ > \text{print}(\phi); \]
\[ t \rightarrow t^2 \]

Create an entry in the remember table for procedure \( \phi \).

\[ > \phi(t) := t^2; \]

\[ \phi(t) := t^2 \]

\[ > \phi; \]
\[ \phi \]

\[ > \text{print}(\phi); \]

\[ \text{proc()} \text{ option remember; 'procname(args)' end proc} \]

\[ > \text{op}(4, \text{eval}(\phi)); \]
\[ \text{table}([t = t^2]) \]

\[ > \phi(t); \]
\[ t^2 \]
6.12 Exercises

1. Improve the general Max procedure on page 224 so that Max(3,x,1,4) returns Max(x,4). That is, the procedure returns the maximum numerical value along with all nonnumerical values.

2. Implement the function \( f(x) = \sqrt{1 - x^{23}} - 1 \), first as a procedure, and then by using the mapping notation. Compute \( f(1/2) \) and \( f(0.5) \), and comment on the different results.

3. You can use \( ab/g \) to compute the least common multiple of two integers, \( a \) and \( b \), where \( g \) is the greatest common divisor of \( a \) and \( b \). For example, the least common multiple of 4 and 6 is 12. Write a Maple procedure, LCM, which takes as input \( n > 0 \) integers \( a_1, a_2, \ldots, a_n \) and computes their least common multiple. By convention, the least common multiple of zero and any other number is zero.

4. Write a Maple procedure called Sigma which, given \( n > 1 \) data values, \( x_1, x_2, \ldots, x_n \), computes their standard deviation. The following equation gives the standard deviation of \( n > 1 \) numbers, where \( \mu \) is the average of the data values.

\[
\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2}
\]

5. Write a Maple procedure which, given a list of lists of numerical data, computes the mean of each column of the data.

6. Write a Maple procedure called Position which returns the position \( i \) of an element \( x \) in a list \( L \). That is, Position(x,L) should return an integer \( i > 0 \) such that \( L[i]=x \). If \( x \) is not in list \( L \), 0 is returned.

7. Demonstrate that the BinarySearch procedure always terminates.

Hint: Suppose the dictionary has \( n \) entries. How many words in the dictionary \( D \) does BinarySearch look at in the worst case?

8. Rewrite BinarySearch to use a while loop instead of a recursive call.

9. The system of cities and roads in Connected Graphs on page 246 splits naturally into two components: the Canadian cities and roads between them, and the European cities and roads between them. In each component you can travel between any two cities, but you cannot
travel between the two components. Write a procedure that, given a
table of neighbors, splits the system into such components.

Hint: Think about the form in which the procedure returns its result.


> Connected( table() );

Error, (in Connected) invalid subscript selector

Correct this shortcoming.

6.13 Conclusion

This chapter discussed the various parts of a Maple procedure when it
is created by using the proc...end proc construct. Alternate methods
to create a procedure, such as using functional operators, the unapply
command, and the map command were also discussed.

Once a procedure is defined, it is beneficial to format and docu-
ment the code for readability. This saves time in the future when error-
correcting or adding enhancements.

When a procedure is invoked, Maple uses particular evaluation rules.
Local variables are, generally, evaluated to one level, and global variables
are evaluated fully. The arguments to a procedure are evaluated at the
time it is invoked. The manner in which they are evaluated depends upon
the environment in which the call occurs, and in some cases, the types
specified in the procedure definition. Once evaluated, Maple substitutes
the values into the procedure, and then executes it. Maple does no further
evaluation on the values which it substituted, unless you specifically use
an evaluation command, for example, eval. This rule makes it impractical
to use parameters to store temporary results, as you would use local
variables.
7  Input and Output

Input and output (I/O) in Maple can be divided into two categories: screen and keyboard I/O (for communication within Maple, and between Maple and the user), and standard disk I/O (for communication between Maple and other software). This chapter discusses both types of I/O in relation to writing procedures.\footnote{The \texttt{Sockets} package contains routines that perform data exchanges between processes on remote hosts on a network. This allows I/O operations over Intranets and the Internet. For more information, refer to \texttt{?Sockets}.}

In This Chapter
- Displaying output to the screen
- Collecting input interactively from users
- Reading data from files
- Writing data to files

7.1   Screen and Keyboard I/O

This section describes how to present output in various forms on the screen, and how to request input directly from the user during procedure execution. These features make it possible to write interactive procedures.

Printing Output to the Screen

As each complete statement is entered in a Maple session, it is evaluated and the result is printed on the output device, usually the computer terminal. The printing of expressions is normally presented in a two-dimensional, multi-line format (in command-line versions of Maple),
or as close to typeset mathematical notation as possible (in graphical user interface versions with Maple worksheets). There are several methods for altering the display of expressions: change the interface variable prettyprint, set the global variable printlevel, or use one of the numerous Maple commands that controls the appearance of output, for example, print, lprint, and printf.

The *interface* Command  The *interface* command is a mechanism for communication between the computational component and user interface. It is used to set and query all variables that affect the format of the output. However, it does not affect the computation. In particular, the *interface* variable prettyprint controls the method that Maple uses to render Maple expressions in the user interface. If the *interface* variable prettyprint is set to 0 (zero) by using the command

\[
\text{interface(prettyprint}=0)\nonumber
\]

expressions are printed in one-dimensional line-printing mode.\(^2\)

\[
> \text{interface(prettyprint}=0)\nonumber
\]

\[
4
\]

\[
> \text{Int}(x^2, \ x=1..10)\nonumber
\]

\[
\text{Int}(x^2,x = 1 .. 10)
\]

If the *interface* variable prettyprint is set to 1 (one) by using the command

\[
\text{interface(prettyprint}=1)\nonumber
\]

expressions are printed in two-dimensional character-based format.

\[
> \text{interface(prettyprint}=1)\nonumber
\]

\[
0
\]

\[
> \text{Int}(x^2, \ x=1..10)\nonumber
\]

\(^2\)For greater control over the format of output, you can set other *interface* variables such as verboseproc and screenwidth. For more information about the *interface* command and these variables, refer to ?interface.
If you are using Maple in a graphical user interface (GUI), you can set the value of `prettyprint` to higher values, or you can use the **Options Dialog** in the Maple worksheet to set the **Output Display**.³

> interface(prettyprint=2);

1

> Int(x^2, x=1..10);

³For more information about the **Options Dialog**, refer to ?worksheet,managing,preferences.
The difference between `prettyprint=2` (*Typeset Notation*) and `prettyprint=3` (*Standard Math Notation*) is that with `prettyprint=3` you can select and edit subexpressions of the output.

You can always determine the current value of the `prettyprint` variable by entering the command `interface(prettyprint)`. The default value of the `prettyprint` variable is 1 (for command-line versions) or 3 (for GUI environments). The `prettyprint` variable is reset to its default value if the `restart` command is entered or the Maple session is exited.

**The printlevel Global Variable** The global variable `printlevel` determines how much output is displayed. The default value of `printlevel` is 1. Setting this variable to -1 prevents the printing of any results. If `printlevel` is set to a value larger than 1, more information, for example, the routines used when execution errors are encountered, is displayed.\(^4\)

**The print Command** The `print` command displays the values of the expressions that appear as its arguments, separated by a comma and a blank space.

```
print( expr1, expr2, ... );
```

It is important to note that the value returned by a call to `print` is NULL. For this reason, the output of the `print` command is not recalled by using a ditto operator.

```
> print(apple, banana, orange);
    apple, banana, orange
```

\(^4\)For more information about `printlevel`, see *Tracing a Procedure* on page 315 or refer to `?printlevel`. 
Maple checks the interface variable prettyprint to determine the print format for expressions. Because the default value for prettyprint is 1 for Maple command-line versions, the print command normally returns output in a character-based format. Similarly, for Maple GUI versions, the default value of prettyprint is 3 so the print command normally returns output to a worksheet in an editable typeset format. For more information about these formats and the prettyprint variable, see page 258 or refer to ?interface.

The print command is most useful for displaying special objects such as tables, arrays, modules, and procedures. These objects have last name evaluation rules. If they are referenced by name, only their name is printed. The print command enables you to see the contents of these objects.

```plaintext
T := table([w,x,y,z]);
T := table([1 = w, 2 = x, 3 = y, 4 = z])

T;

P := proc(x) x^2 end proc;
P := proc(x) x^2 end proc

P;

print(T);
```

```plaintext
  table([1 = w, 2 = x, 3 = y, 4 = z])
```
The `lprint` Command  The `lprint` command displays the values of the expressions `expr1, expr2, ...` as valid Maple syntax in a one-dimensional format.

```maple
lprint( expr1, expr2, ... );
```

Like the `print` command, the value returned by a call to `lprint` is `NULL`. For this reason, the output of the `lprint` command is not recalled by using a `ditto` operator.

```maple
> lprint(apple, banana, orange);
apple, banana, orange

> %;
```

By setting `interface(prettyprint)=0`, Maple uses `lprint` to print all expressions in the interface. For more information, see page 258 or refer to `?interface`.

The `lprint` command is intended for device-independent printing. To achieve formatted output, use the `printf` command.

The `printf` Command  The `printf` command is used to display output in accordance with indicated formatting specifications. The values of the expressions that appear in the calling sequence are formatted according to the format string in the first argument.\(^5\)

```maple
printf( formatString, expr1, expr2, expr3, ... );
```

The `formatString` is a Maple symbol or string consisting of a sequence of formatting specifications, possibly separated by other characters. Each formatting specification has the following structure:

\[
\%[flags][width][.precision][modifiers]code
\]

\(^5\)The `printf` function is based on a C standard library function of the same name.
where the options in brackets are optional.

The % symbol begins the format specification. One or more of the specifications can optionally follow the % symbol. Also, it is important to note that a new line is not automatically started at the end of the output. If a new line is required, the formatString must contain a new line character ("\n").

The flags specify how to format the output. For example, + causes the output to appear with a leading positive or negative sign; - causes left-justified output.

The width indicates the minimum number of characters to display. If the formatted value has fewer characters than width, the output is padded with blank spaces.

The precision specifies the number of digits that appear after the decimal point for floating-point formats, or the maximum field width for string formats.

The modifiers specify the type of the value. For example, Z indicates the formatting of a complex number.

The code specifies the format of the output object for integer, floating-point number, string, or algebraic formats. For example, d indicates a signed decimal integer, e indicates a floating-point number in scientific notation, s indicates a Maple symbol or string, and c indicates a Maple symbol or string with exactly one character.

For a comprehensive list of permissible formatting specifications, refer to ?printf.

Study the following examples to gain a better understanding of how to use formatting strings in printf.

\[
> \text{printf("%d\n", 12345);}
\]

\[
12345
\]

\[
> \text{printf("%10d\n", 12345);}
\]

\[
12345
\]

\[
> \text{printf("%+10d\n", 12345);}
\]

\[
+12345
\]

\[
> \text{printf("%-10d %d\n", 12345, 6789);}
\]

\[
12345
\]
> printf("%+10d %+5s %-c \n", 12345, hi, z );
  +12345   hi z

> printf("The %-s of the %-s\nis %-a dollars.\n", price, hat, 10);

The price of the hat
is 10 dollars.

> printf("%5s is approximately %5.2f.\n", Pi, 3.14159);
Pi    is approximately 3.14.

> printf("The least common multiple of %a and %a is %a.\n", 4, 6,
>    lcm(4,6));

The least common multiple of 4 and 6 is 12.

Interactive Input
Input is normally passed to Maple procedures by using parameters. However, you can write a procedure that requests input from the user while the procedure is executing. For example, you can write a procedure that quizzes a student (for example, generates random problems, accepts input, and then verifies the student’s answers). The input can be the value of a certain parameter or whether a number is positive. Maple has two commands for reading input from the terminal: the \texttt{readline} and \texttt{readstat} commands.

Reading Text from the Terminal The \texttt{readline} command reads one line of text from a file or the keyboard. It has the following syntax.

\begin{verbatim}
readline( filename )
\end{verbatim}

If \texttt{filename} is the special name \texttt{terminal}, \texttt{readline} reads a line of text from the keyboard. The text is returned as a string. For example, enter the following statement.
7.1 Screen and Keyboard I/O

> s := readline( terminal );

At the next Maple prompt, enter a line of text.
> This is my line of text

\[
s := "This is my line of text"
\]

A simple application of the `readline` command\(^6\) is the following example that prompts the user for an answer to a question.\(^7\)

```maple
> DetermineSign := proc(a::algebraic) local s;
> > printf("Is the sign of \%a positive? ",a);
> > "Answer yes or no:\n",a);
> > s := readline(terminal);
> > evalb( s="yes" or s = "y" );
> > end proc:
>
> DetermineSign(u-1);

Is the sign of u-1 positive? Answer yes or no:

> y

true
```

**Reading Expressions from the Terminal** You can also write procedures that require the user to input an expression rather than a string. The `readstat` command reads one Maple statement from the keyboard.

---

\(^6\)You can also use the `readline` command to read a single line of text from a file. For more information, see page 274 or refer to `?readline`.

\(^7\)The `printf` command is used to print expressions. For more information about `printf`, see page 262 or refer to `?printf`. 
readstat( promptString )

If the optional string promptString is included, it is displayed as the prompt when the readstat command is executed. If you omit this argument, Maple uses a blank prompt.\(^8\)

For example, when the following statement is entered, the promptString is immediately displayed, and Maple waits for the user to enter a statement that terminates in a semicolon (or colon). In this case, n-1; was entered.

> readstat("Enter degree: ");

Enter degree: n-1;

n-1

If an incomplete statement is entered (for example, one that does not end in a semicolon or colon), readstat redisplay the promptString to indicate that further input is required. This continues until a complete statement is entered. If a syntax error is discovered, an error message is returned and promptString is redisplayed.

Unlike the readline command, which reads only one line, the readstat command accepts a large expression broken across multiple input lines.

Another advantage of using the readstat command is that if the user makes a mistake in the input, the readstat command automatically prompts the user for new input, providing an opportunity to correct the error.

> readstat("Enter a number: ");

Enter a number: 5^^8;
syntax error, ‘^’ unexpected:
5^^8;
\(^\)

Enter a number: 5^8;

390625

The following procedure shows how to use readstat to prompt a user about the derivative of a function.

\(^8\)For information about other options available to readstat, refer to ?readstat.
> InteractiveDiff :=
> proc() local a, b;
> a := readstat("Please enter an expression: ");
> b := readstat("Differentiate with respect to: ");
> printf("The derivative of %a with respect to %a is %a\n", a, b, diff(a,b))
> end proc:

> InteractiveDiff();

Please enter an expression: x^2 + 1/x;
Differentiate with respect to: x;
The derivative of x^2 + 1/x with respect to x is 2*x-1/x^2

The final example in this section is an application of the readstat command that implements an interface to the limit command. The procedure does the following: Given the function f(x), assume x is the variable if only one variable is present. Otherwise, ask the user to enter the variable. Ask the user for the limit point.\(^9\)

> GetLimitInput :=
> proc(f::algebraic) local x, a, K;
> # choose all variables in f
> K := select(type, indets(f), name);
> if nops(K) = 1 then
> x := K[1];
> else
> x := readstat("Input limit variable: ");
> while not type(x, name) do
> printf("A variable is required: received %a\n", x);
> x := readstat("Please reinput limit variable: ");
> end do;
> end if;
> a := readstat("Input limit point: ");
> x = a;
> end proc:

The expression \(\sin(x) = x\) depends only on one variable, so GetLimitInput does not ask for a limit variable.

> GetLimitInput( sin(x)/x );

Input limit point: 0;

\[
x = 0
\]

\(^9\)The \texttt{indets} command finds the indeterminates of an expression. For more information, refer to \texttt{?indets}. 
In the next call to \texttt{GetLimitInput}, the number 1 is entered as the limit variable. Since 1 is not a name, \texttt{GetLimitInput} asks for another limit variable.

\begin{verbatim}
> GetLimitInput( \exp(u*x) );
\end{verbatim}

Input limit variable: 1;
A variable is required: received 1

Please re-input limit variable: \( x \);

Input limit point: infinity;

\begin{verbatim}
x = infinity
\end{verbatim}

The \texttt{readstat} command can also be used to read one statement from a file. For more information, refer to \texttt{?readstat}.

\textbf{Controlling the Evaluation of Procedure Input} Occasionally, more control is required over how and when Maple evaluates user input to a procedure than the \texttt{readstat} command permits (see page 265). In such cases, use the \texttt{readline} command to read the input as a string, and then the \texttt{parse} command to convert the string to an expression. For information on the \texttt{readline} command, see page 264. The \texttt{parse} command has the following syntax.

\begin{verbatim}
parse( \texttt{string}, \texttt{options} )
\end{verbatim}

The string must represent a complete Maple expression. When the string is parsed, it becomes an expression.

In the following example, the string “\( a \times x^2 + 1 \)” is parsed as an expression.

\begin{verbatim}
> s := "a*x^2 + 1";
    s := "a*x^2 + 1"

> y := parse( s );
\end{verbatim}
\[ y := a \times + 1 \]

In this case, the expression is a sum.

> type(s, string), type(y, ‘+’);
true, true

The `parse` command does not evaluate the expression it returns. You must use the `eval` command to evaluate the expression explicitly. For example, Maple does not evaluate the variable `a` to its value 2 until you explicitly use the `eval` command.

> a := 2;

\[ a := 2 \]

> z := parse( s );

\[ z := a \times + 1 \]

> eval(z);

\[ 2 x + 1 \]

For more information about the `parse` command, refer to chapter 3 of the *Maple Advanced Programming Guide* or `?parse`.

### 7.2 Standard Disk I/O

Previously, the procedures discussed use keyboard input. However, if a large amount of data must be processed, a different method is recommended. To improve efficiency, use data files to store input and output.
This section discusses some basic tools for accessing and storing data in files.

**Readable File Formats in Maple**

Maple accepts a variety of different file formats. The remainder of this chapter discusses how to use the Maple I/O utilities with the following file types.

- Maple worksheet files
- Maple language files
- general data files created by a text editor

**Worksheet Files** If you are running Maple in a GUI environment, you can save your session as a worksheet file. Worksheet files are identified by filenames that end with the ".mw" extension. You can save and retrieve worksheet files in Maple by using the menus in the GUI environment.

The `Worksheet` package contains tools for generating and manipulating worksheets. For more information, refer to ?Worksheet.

**Note:** Maple worksheets use an .mw file extension. Previous releases of Maple create worksheets as .mws files. The two formats are different, but Maple can open and run both file types. Older worksheets might not behave exactly as they would in the version they were created because improvements to the system sometimes result in different forms of responses.

**Language Files** A Maple language file is one that contains statements conforming to the syntax of the Maple language. These statements are the same as those that are entered interactively during a Maple session. Reading a language file is identical to entering the same sequence of statements into a Maple session—each statement is executed and its result is displayed.

Maple language files can be created by using a text editor, or by using the save statement in Maple. Any filename consisting of letters, decimal numbers, periods, and the underscore (_) can be used to save a Maple language file. The extension cannot be "m", which indicates a

---

10 For more information about other file formats in Maple, refer to ?filetypes and ?files.

11 For more information about file management with worksheet files in a GUI environment, refer to the Maple Getting Started Guide or ?managing.
Maple internal file (refer to ?filenames). In addition, like most software programs, special characters are not permitted in filenames. This excludes the dollar sign ($), question mark (?), and exclamation point (!), which can be used.

**General Data Files** Maple can import and export files that contain formatted numerical data, and files that contain both formatted textual and numerical data. You can create these general data files by using a text editor, and then read them into Maple to perform computations.

**Using Filenames in Maple Statements**
If a Maple statement expects a filename as a parameter, enter it as a string. However, if you are entering the filename and it does not contain a slash (\ or / to indicate the directory location) or a period (to indicate a file extension), you can enter the filename as a symbol. In other words, if there are no special characters in the filename, neither single nor double quotes are necessary. For example, MyFile, ‘MyFile’, and “MyFile” represent the same valid filename.

If a filename contains slashes or periods, then the name must be enclosed in double quotes ("). Backslashes that appear in a filename (the directory separator character on some operating systems) must be escaped by “doubling up” (the first backslash acts as an escape character to the second backslash). For example, the filename C:\Maple\Data\MyFile must be written as "C:\\Maple\\Data\\MyFile". Alternatively, you can use the canonical form "C://Maple/Data/MyFile".12

**Reading and Saving Data in Maple**
You can use Maple without understanding how to access data files for input. However, accessing files from a Maple session, instead of re-entering commands and data, is an important feature (especially for command-line versions). You can save time by reading files that contain Maple statements saved from a previous session, or by accessing general data files that contain formatted data to use in Maple and other applications.

You can interact with the file system either explicitly by using the read and save statements, or implicitly by executing a command that automatically accesses information from a general data file.

**Saving Assignment Statements in a Maple Session** You can save the values of a sequence of variables by using the save statement. It has

---
12For more information about backslashes and filenames, refer to ?backslash and ?filename.
the following syntax where \texttt{nameseq} is a sequence of names of assigned variables, and \texttt{filename} is a valid filename.

\begin{verbatim}
save nameseq, filename;
\end{verbatim}

Maple saves each variable (and its value) in \texttt{nameseq} in the file \texttt{filename}. Maple evaluates each argument in \texttt{nameseq} to a name. The last argument (\texttt{filename}) is fully evaluated. It must evaluate to a string (or name) that specifies a valid filename.

For example, clear the Maple internal memory by using the \texttt{restart} command and assign new values.

\begin{verbatim}
> r0 := x^3:
> r1 := diff(r0,x):
> r2 := diff(r1,x):
> r3 := proc(y) local a;
> a := irem(y, 2);
> if a=0 then
> "Even"
> else
> "Odd"
> end if
> end proc:
\end{verbatim}

The following statement saves \texttt{r0}, \texttt{r1}, \texttt{r2}, and \texttt{r3} in the ASCII file named \texttt{my_file} in the current working directory.\footnote{To determine the current working directory, use the \texttt{currentdir} function.}

\begin{verbatim}
> save r0, r1, r2, r3, "my_file";
\end{verbatim}

Once this statement is executed, the following lines form the contents of file \texttt{my_file}.\footnote{You can also append commands entered in a Maple session to an existing file. For more information, refer to \texttt{?appendto}.}

\begin{verbatim}
r0 := x^3;

r1 := 3*x^2;

r2 := 6*x;

r3 := proc (y) local a; a := irem(y,2);
if a = 0 then "Even" else "Odd" end if end proc;
\end{verbatim}
Reading Files Containing Maple Assignments into a Maple Session

You can read files that contain Maple statements into a Maple session by using the `read` statement. The `read` statement has the following syntax, where `filename` is any Maple language file (a sequence of valid Maple statements, separated by semicolons or colons, entered during a Maple session and saved by using the `save` statement, or created in a text editor and saved as a text file).\(^\text{15}\)

```
read filename;
```

The `filename` must evaluate to the name of a valid file. Reading the file is identical to entering the same sequence of statements interactively into a Maple session, except that the input statements are not echoed to the screen. The system only displays the result of executing each statement read from the file.

```
> read "my_file";

r0 := x^3
r1 := 3 x^2
r2 := 6 x

r3 := proc(y)
    local a;
    a := irem(y, 2); if a = 0 then "Even" else "Odd" end if
end proc
```

If you prefer to display both the input assignment statements and their results, you must set the `interface` variable `echo` to 2 or higher. For more information, refer to `?interface.`\(^\text{16}\)

---

\(^{15}\)The `read` statement is particularly useful for reading long procedure definitions. If you must enter a long procedure that requires much formatting and indentation, it is easier to enter it in a text editor that has a tabulation feature, and then read it into Maple.

\(^{16}\)Because the Maple language contains enhancements with each new release, a conversion utility that translates Maple language files from previous releases is shipped with each new version of Maple that requires it. For more information, refer to `?updtsrc`. 
> interface(echo=2);
> read "my_file";

> r0 := x^3;

\[ r0 := x^3 \]

> r1 := 3*x^3;

\[ r1 := 3x^2 \]

> r2 := 6*x;

\[ r2 := 6x \]

> r3 := proc (y)
>     local a;
>     a := irem(y,2);
>     if a = 0 then
>         "Even"
>     else
>         "Odd"
>     end if
> end proc;

\[ r3 := \text{proc}(y) \text{ local } a; \text{ a := irem(y, 2); if } a = 0 \text{ then } \text{"Even" else } \text{"Odd" end if } \text{end proc} \]

**Importing Data from Files**

You have already seen how to use readline to read text from the keyboard. You can also use readline to read one line of data from a specified file. Alternatively, you can use the readdata command to read numerical data arranged in columns from a file.

**The readline Command** The readline command has the following syntax.
If a `readline` command is entered, Maple attempts to read the next line from the specified file. If successful, a string consisting of the line read from the file is returned. However, the new line character at the end of the line is excluded from the string. If there are no more lines in the file to read, `readline` returns 0 to indicate that the end of the file has been reached.\(^\text{17}\)

Study the following example to understand how to use `readline` for reading files.

Suppose that the following three lines of data were saved in the file `mydata.txt` in the current working directory.\(^\text{18}\)

```
1   -5.2
2   6.0
3   7.8
```

You could read this data by entering the following statements.

```
> readline("mydata.txt");
```

```
"1  -5.2"
```

The `readline` statement opens file `mydata.txt` for reading, and reads the first line of the file. The `readline` statement returns the data as a string. Numerical data is not very useful to Maple in the form of a string. Thus, if you want to use this data for further calculations, use the string formatting command `sscanf` to convert the string to a list of values.

```
> sscanf(%, "%d%f");
```

```
[1, -5.2]
```

\(^{17}\)If the file was not opened for reading when the `readline` command is entered, Maple opens it. Once `readline` returns 0 to indicate that the end of the file has been reached, the file is automatically closed.

\(^{18}\)To determine the current working directory, use the `currentdir` function.
The arguments to the `sscanf` command are very similar to the `printf` command. Here, `d` indicates a decimal integer and `f` indicates a floating-point value.\(^1\)

To read the remaining lines in the file, continue to enter `readline` statements.

```
> readline("mydata.txt");

"2   6.0"

> readline("mydata.txt");

"3   7.8"

> readline("mydata.txt");

0
```

The value 0 is returned because `readline` reached the end of the file. You can automate the process of reading this data by using the following procedure.

```
> readfile := proc(s::string) local line;
    >
    > do
    >     line:=readline(s);
    >     if line = 0 then
    >       break
    >     end if;
    >     print(line);
    >     end do;
    > end proc:
```

\(^1\)For more information about `sscanf`, refer to chapter 3 of the *Maple Advanced Programming Guide* or `?sscanf`. 
> readfile("mydata.txt");

    “1  -5.2”
    “2   6.0”
    “3   7.8”
    0

Since `readline` returns lines read from a file in the form of a string, it is more useful for reading textual data. If a file contains numerical data formatted into columns, it is more appropriate to use the `readdata` command.

**The readdata Command** The `readdata` command has the following syntax where `filename` is the file that contains numeric data formatted into columns, `format` specifies whether the data is to be read as integer or floating-point values, and `n` is the number of columns of data in `filename`.

```
readdata( filename, format, n );
```

If only one column of data is read, the output is a list of the data. If more than one column is read, the output is a list of lists of data corresponding to the rows of data in `filename`.

> readdata("mydata.txt", 2);

    [[1., -5.2], [2., 6.0], [3., 7.8]]

The data is automatically returned as floating-point values because no `format` was specified in the calling sequence and Maple detected a floating-point value in the file. You can specify the format of the values in each column by using the following version of the `readdata` command.

> readdata("mydata.txt", [integer, float]);

    [[1, -5.2], [2, 6.0], [3, 7.8]]
Once you have read the data from a file, use Maple commands to process the data. For example, plot the data points from file `mydata.txt`.

> points := %;

\[
\text{points := } [[1, -5.2], [2, 6.0], [3, 7.8]]
\]

> plot(points, style=point, color=black);

**Importing Files Containing Non-uniform Data**  Data files frequently contain both numbers and text. These types of data files can be read as formatted input by using the `fscanf` command and the low-level I/O routines. For more information about the low-level routines, refer to chapter 3 of the *Maple Advanced Programming Guide* or the Maple online help system.

**Exporting Data to Files**  You can use the `writeline` command to write *strings* to a specified file. Alternatively, you can use the `writedata` command to write *numerical data* to a text file.

**The writeline Command**  The `writeline` command writes a sequence of strings as strings separated by new line characters to `filename`. A new line character is also inserted after the last string.

```
writeline( filename, string1, string2, ... );
```

A count of the number of characters written to `filename` is returned. If no string sequence is specified in the calling sequence, `writeline` writes a blank line to `filename`. If `filename` exists, the file is overwritten.
The *writedata* Command  The *writedata* command writes *data* to a specified *filename*. The *data* can be in the form of a list or an array. The *format* of the entries in *data* can be *integer*, *float*, or *string*.

\[
\text{writedata( filename, data, format )};
\]

If *filename* exists, the file is overwritten. If *data* is a list of values, each value is printed on a separate line. If *data* is an array or a list of lists of values, the data is printed one row per line, with values separated by a \text{TAB}. If no format is specified, *float* is used.\(^2\)

For example, consider the following array.

\[
> a := \text{array}(1..3, 1..3):
> \text{for } i \text{ to } 3 \text{ do}
> \quad \text{for } j \text{ to } 3 \text{ do}
> \quad a[i,j] := \text{evalf}(i/j,3)
> \quad \text{end do}
> \text{end do};
> \text{print}(a);
\]

\[
\begin{bmatrix}
1 & 0.500 & 0.333 \\
2 & 1 & 0.667 \\
3 & 1.50 & 1
\end{bmatrix}
\]

To save the entries of *a* to file *arraydata*, enter the following command.

\[
> \text{writedata("arraydata", a, float)}:
\]

File *arraydata* contains the following data.

\[
> \text{writedata(terminal, a, float)};
\]

\[
\begin{array}{ccc}
1 & .500 & .333 \\
2 & 1 & .667 \\
3 & 1.50 & 1
\end{array}
\]

C, Fortran, Java\textsuperscript{TM}, MATLAB, and Visual Basic *Code Generation*  Maple also provides commands to translate Maple expressions to other programming languages: C, Fortran, Java\textsuperscript{TM}, MATLAB\textsuperscript{®}, and Visual Basic\textsuperscript{®}. These commands are available in the Maple *codegen* and *CodeGeneration* packages. For more information, refer to chapter 6 of the *Maple Advanced Programming Guide*, \texttt{?codegen}, or \texttt{?CodeGeneration}.

\(^2\)For more information about *writedata*, refer to \texttt{?writedata}. 
7.3 Troubleshooting

This section provides you with a list of common mistakes, examples, and hints that will help you understand and avoid common errors. Use this section to study the errors that you may encounter when entering the examples from this chapter in a Maple session.\footnote{For more information about I/O errors in Maple, refer to \texttt{?IO_errors}.}

**Syntax Error in readstat**

Maple generates an error if the input statement supplied to a \texttt{readstat} command contains a syntax error.\footnote{For more information about the \texttt{readstat} statement, refer to \texttt{?readstat}.}

```maple
> readstat("Bad Input: ");
Bad Input: a ++ b;

syntax error, ‘+’ unexpected:
a ++ b;
^ 
```

The Bad Input prompt redisplay to request new input. To correct this error, enter a valid statement that ends in a colon or semicolon.

```
Bad Input: a + b;
```

\[
a + b
\]

**Extra Characters at End ofParsed String**

The \texttt{readstat} routine reads as many lines of characters as necessary to parse a complete statement. Maple generates a warning if the last line contains additional characters after the semicolon or colon. Therefore, multiple statements per line are permitted, but those after the first are ignored.

```
> readstat("Extra Input: ");
Extra Input: a+b; c+d;
```

Warning, extra characters at end of parsed string

\[
a + b
\]
Unable to Read Filename
Maple returns an error message if the filename supplied to the `read` statement is invalid or if the file does not exist. To correct this error, check that the spelling of the filename is correct and that the file exists in the directory that you have specified.

```maple
> read MyDataFile;

Error, unable to read ‘MyDataFile’
```

7.4 Exercises

1. Write a loop (with a single statement in its body) that prints strings listing the cubes of the integers 1 to 10.

2. Create a file in a text editor that contains the following lines.

```maple
x := 1;    # valid input line
if := 2;}   # invalid assignment
y := 3;    # valid input line
two words := 4;  # invalid assignment
```

Save the file. In a Maple session, open the file by using the `read` statement. Observe how Maple reacts to invalid statements.

3. Create a data file in a text editor that contains the following information.

```
1 2 3
4 5 6
```

Save the file. Read this file into a Maple session, convert the data to a list, and reverse its order. Write the reversed data in the same format to a different file.
Chapter 7: Input and Output

7.5 Conclusion

The techniques in this chapter illustrated some of the basic I/O facilities that are available in Maple—how to incorporate interactive input in your procedures, and how to import and export data from Maple. In addition to the commands discussed in this chapter, Maple has many low-level I/O routines, which are useful for reading formatted and non-uniform data from text files.

In Maple, there are packages that generate code in the C, Fortran, Java, MATLAB, and Visual Basic programming languages. For more information, refer to chapter 6 of the Maple Advanced Programming Guide. The Sockets and Worksheet packages also contain important I/O routines.
New programs, whether developed in Maple or any other language, often work incorrectly. Problems that occur in the execution of a program are usually due to syntax errors introduced during implementation, or logic errors in the design of the algorithm. Most errors are subtle and hard to find by visual inspection of the program. Maple provides error detection commands and a debugger to help you find these errors.

Maple provides a variety of commands to help you find errors in procedures. Among these are commands to trace procedure execution, check assertions, raise exceptions and trap errors, and verify procedure semantics and syntax.

Alternatively, the Maple debugger lets you stop execution in a Maple procedure, inspect and modify the values of local and global variables, and continue execution, either to completion, or one statement or block at a time. You can stop execution when Maple reaches a particular statement, when it assigns a value to a specified local or global variable, or when a specified error occurs. This facility lets you investigate the inner workings of a program.

Even when a program is working correctly, you may want to analyze its performance to try to improve its efficiency. Maple commands are available to analyze the time and memory consumption involved in running the program.

In This Chapter
- Using the Maple debugger
- Detailed debugger information
- Additional commands for error detection
- Measuring and improving program efficiency
Chapter 8: Debugging and Efficiency

8.1 A Tutorial Example

The Maple debugger is a tool that you can use to detect errors in your procedures. Using this facility, you can follow the step-by-step execution of your program to determine why it is not returning the results that you expect.

This section illustrates how to use the Maple debugger as a tool for debugging a Maple procedure. The debugger commands are introduced and described as they are applied. Additional information about the commands is provided in 8.2 Maple Debugger Commands.

As an alternative to the command-line Maple debugger, you can use the interactive Maple debugger available in the graphical user interface (GUI) version of Maple.

The interactive Maple debugger is invoked automatically by Maple when a breakpoint or watchpoint is encountered. An interactive debugger window is displayed, which contains the following:

• a main text box that displays a procedure name and the debugger output
• a text box for entering commands and an associated **Execute** button
• buttons that perform common debugging functions

The interactive Maple debugger functions identically to the command-line Maple debugger. For more information, refer to ?InteractiveDebugger.

**Example**  Consider the following procedure, sieve, which is used as a case study. It implements the *Sieve of Eratosthenes*: Given a parameter n, return a count of the prime numbers less than n (inclusive). To debug the sieve procedure, use breakpoints and watchpoints, which cause Maple to stop the execution of the procedure.

```maple
> sieve := proc(n::integer)
  local i, k, flags, count, twicei;
  count := 0;
  for i from 2 to n do
    flags[i] := true
    end do;
  for i from 2 to n do
    if flags[i] then
      twicei := 2*i;
      for k from twicei by i to n do
        flags[k] = false;
        end do;
      count := count+1
    end if;
```

```maple
> sieve := proc(n::integer)
  local i, k, flags, count, twicei;
  count := 0;
  for i from 2 to n do
    flags[i] := true
    end do;
  for i from 2 to n do
    if flags[i] then
      twicei := 2*i;
      for k from twicei by i to n do
        flags[k] = false;
        end do;
      count := count+1
    end if;
```
8.1 A Tutorial Example

Numbering the Procedure Statements

To use the Maple debugger, you must enter a variety of debugger commands. Many of these debugger commands refer to statements in the procedures that you are debugging. Statement numbers allow such references. The `showstat` command displays a Maple procedure along with numbers preceding each line that begins a new statement.

```
> showstat(sieve);
```

```
sieve := proc(n::integer)
    local i, k, flags, count, twicei;
    1    count := 0;
    2    for i from 2 to n do
    3        flags[i] := true
            end do;
    4    for i from 2 to n do
    5        if flags[i] then
    6            twicei := 2*i;
    7            for k from twicei by i to n do
    8                flags[k] = false
                    end do;
    9            count := count+1
                end if
            end do;
    10   count
    end proc
```

**Note:** The numbers preceding each line differ from line numbers that may display in a text editor. For example, keywords that end a statement (such as `end do` and `end if`) are not considered separate commands and are therefore not numbered.

**Invoking the Debugger**

To invoke the Maple debugger, start the execution of a procedure, and stop the execution within the procedure. To execute a Maple procedure, call it by using a Maple command at the top-level, or call it from another procedure. The simplest way to cause execution to stop within the procedure, is to set a `breakpoint` in the procedure.
Setting a Breakpoint  Use the stopat command to set a breakpoint in procedure sieve.

> stopat(sieve);

\[\text{sieve}\]

This command sets a breakpoint before the first statement in procedure sieve. When you subsequently execute sieve, Maple stops before executing the first statement. When execution stops, the debugger prompt appears (DBG>).

The following example demonstrates an initial execution of sieve.

> sieve(10);

sieve:
  i* count := 0;

Preceding the debugger prompt are several pieces of information.

- The previously computed result (This particular execution stopped at the first statement before making any computations, so no result appears.)

- The name of the procedure in which execution has stopped (sieve)

- Execution stopped before statement number 1. An asterisk (*) follows this statement number to indicate that a breakpoint was set before the statement.

At the debugger prompt, you can evaluate Maple expressions and invoke debugger commands. Maple evaluates expressions in the context of the stopped procedure. You have access to the same procedure parameters, and local, global, and environment variables, as the stopped procedure. For example, since sieve was called with parameter value 10, the formal parameter n has the value 10.

\[1\] If a procedure has a remember table, you may have to execute a restart command before issuing a second or subsequent stopat command. For more information about remember tables, see page 210 or refer to ?remember.
8.1 A Tutorial Example

Notice that for each expression that Maple evaluates, it displays:

- the result of the expression,
- the name of the stopped procedure,
- the statement number where the procedure stopped followed by the statement, and
- a new debugger prompt.

**Note:** To remove a breakpoint from a procedure, use the `unstopat` command.

**Controlling the Execution of a Procedure During Debugging**

Debugger commands control the execution of the procedure once the debugger is active. Some commonly used debugger commands are `next`, `step`, `into`, `list`, `outfrom`, and `cont`.

The `next` command executes the next statement at the current nesting level. After the statement is executed, control is returned to the debugger. If the statement is a control structure (an `if` statement or a loop), the debugger executes any statements within the control structure that it would normally execute. It stops execution before the next statement after the control structure. Similarly, if the statement contains calls to procedures, the debugger executes these procedure calls in their entirety before execution stops.

```
DBG > next
0
sieve:
  1* count := 0;
```

The 0 in the first line of the output represents the result of the executed statement—that is, the result of \texttt{count := 0}. A “*” does not appear next to the statement number because there is no breakpoint set immediately before statement 2. The debugger does not show the body of the \texttt{for} loop, which itself consists of statements with their own statement numbers, unless execution actually stops within its body. Maple represents the body of compound statements by ellipses (\ldots).

Executing the \texttt{next} command again results in the following output.

\begin{verbatim}
DBG > next
true
sieve:
  4  for i from 2 to n do
    ... 
  end do;
\end{verbatim}

Execution now stops before statement 4. Statement 3 (the body of the previous \texttt{for} loop) is at a deeper nesting level. Therefore, the loop is executed \(n-1\) times. The debugger displays the last result computed in the loop (the assignment of the value \texttt{true} to \texttt{flags[10]}).

To step into a nested control structure (such as an \texttt{if} statement or \texttt{for} loop) or a procedure call, use the \texttt{step} debugger command.

\begin{verbatim}
DBG > step
true
sieve:
  5  if flags[i] then
    ... 
  end if
\end{verbatim}

\begin{verbatim}
DBG > step
true
sieve:
  6  twicei := 2*i;
\end{verbatim}
If you use the `step` debugger command when the next statement to execute is not a deeper structured statement or procedure call, it has the same effect as the `next` debugger command.

```
DBG > step
4
sieve:
7    for k from twicei by i to n do
    ... 
    end do;
```

At any time during the debugging process, you can use the `showstat` debugger command to display the current status of the debugging process.

```
DBG > showstat

sieve := proc(n::integer)
local i, k, flags, count, twicei;
  1* count := 0;
  2   for i from 2 to n do
  3     flags[i] := true
  end do;
  4   for i from 2 to n do
  5     if flags[i] then
  6       twicei := 2*i;
  7     !   for k from twicei by i to n do
  8       flags[k] := false
  end do;
  9     count := count+1
  end if
  end do;
  10  count
end proc
```

Maple displays a debugger prompt to indicate that you are still working inside the Maple debugger. The asterisk (*) marks the unconditional breakpoint. An exclamation point (!) that follows a statement number (see line 7) indicates the statement at which the procedure is stopped.
To continue the debugging process, issue another debugger command. For example, you can use `into` or `step` to enter the innermost loop.

The behavior of the `into` debugger command is between that of the `next` and `step` commands. Execution stops at the next statement in the current procedure independent of whether it is at the current nesting level or in the body of a control structure (an `if` statement or a loop). That is, the `into` command steps into nested statements, but not procedure calls. It executes called procedures completely, then stops. The `into` command is a very useful debugging tool.

```
DBG > into

4 sieve:
  8  flags[k] = false
```

A debugger command that is related to `showstat` is the `list` command. It displays the previous five statements, the current statement, and the next statement, to provide an indication of where the procedure has stopped.

```
DBG > list

sieve := proc(n::integer)
  local i, k, flags, count, twicei;
  ...
  3  flags[i] := true
  end do;
  4  for i from 2 to n do
  5  if flags[i] then
  6    twicei := 2*i;
  7    for k from twicei by i to n do
  8      flags[k] = false
      end do;
  9    count := count+1
  end if
  end do;
  ...
end proc
```
You can use the `outfrom` debugger command to finish execution at the current nesting level or deeper. Execution of the procedure is stopped once a statement at a shallower nesting level is reached, that is, after a loop terminates, a branch of an `if` statement executes, or the current procedure call returns.

```
DBG > outfrom
true = false
sieve:
  9       count := count+1
```

```
DBG > outfrom
1
sieve:
  5       if flags[i] then
           ...
    end if
```

The `cont` debugger command continues execution, until either the procedure terminates normally or it encounters another breakpoint.

```
DBG > cont
```

```
9l
```

You can now see that the procedure does not give the expected output. Although you may find the reason obvious from the previous debugger command examples, in most cases it is not easy to find procedure errors. Therefore, continue to use the debugger. First, use the `unstopat` command to remove the breakpoint from `sieve`.

```
> unstopat(sieve);
```
Chapter 8: Debugging and Efficiency

Invoking the Debugger II

The procedure \texttt{sieve} maintains the changing result in the variable \texttt{count}. Therefore, a logical place to look during debugging is wherever Maple modifies \texttt{count}. The easiest way to do this is by using a \textit{watchpoint}, which invokes the debugger whenever Maple modifies a watched variable.

**Setting a Watchpoint**  Use the \texttt{stopwhen} command to set watchpoints. In this case, stop execution whenever Maple modifies the variable \texttt{count} in the procedure \texttt{sieve}.

\begin{verbatim}
> stopwhen([sieve,count]);

[[sieve, count]]
\end{verbatim}

The \texttt{stopwhen} command returns a list of all the currently watched variables.

Execute the \texttt{sieve} procedure again.

\begin{verbatim}
> sieve(10);

count := 0
sieve:
  2  for i from 2 to n do
     ...
  end do;
\end{verbatim}

Execution stops because Maple has modified \texttt{count}, and the debugger displays the assignment statement \texttt{count := 0}. As in the case of breakpoints, the debugger then displays the name of the procedure and the next statement to be executed in the procedure. Note that execution stops after Maple has assigned a value to \texttt{count}.

This first assignment to \texttt{count} is correct. Use the \texttt{cont} debugger command to continue execution of the procedure.

\begin{verbatim}
DBG > cont

count := 1
sieve:
  5  if flags[i] then
     ...
  end if
\end{verbatim}
If you do not look carefully, this also looks correct. Assume that nothing is wrong and continue execution.

```
DBG > cont

count := 2*l
sieve:
  5   if flags[i] then
      ...
   end if
```

This output is suspicious because Maple should have simplified 2*1. Notice that it has printed 2*l (two times the letter l) instead. By studying the source text for the procedure, you can see that the letter “l” was entered instead of the number “1”. Since the source of the error has been discovered, there is no reason to continue the execution of the procedure. Use the quit debugger command to exit the debugger, and then use the unstopwhen command to remove the watchpoint from the procedure.

```
DBG > quit

Warning, computation interrupted
```

```
> unstopwhen();
```

After correcting the source text for sieve, issue a restart command, read the corrected version of sieve into Maple, and execute the procedure again.

```
> sieve(10);
```
This result is still incorrect. There are four primes less than 10, namely 2, 3, 5, and 7. Therefore, invoke the debugger once more, stepping into the innermost parts of the procedure to investigate. Since you do not want to start at the beginning of the procedure, set the breakpoint at statement 6.

> stopat(sieve,6);

[sieve]

> sieve(10);

true
sieve:
  6* twicei := 2*i;

DBG > step

4
sieve:
  7    for k from twicei by i to n do
        ...
    end do;

DBG > step

4
sieve:
  8    flags[k] = false

DBG > step

true = false
sieve:
  8    flags[k] = false
The last step reveals the error. The previously computed result should have been false (from the assignment of flags[k] to the value false), but instead true = false was returned. An equation was used instead of an assignment. Therefore, Maple did not set flags[k] to false.

Once again, exit the debugger and correct the source text.

DBG > quit

Warning, computation interrupted

The following code represents the corrected procedure.

```maple
> sieve := proc(n::integer)
>     local i, k, flags, count, twicei;
>     count := 0;
>     for i from 2 to n do
>         flags[i] := true
>     end do;
>     for i from 2 to n do
>         if flags[i] then
>             twicei := 2*i;
>             for k from twicei by i to n do
>                 flags[k] := false;
>             end do;
>             count := count+1
>         end if;
>     end do;
>     count;
> end proc:
```

Execute procedure sieve again to test the corrections.

```maple
> sieve(10);
```

4

The sieve procedure returns the correct result.
8.2 Maple Debugger Commands

This section provides additional details about the commands used in the tutorial in 8.1 A Tutorial Example and a description of other debugger commands.

Numbering the Procedure Statements II

The showstat command has the following syntax. The procedureName parameter is optional.

\[
\text{showstat( procedureName );}
\]

If showstat is called with no arguments, all procedures that contain breakpoints are displayed.

You can also use the showstat command to display a single statement or a range of statements by using the following syntax.

\[
\text{showstat( procedureName, number );}
\]
\[
\text{showstat( procedureName, range );}
\]

In these cases, the statements that are not displayed are represented by ellipses (\ldots). The procedure name, its parameters, and its local and global variables are always displayed.

\[
> f := \text{proc}(x)
> \quad \text{if } x \leq 2 \text{ then}
> \quad \text{print}(x);
> \quad \text{end if;}
> \quad \text{print}(-x)
> \quad \text{end proc:}
\]

\[
> \text{showstat}(f, 2..3);
\]

\[
f := \text{proc}(x)
\quad \ldots
\quad 2 \quad \text{print}(x)
\quad \text{end if;}
\quad 3 \quad \text{print}(-x)
\quad \text{end proc}
\]

Invoking the Debugger III

This section provides additional information about breakpoints and watchpoints.
Setting Breakpoints  The \texttt{stopat} command has the following syntax where \texttt{procedureName} is the name of the procedure in which to set the breakpoint, \texttt{statementNumber} is the line number of the statement in the procedure \textit{before} which the breakpoint is set, and \texttt{condition} is a Boolean expression which must be \textit{true} for execution to stop. The \texttt{statementNumber} and \texttt{condition} arguments are optional.

\begin{verbatim}
  stopat( procedureName, statementNumber, condition );
\end{verbatim}

This \texttt{condition} argument can refer to any global variable, local variable, or parameter of the procedure. These \textit{conditional} breakpoints are marked by a question mark (\?) if \texttt{showstat} is used to display the procedure.

Since the \texttt{stopat} command sets the breakpoint before the specified statement, when Maple encounters a breakpoint, execution stops and Maple engages the debugger \textit{before} the statement. \textit{This means that you cannot set a breakpoint after the last statement in a statement sequence}—\textit{that is, at the end of a loop body, an if statement body, or a procedure.}

If two identical procedures exist, depending on how you created them, they may share breakpoints. If you entered the procedures individually, with identical procedure bodies, then they do not share breakpoints. If you created a procedure by assigning it to the body of another procedure, then their breakpoints are shared.

\begin{verbatim}
> f := proc(x) x^2 end proc:  
> g := proc(x) x^2 end proc:  
> h := op(g):  
> stopat(g);  

[g, h]

> showstat();

g := proc(x)  
   1* x^2  
end proc

h := proc(x)  
   1* x^2  
end proc
\end{verbatim}
Removing Breakpoints  The unstopat command has the following syntax where procedureName is the name of the procedure that contains the breakpoint, and statementNumber is the line number of the statement where the breakpoint is set. The statementNumber parameter is optional.

\[
\text{unstopat}( \text{procedureName}, \text{statementNumber} );
\]

If statementNumber is omitted in the call to unstopat, then all breakpoints in procedure procedureName are cleared.

Setting Explicit Breakpoints  You can set an explicit breakpoint by inserting a call to the DEBUG command in the source text of a procedure. The DEBUG command has the following syntax. The argument parameter is optional.

\[
\text{DEBUG}( \text{argument} );
\]

If no argument is included in the DEBUG command, execution in the procedure stops at the statement following the location of the DEBUG command, and then the debugger is invoked.\(^2\)

```plaintext
> f := proc(x,y) local a;
>     a := x^2;
> DEBUG();
>     a := y^2;
> end proc:

> showstat(f);

f := proc(x, y)
local a;
  1 a := x^2;
  2 DEBUG();
  3 a := y^2
end proc

> f(2,3);
```

\(^2\)The showstat command does not mark explicit breakpoints with a “*” or a “?”. 
4
f:
  3  a := y^2

DBG > quit

Warning, computation interrupted

If the argument of the `DEBUG` command is a Boolean expression, execution stops only if the Boolean expression evaluates to `true`. If the Boolean expression evaluates to `false` or `FAIL`, the `DEBUG` command is ignored.

> f := proc(x,y) local a;
>     a:=x^2;
>     DEBUG(a<1);
>     a:=y^2;
>     DEBUG(a>1);
>     print(a);
> end proc:

> f(2,3);

9
f:
  5  print(a)

DBG > quit

Warning, computation interrupted

If the argument of the `DEBUG` command is anything but a Boolean expression, the debugger prints the value of the argument (instead of the last result) when execution stops at the following statement.
Chapter 8: Debugging and Efficiency

```plaintext
> f := proc(x)
> x^2;
> DEBUG("This is my breakpoint. The current value of x is:", x);
> x^3
> end proc:

> f(2);

"This is my breakpoint. The current value of x is:", 2
f:
  3 x^3
```

Removing Explicit Breakpoints The `unstopat` command cannot remove explicit breakpoints. You must remove breakpoints that were set by using `DEBUG` by editing the source text for the procedure.

```
DBG > unstopat

[f, g, h]
```

```
f:
  3 x^3
```

```
DBG > showstat

f := proc(x)
  1  x^2;
  2  DEBUG("This is my breakpoint. The current value of x is:", x);
  3  x^3
end proc

DBG > quit

Warning, computation interrupted
```
Note: If you display the contents of a procedure by using print (or lprint) and the procedure contains a breakpoint that was set by using stopat, the breakpoint appears as a call to DEBUG.

```maple
> f := proc(x) x^2 end proc:
> stopat(f);

[f, g, h]
```

```maple
> print(f);
proc(x) DEBUG(); x^2 end proc
```

**Setting Watchpoints** The stopwhen command has the following syntaxes.

```
stopwhen( globalVariableName );
stopwhen( [procedureName, variableName] );
```

The first form specifies that the debugger should be invoked when the global variable `globalVariableName` is changed. Maple environment variables, such as `Digits`, can also be monitored by using this method.

```maple
> stopwhen(Digits);
[Digits]
```

The second form invokes the debugger when the (local or global) variable `variableName` is changed in the procedure `procedureName`.

When any form of stopwhen is called, Maple returns a list of the current watchpoints.

Execution stops after Maple assigns a value to the watched variable. The debugger displays an assignment statement instead of the last computed result (which would otherwise be the right-hand side of the assignment statement).

**Clearing Watchpoints** The syntax to call unstopwhen is the same as that for stopwhen. Similar to stopwhen, unstopwhen returns a list of all (remaining) watchpoints.

If no arguments are entered in the call to unstopwhen, then all watchpoints are cleared.
Setting Watchpoints on Specified Errors  You can use an error watchpoint to invoke the debugger when Maple returns a specified error message. When a watched error occurs, execution of the procedure stops and the debugger displays the statement in which the error occurred.

Error watchpoints are set by using the \texttt{stoperror} command. The \texttt{stoperror} command has the following syntax

\begin{verbatim}
stoperror( "errorMessage" );
\end{verbatim}

where \texttt{errorMessage} is a \texttt{string} or a \texttt{symbol} that represents the error message returned from a procedure. A list of the current error watchpoints is returned.

If no argument is entered in the call to \texttt{stoperror}, the list of current watchpoints is returned.

\begin{verbatim}
> stoperror();
[]
\end{verbatim}

\begin{verbatim}
> stoperror( "numeric exception: division by zero" );
[“numeric exception: division by zero”]
\end{verbatim}

\begin{verbatim}
> stoperror();
[“numeric exception: division by zero”]
\end{verbatim}

If the special name \texttt{'all'} is used instead of a specific error message as the parameter to the \texttt{stoperror} command, execution of a procedure stops when \textit{any} error that would \textit{not} be trapped occurs.

Errors trapped by a traperror construct (\texttt{try...catch} statement) do not generate an error message. Hence, \texttt{stoperror} cannot be used to catch them. For more information about the \texttt{try...catch} structure, see Trapping Errors on page 325. If the special name \texttt{'traperror'} is used instead of a specific error message as the parameter to the \texttt{stoperror} command, execution of a procedure stops when \textit{any} error that \textit{is} trapped occurs. If the \texttt{errorMessage} parameter is entered in the form \texttt{traperror["message"]} to \texttt{stoperror}, the debugger is invoked only if the error specified by \texttt{"message"} \textit{is} trapped.
When execution of a procedure stops due to an error which causes an exception, continued execution is not possible. Any of the execution control commands, such as next or step (see Controlling the Execution of a Procedure During Debugging I on page 287 and Controlling the Execution of a Procedure During Debugging II on page 305), process the error as if the debugger had not intervened. For example, consider the following two procedures. The first procedure, \( f \), calculates \( 1/x \). The other procedure, \( g \), calls \( f \) but traps the "division by zero" error that occurs when \( x = 0 \).

\[
> f := \text{proc}(x) \ 1/x \ \text{end:} \\
> g := \text{proc}(x) \ \text{local r}; \\
> \quad \text{try} \\
> \quad \quad f(x); \\
> \quad \quad \text{catch:} \\
> \quad \quad \quad \text{infinity}; \\
> \quad \quad \quad \text{end try}; \\
> \quad \text{end proc:}
\]

If procedure \( g \) is executed at \( x=9 \), the reciprocal is returned.

\[
> g(9);
\]

\[
\frac{1}{9}
\]

At \( x=0 \), as expected, infinity is returned.

\[
> g(0);
\]

\[
\infty
\]

The stoperror command stops execution when you call \( f \) directly.

\[
> \text{stoperror("numeric exception: division by zero")};
\]

[“numeric exception: division by zero”]

\[
> f(0);
\]

Error, numeric exception: division by zero

\[
f: \quad 1 \quad 1/x
\]
Chapter 8: Debugging and Efficiency

DBG > cont
Error, (in f) numeric exception: division by zero

The call to f from g is inside a traperror (try...catch statement), so the "division by zero" error does not invoke the debugger.
> g(0);

∞

Instead, try to use stoperror(traperror).
> unstoperror( "numeric exception: division by zero" );

[]

> stoperror( 'traperror' );

[traperror]

This time Maple does not stop at the error in f.
> f(0);
Error, (in f) numeric exception: division by zero

However, Maple invokes the debugger when the trapped error occurs.
> g(0);
Error, numeric exception: division by zero
f:
    1   1/x
8.2 Maple Debugger Commands

DBG > step
Error, numeric exception: division by zero
g:
    3    infinity

DBG > step
∞

In the case that a particular error message is specified in the form traperror["message"], the debugger is invoked only if the error specified by "message" is trapped.

Clearing Watchpoints on Specified Errors Error watchpoints are cleared by using the top-level unstoperror command. The syntax to call unstoperror is the same as for stoperror. Like stoperror, unstoperror returns a list of all (remaining) error watchpoints.

If no argument is entered in the call to unstoperror, then all error watchpoints are cleared.

> unstoperror();
[
]

Controlling the Execution of a Procedure During Debugging II
Once the execution of a procedure is stopped and the debugger is invoked, you can examine the values of variables or perform other experiments (see the following section, Changing the State of a Procedure During Debugging). After you have examined the state of the procedure, you can cause execution to continue by using a number of different debugger commands.

The most commonly used debugger commands are into, next, step, cont, outfrom, return, and quit.

The return debugger command causes execution of the currently active procedure call to complete. Execution stops at the first statement after the current procedure.
Chapter 8: Debugging and Efficiency

The other commands are described in the tutorial in 8.1 A Tutorial Example. For more information about these and other debugger commands, refer to ?debugger.

Changing the State of a Procedure During Debugging

When a breakpoint or watchpoint stops the execution of a procedure, the Maple debugger is invoked. In the debugger mode, you can examine the state of the global variables, local variables, and parameters of the stopped procedure. You can also determine where execution stopped, evaluate expressions, and examine procedures.

While in the debugger mode, you can evaluate any Maple expression and perform assignments to local and global variables. To evaluate an expression, enter the expression at the debugger prompt. To perform assignments to variables, use the standard Maple assignment statement.

```maple
> f := proc(x) x^2 end proc:
> stopat(f);

[f]

> f(10);

f:
   1* x^2

DBG > sin(3.0)

.1411200081
f:
   1* x^2

DBG > cont

100
```

The debugger evaluates any variable names that you use in the expression in the context of the stopped procedure. Names of parameters or local variables evaluate to their current values in the procedure. Names of global variables evaluate to their current values. Environment variables,
such as \texttt{Digits}, evaluate to their values in the stopped procedure's environment.

If an expression corresponds to a debugger command (for example, your procedure has a local variable named \texttt{step}), you can still evaluate it by enclosing it in parentheses.

\begin{verbatim}
> f := proc(step) local i;
>     for i to 10 by step do
>         i^2
>     end do;
> end proc:

> stopat(f,2);

[f]

> f(3);

f:
   2* i^2

DBG > step

1
f:
   2* i^2

DBG > (step)

3
f:
   2* i^2

DBG > quit

Warning, computation interrupted
While execution is stopped, you can modify local and global variables by using the assignment operator (:=). The following example sets a breakpoint in the loop only when the index variable is equal to 5.

```maple
> sumn := proc(n) local i, sum;
  sum := 0;
  for i to n do
    sum := sum + i
  end do;
end proc:

> showstat(sumn);

sumn := proc(n)
local i, sum;
  1 sum := 0;
  2 for i to n do
  3    sum := sum+i
  end do
end proc

> stopat(sumn,3,i=5);

[sumn]

> sumn(10);

10

sumn:
 3? sum := sum+i

Reset the index to 3 so that the breakpoint is encountered again.

DBG > i := 3

sumn:
 3? sum := sum+i
8.2 Maple Debugger Commands

```plaintext
DBG > cont
17
sumn:
  3? sum := sum+i
```

Maple has added the numbers 1, 2, 3, 4, 3, and 4 and returned 17 as the result. Continuing the execution of the procedure, the numbers 5, 6, 7, 8, 9, and 10 are added and 62 is returned as the result.

```
DBG > cont
```

62

**Examining the State of a Procedure During Debugging**

There are two debugger commands available that return information about the state of the procedure execution. The `list` debugger command shows you the location within a procedure where execution stopped, and the `where` debugger command shows you the stack of procedure activations.

The `list` debugger command has the following syntax.

```
list procedureName statementNumber[..statNumber]
```

The `list` debugger command is similar to `showstat`, except in the case that you do not specify any arguments. If no arguments are included in the call to `list`, only the five previous statements, the current statement, and the next statement to be executed are displayed. This provides some context in the stopped procedure. In other words, it indicates the static position where execution stopped.

The `where` debugger command shows you the stack of procedure activations. Starting from the top-level, it shows you the statement that is executing and the parameters it passed to the called procedure. The `where` debugger command repeats this for each level of procedure call until it reaches the current statement in the current procedure. In other words, it indicates the dynamic position where execution stopped. The `where` command has the following syntax.
To illustrate these commands, consider the following example. The procedure `check` calls the `sumn` procedure from the previous example.

```maple
> check := proc(i) local p, a, b;
> p := ithprime(i);
> a := sumn(p);
> b := p*(p+1)/2;
> evalb( a=b );
> end proc:
```

There is a (conditional) breakpoint in `sumn`.

```maple
> showstat(sumn);
```

```
sumn := proc(n)
local i, sum;
1 sum := 0;
2 for i to n do
3? sum := sum+i
end do
end proc
```

When `check` calls `sumn`, the breakpoint invokes the debugger.

```maple
> check(9);
```

```
10
sumn:
3? sum := sum+i
```

The `where` debugger command reveals that:

- `check` was invoked from the top-level with argument 9,
- `check` called `sumn` with argument 23, and
- execution stopped at statement number 3 in `sumn`.

`DBG > where`
TopLevel: check(9)
[9]
check: a := sumn(p)
[23]
sumn:
  3? sum := sum+i

DBG > cont

true

The next example illustrates the use of where in a recursive function.

> fact := proc(x)
>       if x <= 1 then
>         1
>       else
>         x * fact(x-1)
>       end if;
> end proc:

> showstat(fact);

fact := proc(x)
  1 if x <= 1 then
  2 1
  3 else
  4   x*fact(x-1)
    end if
end proc

> stopat(fact,2);

[ fact ]

> fact(5);

fact:
  2* 1
If you are not interested in the entire history of the nested procedure calls, use the \texttt{numLevels} parameter in the call to \texttt{where} to print a specified number of levels.

\begin{verbatim}
DBG > where 3
fact: x*fact(x-1)
[2]
fact: x*fact(x-1)
[1]
fact:
  \ 2*  \ 1
\end{verbatim}

\begin{verbatim}
DBG > quit
Warning, computation interrupted
\end{verbatim}

The \texttt{showstop} command (and the \texttt{showstop} debugger command) displays a report of all currently set breakpoints, watchpoints, and error watchpoints. Outside the debugger at the top-level, the \texttt{showstop} command has the following syntax.
8.2 Maple Debugger Commands

showstop();

The next example illustrates the use of showstop.

```maple
> f := proc(x) local y;
> if x < 2 then
> y := x;
> print(y^2);
> end if;
> print(-x);
> x^3;
> end proc:

Set breakpoints.

> stopat(f):
> stopat(f,2):
> stopat(int);

[f, int]

Set watchpoints.

> stopwhen(f,y):
> stopwhen(Digits);

[Digits, [f, y]]

Set an error watchpoint.

> stoperror( "numeric exception: division by zero" );

[“numeric exception: division by zero”]

The showstop command reports all the breakpoints and watchpoints.

> showstop();

Breakpoints in:
  f
  int

Watched variables:
  Digits
  y in procedure f

Watched errors:
"numeric exception: division by zero"

**Using Top-Level Commands at the Debugger Prompt**

The `showstat`, `stopat`, `unstopat`, `stopwhen`, `unstopwhen`, `stoperror`, and `showstop` commands can be used at the debugger prompt. The following list describes the syntax rules for top-level commands used at the debugger prompt.

- Do *not* enclose the arguments of the command in parentheses.

- Do *not* separate the arguments of the command with a comma. The arguments must be separated by a space character.

- Do *not* use colons or semicolons to end statements.

- The procedure name is *not* required by any command. Commands that use a procedure name assume the currently stopped procedure if one is not specified.

- For the `stoperror` command, double quotes are *not* required.

Except for these rules, the debugger prompt call for each command is of the same form and takes the same arguments as the corresponding top-level command call.

**Restrictions**

At the debugger prompt, the only permissible Maple statements are debugger commands, expressions, and assignments. The debugger does not permit statements such as `if`, `while`, `for`, `read`, and `save`. However, you can use `if` to simulate an `if` statement, and `seq` to simulate a loop.

The debugger cannot set breakpoints in, or step into, built-in routines, such as `diff` and `has`. These routines are implemented in C and compiled into the Maple kernel. Debugging information about these routines is not accessible to Maple. However, if a built-in command calls a library routine, for example, the `diff` command calling `diff/sin`, you can use a breakpoint to stop in the latter.

If a procedure contains two identical statements that are expressions, the debugger cannot determine with certainty the statement at which execution stopped. If this situation occurs, you can still use the debugger
and execution can continue. The debugger merely issues a warning that the displayed statement number may be incorrect.\footnote{This problem occurs because Maple stores all identical expressions as a single occurrence of the expression. The debugger cannot determine at which invocation execution stopped.}

### 8.3 Detecting Errors

This section describes some simple commands that you can use for detecting errors in procedures that are written in Maple. If you are not successful in finding the error by using these commands, you can use the Maple debugger, which is discussed in 8.1 A Tutorial Example and 8.2 Maple Debugger Commands, to display the stepwise execution of a procedure.

#### Tracing a Procedure

The simplest tools available for error detection in Maple are the `printlevel` global variable, and the `trace` and `tracelast` commands. These facilities enable you to trace the execution of both user-defined and Maple library procedures. However, they differ in the type of information that is returned about a procedure.

The `printlevel` variable is used to control how much information is displayed when a program is executed. By assigning a large integer value to `printlevel`, you can monitor the execution of statements to selected levels of nesting within procedures. The default value of `printlevel` is 1. Larger, positive integer values cause the display of more intermediate steps in a computation. Negative integer values suppress the display of information.

The `printlevel` global variable is set by using the following syntax, where `n` is the level to which Maple commands are evaluated.

```maple
printlevel := n;
```

To determine what value of `n` to use, remember that statements within a particular procedure are recognized in levels that are determined by the nesting of conditional or repetition statements, and by the nesting of procedures. Each loop or `if` condition increases the evaluation level by 1, and each procedure call increases the evaluation level by 5. Alternatively, you can use a sufficiently large value of `n` to ensure that all levels are traced.
For example, `printlevel := 1000` displays information in procedures up to 200 levels deep.

```maple
> f := proc(x) local y; y := x^2; g(y) / 4; end proc;

f := proc(x) local y; y := x^2; 1/4 * g(y) end proc

> g := proc(x) local z; z := x^2; z * 2; end proc;

g := proc(x) local z; z := x^2; 2 * z end proc

> f(3);

81
2

> printlevel := 5;

printlevel := 5

> f(3);

{--> enter f, args = 3

y := 9

81
2

<-- exit f (now at top level) = 81/2}

> printlevel := 10;

printlevel := 10
8.3 Detecting Errors

> f(3);

{--> enter f, args = 3

   y := 9

{--> enter g, args = 9

   z := 81

   162

<-- exit g (now in f) = 162}

   \frac{81}{2}

<-- exit f (now at top level) = 81/2}

   \frac{81}{2}

The amount of information that is displayed depends on whether the call to the procedure was terminated with a colon or a semicolon. If a colon is used, only entry and exit points of the procedure are printed. If a semicolon is used, the results of the statements are also printed.

To reset the value of the \texttt{printlevel} variable, reassign its value to 1.

> printlevel := 1;

\texttt{printlevel := 1}

By assigning a large value to \texttt{printlevel}, the trace of all subsequent Maple procedure calls is displayed. To display the trace of specific procedures, you can use the \texttt{trace} command. The \texttt{trace} command has the following syntax, where \texttt{arguments} is one or more procedure names.
The `trace` command returns an expression sequence containing the names of the traced procedures. To begin tracing, you must call the procedure.

```
> trace(f,g);

f, g
```

```
> f(3):

{--> enter f, args = 3
{--> enter g, args = 9
<-- exit g (now in f) = 162}
<-- exit f (now at top level) = 81/2}

> f(3);

{--> enter f, args = 3

y := 9

{--> enter g, args = 9

z := 81

162

<-- exit g (now in f) = 162}

81
\[\frac{81}{2}\]

<-- exit f (now at top level) = 81/2}
```
Like printlevel, the amount of information that is displayed during tracing when trace is used depends on whether the call to the procedure was terminated with a colon or a semicolon. If a colon is used, only entry and exit points of the procedure are printed. If a semicolon is used, the results of the statements are also printed.

To turn off the tracing of specific procedures, use the untrace command.

```plaintext
> untrace(f,g);

f, g

> f(3);
```

If a procedure returns an error message, you can use the tracelast command to determine the last statement executed and the values of variables at the time of the error. The tracelast command has the following syntax.

```
tracelast;
```

When a procedure returns an error message, the following information is returned from a call to tracelast.

- The first line displays which procedure was called and what parameter was used.
- The second line displays the # symbol, the procedure name with the line number of the statement that was executed, and the statement that was executed.
- Finally, if there are any local variables in the procedure, they are displayed with their corresponding values.

---

4You can use debug and undebug as alternate names for trace and untrace.
Chapter 8: Debugging and Efficiency

> f := proc(x) local i,j,k;
> i := x;
> j := x^2;
> seq(k, k=i..j);
> end proc;

\[ f := \text{proc}(x) \text{local} i, j, k; \]
\[ i := x; \]
\[ j := x^2; \]
\[ \text{seq}(k, k=i..j); \]
\[ \text{end proc} \]

\[ f(2,3); \]
\[ \text{Error, (in f) unable to execute seq} \]

> tracelast;

\[ f \text{ called with arguments: 2, 3} \]
\[ #(f2,3): \text{seq}(k,k = i .. j) \]
\[ \text{Error, (in f) unable to execute seq} \]

locals defined as: i = 2, j = j, k = k

You can find the error in this procedure by studying the results of the \text{tracelast} command—the assignment to the local variable \( j \) incorrectly uses an equal sign (=) instead of an assignment operator (:=).

The information provided by \text{tracelast} can become unavailable whenever Maple does a garbage collection. Therefore, it is advisable to use \text{tracelast} immediately after an error occurs.  

Using Assertions
An assertion is a statement about a procedure that you assert to be true. You can include assertions in your procedure to guarantee pre- and post-conditions, and loop invariants during execution by using the \text{ASSERT} command. You can also use assertions to guarantee the value returned by a procedure or the value of local variables inside a procedure. The \text{ASSERT} command has the following syntax.

\[ \text{assertions:} \]

\[ \text{assertions:} \]

5For more information about garbage collection in Maple, see page 335 or refer to \texttt{?gc}.
8.3 Detecting Errors

\begin{verbatim}
ASSERT( condition, message );
\end{verbatim}

If \texttt{condition} evaluates to \texttt{false}, an error is generated and \texttt{message} is printed. If the first argument evaluates to \texttt{true}, \texttt{ASSERT} returns \texttt{NULL}.

To check assertions, turn on assertion checking prior to executing a procedure that contains an \texttt{ASSERT} command. To query the current state of assertion checking, or turn assertion checking on or off, use the \texttt{kernelopts} command.\(^6\)

The default state for assertion checking is \texttt{false}.

\begin{verbatim}
> kernelopts(ASSERT); #query the current state
false
\end{verbatim}

If you enter a \texttt{kernelopts} command to turn assertion checking on, \texttt{kernelopts} returns its \textit{previous} value.

\begin{verbatim}
> kernelopts(ASSERT=true);
false
\end{verbatim}

At any time during the Maple session, you can confirm whether assertion checking is on by entering the following command.

\begin{verbatim}
> kernelopts(ASSERT);
true
\end{verbatim}

If assertion checking is on and a procedure that contains an \texttt{ASSERT} statement is executed, the condition represented by the \texttt{ASSERT} statement is checked.

\begin{verbatim}
> f := proc(x,y) local i,j;
> i:=0;
> j:=0;
> while (i <> x) do
>     ASSERT(i > 0,"invalid index");
>     j := j + y;
>     i := i + 1;
> end do;
> j;
> end proc;
\end{verbatim}

\(^6\)For more information about \texttt{kernelopts}, refer to \texttt{?kernelopts}.
Chapter 8: Debugging and Efficiency

\[ f := \text{proc}(x, y) \]
\[ \text{local} i, j; \]
\[ i := 0; \]
\[ j := 0; \]
\[ \text{while } i \neq x \text{ do} \]
\[ \text{ASSERT}(0 < i, "invalid index"); j := j + y; i := i + 1 \]
\[ \text{end do}; \]
\[ j \]
\[ \text{end proc} \]

> f(2,3);

Error, (in f) assertion failed, invalid index

Use the \texttt{kernelopts} command again to turn assertion checking off. (Again, \texttt{kernelopts} returns its previous value.) When assertion checking is off, the overhead of processing an \texttt{ASSERT} statement in a procedure is negligible.

> kernelopts(ASSERT=false);

\texttt{true}

Related to assertions are Maple warning messages. The \texttt{WARNING} command causes a specified warning, preceded by the string "Warning," to display. The \texttt{WARNING} command has the following syntax.

\texttt{WARNING( msgString, msgParam1, msgParam2, ... );}

The \texttt{msgString} parameter is the text of the warning message and \texttt{msgParami} are optional parameters to substitute into \texttt{msgString}, if any.

> f := \text{proc}(x) \]
> if x < 0 then \]
> \hspace{1em} \text{WARNING("the result is complex")} \]
> \hspace{1em} \text{end if;} \]
> \hspace{1em} \text{sqrt(x)} \]
> \hspace{1em} \text{end proc;}
8.3 Detecting Errors

\[f := \text{proc}(x)\]
\[\text{if } x < 0 \text{ then WARNING(“the result is complex”) end if;}\]
\[\sqrt{x}\]
\[\text{end proc}\]

\(> f(-2)\);

Warning, the result is complex

\[\sqrt{2}i\]

You can turn the WARNING command off by using \text{interface(warnlevel=0)}. In this case, the warning is not displayed and the call to \text{WARNING} has no effect.

\(> \text{interface(warnlevel=0)};\)

3

\(> f(-2);\)

\[\sqrt{2}i\]

Handling Exceptions

An \textit{exception} is an event that occurs during the execution of a procedure that disrupts the normal flow of instructions. Many kinds of errors can cause exceptions, for example, attempting to read from a file that does not exist. Maple has two mechanisms available when such situations arise:

- the \texttt{error} statement to raise an exception, and
- the \texttt{try...catch...finally} block to handle exceptions.

Raising Exceptions The \texttt{error} statement raises an exception. Execution of the current statement sequence is interrupted, and the block and procedure call stack is popped until either an exception handler is encountered, or execution returns to the top-level (in which case the exception becomes an error). The \texttt{error} statement has the following syntax.
The `msgString` parameter is a string that gives the text of the error message. It can contain numbered parameters of the form `%n` or `%-n`, where `n` is an integer. These numbered parameters are used as placeholders for actual values. In the event that the exception is printed as an error message, the actual values are specified by the `msgParams`.

For example, the error message "f has a 4th argument, x, which is missing" is specified by the following `error` statement.

```
error "%1 has a %-2 argument, %3, which is missing", f, 4, x
```

A numbered parameter of the form `%n` displays the `n`th `msgParam` in line-printed notation (that is, as `lprint` would display it). A numbered parameter of the form `%-n` displays the `n`th `msgParam`, assumed to be an integer, in ordinal form. For example, the `%-2` in the error statement above is displayed as “4th”. The special parameter `%0` displays all the `msgParams`, separated by a comma and a space.

The `error` statement evaluates its arguments, and then creates an exception object which is an expression sequence with the following elements.

- The name of the procedure in which the exception was raised, or the constant 0 if the exception was raised at the top-level.
- The `msgString`.
- The `msgParams`, if any.

The created exception object is assigned to the global variable `lastexception` as an expression sequence.\(^7\)\(^8\)

The `error` statement normally causes an immediate exit from the current procedure to the Maple session. Maple prints an error message of the following form.

---

\(^7\)The actual arguments to the `error` statement are also assigned to `lasterror` for compatibility with older versions of Maple. For more information, refer to `?traperror`.

\(^8\)To view the value of the `lastexception` variable inside the debugger, use the `showexception` debugger command.
8.3 Detecting Errors

Error, (in procName) msgText

In this case, msgText is the text of the error message (which is constructed from the msgString and optional msgParams of the error statement), and procName is the procedure in which the error occurred. If the procedure does not have a name, procName is displayed as unknown. If the error occurs at the top-level, outside of any procedure, the (in procName) part of the message is omitted.

The error statement is commonly used when parameter declarations are not sufficient to check that the actual parameters to a procedure are of the correct type. The following pairup procedure takes a list L of the form \([x_1, y_1, x_2, y_2, \ldots, x_n, y_n]\) as input, and creates from it a list of the form \([[[x_1, y_1], [x_2, y_2], \ldots, [x_n, y_n]]\). A simple type check cannot determine if list L has an even number of elements, so you need to check this explicitly by using an error statement.

```plaintext
> pairup := proc(L::list)
>     local i, n;
>     n := nops(L);
>     if irem(n,2) = 1 then
>         error "list must have an even number of ", n
>     end if;
>     [seq( [L[2*i-1],L[2*i]], i=1..n/2 )]
> end proc:

> pairup([1, 2, 3, 4, 5]);
Error, (in pairup) list must have an even number of entries, but had 5

> pairup([1, 2, 3, 4, 5, 6]);

[[1, 2], [3, 4], [5, 6]]
```

**Trapping Errors** The try statement is a mechanism for executing procedure statements in a controlled environment so that if an error occurs, it does not immediately terminate the procedure. The try statement has the following syntax (the finally clause is optional).
try tryStatSeq
  catch catchStrings : catchStatSeq
  finally finalStatSeq
end try

If procedure execution enters a try...catch block, the tryStatSeq is executed. If no exceptions occur during the execution of tryStatSeq, procedure execution continues with the statement after end try.

If procedure execution enters a try...catch...finally block, the tryStatSeq is executed. If no exceptions occur during the execution of tryStatSeq, the finalStatSeq in the finally clause is executed. Execution then continues with the statement after end try.

If an exception occurs during the execution of tryStatSeq (in a try...catch or try...catch...finally block), execution of tryStatSeq terminates immediately. The exception object corresponding to the exception is compared against each catchString. Any number of catch clauses can be provided, and each can have any number of catchStrings separated by commas. Alternatively, a catch clause need not have a catch string. Any given catchString (or a catch clause without one) can appear only once in a try...end try construct.

If a matching catch clause is found, or the catch clause contains no catchStrings, the catchStatSeq of that catch clause is executed, and the exception is considered to have been caught. If no matching catch clause is found, the exception is considered not caught, and is re-raised outside the try block.

When Maple is looking for a matching catch clause, the following definition of “matching” is used.

- Neither the exception object nor the catchStrings are evaluated (the exception object has already been evaluated by the error statement that produced it).

- The catchStrings are considered to be prefixes of the exception object’s msgString. If a catchString has n characters, only the first n characters of the msgString need match the catchString. This permits the definition of classes of exceptions.

- A missing catchString matches any exception.

- The “result” of a try statement (the value that % returns if it is evaluated immediately after execution of the try statement) is the result of the last statement executed in the try statement.
A `catchStatSeq` can contain an error statement with no arguments, which also re-raises the exception. When an exception is re-raised, a new exception object is created that records the current procedure name, and the message and parameters from the original exception.

Under normal circumstances, the `finalStatSeq` of the finally clause, if there is one, is always executed before control leaves the try statement. This is true in the case that an exception occurs, independent of whether it is caught or whether another exception occurs in the catch clause.

This is true even if a `catchStatSeq` re-raises the exception, raises a new one, or executes a return, break, or next statement.

Under certain abnormal circumstances, the `finalStatSeq` is not executed:

- If an exception is raised in a catch clause and this exception is caught by the debugger and the user exits the debugger, the user’s command to stop execution overrides everything.

- If one of the following untrappable exceptions occurs, the exception is not caught, and `finalStatSeq` is not executed:

  1. Computation timed out. (This can only be caught by `timelimit`, which raises a “time expired” exception that can be caught. For more information on the `timelimit` command, see page 334.)
  2. Computation interrupted. (In other words, the user pressed CTRL+C, Break, or equivalent.)
  3. Internal system error. (This indicates a bug in Maple itself.)
  4. `ASSERT` or local variable type assertion failure. (Assertion failures are not trappable because they indicate a coding error, not an algorithmic failure.)
  5. Stack overflow. (If a stack overflow occurs, there is generally not enough stack space to do anything, such as running cleanup code.)

If an exception occurs during the execution of a `catchStatSeq` or the `finalStatSeq`, it is treated in the same way as if it occurred outside the try...end try statement.

**Example 1** A useful application of the `try` and `error` statements is to abort an expensive computation as quickly and cleanly as possible. For example, suppose that you are trying to compute an integral by using one of several methods, and in the middle of the first method, you determine
that it will not succeed. You would like to abort that method and try another method. The following code implements this example.

```cpp
> try
> result := MethodA(f,x)
> catch "FAIL":
>   result := MethodB(f,x)
> end try:
```

MethodA can abort its computation at any time by executing the statement `error "FAIL"`. The catch clause will catch that exception, and proceed to try MethodB. If any other error occurs during the execution of MethodA, or if an error occurs during the execution of MethodB, it is not caught.

Another useful application of the `try` statement is to ensure that certain resources are freed when you are done with them, regardless of whether anything went wrong while you were using them.

**Example 2** Use the following code to access the Maple I/O facilities to read the lines of a file and process them in some way.

```cpp
> f := fopen("myfile",TEXT,READ):
> try
>   line := readline(f);
>   while line < 0 do
>     ProcessContentsOfLine(line);
>   line := readline(f)
> end do
> finally
>   fclose(f)
> end try:
```

In this example, if any exception occurs while reading or processing the lines of the file, it is not caught because there is no catch clause. However, `fclose(f)` is executed before execution leaves the `try` statement, regardless of whether there was an exception.

The next example uses both `catch` and `finally` clauses to write to a file instead of reading from one.

```cpp
> f := fopen("myfile",TEXT,WRITE):
> try
>   for i to 100 do
>     fprintf(f,"Result %d is %q\n",i,ComputeSomething(i))
>   end do
> catch:
>   fprintf(f,"Something went wrong: %q\n",lastexception);
>   error
> finally
>   fclose(f)
```
8.3 Detecting Errors

> end try:

If any exception occurs, it is caught with the catch clause that has no catchString, and the exception object is written into the file. The exception is re-raised by executing the error statement with no msgString. In all cases, the file is closed by executing fclose(f) in the finally clause.

Checking Syntax

The Maple maplemint command generates a list of semantic errors for a specified procedure, if any. The semantic errors for which maplemint checks include parameter name conflicts, local and global variable name conflicts, unused variable declarations, and unreachable code. The maplemint command has the following syntax.

maplemint( procedureName );

In the case where the specified procedure is free of semantic errors, maplemint returns NULL.

> f := proc() local a,i; global c;
>     for i from 1 to 10 do
>         print(i);
>         for i from 1 to 5 do
>             if (a=5) then
>                 a:=6;
>                 return true;
>             print('test');
>             end if;
>         end do;
>     end do;
> end proc;

> maplemint(f);

This code is unreachable:
    print(test)
These global variables were declared, but never used:
    c
These local variables were used before they were assigned a value:
    a
These variables were used as the same loop variable for nested loops:
    i
Similar to maplemint, Maple also has an *external* program utility called mint. The mint program is called from outside Maple and it is used to check both semantic and syntax errors in an external Maple source file. For more information about mint, refer to ?mint.

### 8.4 Creating Efficient Programs

After a Maple procedure is debugged, it is often desirable to improve the performance of the code. Maple commands are available to analyze the time and memory consumption involved in executing individual statements. Maple also provides commands to monitor the efficiency of procedures.

During the performance improvement phase, keep in mind that Maple is based on a small kernel written in C, and on large libraries of Maple code which are interpreted. Therefore, whenever possible, it is generally more efficient to perform computations by using the built-in functions in the kernel. The phrase `option builtin` is used to identify the built-in functions. For example, the `add` function is a built-in function in Maple. To determine if a function is built-in, use the `eval` command with the function name as its argument.

```
> eval(add);

proc() option builtin; 114 end proc
```

The `option builtin` phrase identifies this as a built-in function, and the number following `builtin` is a special number that identifies this particular function in the kernel.\(^9\)

### Displaying Time and Memory Statistics

A simple way to measure the time and memory requirements of an executed command at the interactive level is to use the `showtime` command. The `showtime` command has the following syntax.

\(^9\)For more information about efficiency in Maple programming, refer to ?efficiency.
8.4 Creating Efficient Programs

Once the `showtime` command is entered, subsequent Maple statements that are executed are evaluated normally with the exception that the input statements are assigned to the global variables $01$, $02$, $03$, .... Immediately after the output, the amount of CPU time taken and the amount of memory used is displayed.  

The following statements all return the sum of the same sequence of numbers. However, by using the `showtime` command, it is clear that statement $03$, which uses the `add` command, is the most efficient method with respect to time and memory consumption. The `for...do` loop is the least efficient method in this case.

```maple
> S:=0: #initialize sum
> showtime();

01 := for i from 1 to 100 do S := S + 2^i end do:

time = 0.10, bytes = 32166

02 := `+`(seq(2^i, i=1..100)):

time = 0.01, bytes = 13462

03 := add(2^i, i=1..100):

time = 0.01, bytes = 12450
```

To turn `showtime` off, enter the `off` command. Maple returns to its normal interactive mode using the standard prompt.

```maple
04 := off:
```

An alternate method for measuring the time requirements of an executed command at the interactive level is to use the `time` command. The total CPU time used since the start of the Maple session is returned. The units are in seconds and the value returned is a floating-point number. The `time` command has the following syntax.

---

$^{10}$For more information about using the global variables $0i$ at a later time in a Maple session, refer to ?history.
Time;

To find the time used to execute particular statements or groups of statements, use the following assignments.

\[
\text{st} := \text{time}();
\]
\[
\ldots \text{statements to be timed} \ldots
\]
\[
\text{time}() - \text{st};
\]

Therefore, you could use the following set of statements to calculate the amount of time (in seconds) required to add the first 10,000 powers of 2 by using the add command.

\[
> \text{st} := \text{time}(); \text{add}(2^i, i=1..10000); \text{time}() - \text{st};
\]

8.402

Profiling a Procedure

Used in conjunction with the profile command, the showprofile command is used to display run-time information about a procedure. The run-time information is displayed in tabular form and it contains the number of calls to the procedure, the nesting level of each call, the CPU time used, and the number of bytes used by each call. To turn on profiling, use the profile command.

\[
\text{profile( procedureName )};
\]

Then, to display the run-time information collected for the specified procedure, use the showprofile command. If no argument is supplied to showprofile, the run-time information for all profiled procedures is displayed.

\[
\text{showprofile( procedureName )};
\]

To illustrate the use of profiling in Maple, consider the following procedures which compute the nth Fibonacci number. Both procedures contain the same code except that \text{fib1} uses option remember.\textsuperscript{11}

\textsuperscript{11}For more information about option remember, see page 210 or refer to ?remember.
> fib1:=proc(n) option remember;
> if n<2 then
> n
> else
> fib1(n-1)+fib1(n-2)
> end if;
> end proc;

\[
\text{fib1 := proc}(n) \\
\text{option remember;}
\text{if n < 2 then n else fib1(n - 1) + fib1(n - 2) end if}
\text{end proc}
\]

> fib2:=proc(n)
> if n<2 then
> n
> else
> fib2(n-1)+fib2(n-2)
> end if;
> end proc;

\[
\text{fib2 := proc}(n) \\
\text{if n < 2 then n else fib2(n - 1) + fib2(n - 2) end if}
\text{end proc}
\]

Turn on profiling for both procedures.

> profile(fib1);
> profile(fib2);

Execute the procedures.

> fib1(10);

55

> fib2(10);

55

Use \texttt{showprofile} to display the run-time information about \texttt{fib1} and \texttt{fib2}.

> showprofile();
By studying the run-time information, particularly the number of calls to each procedure, you can see that it is more efficient to use option remember in a recursive procedure.

To turn off profiling, use the unprofile command. If no argument is supplied to unprofile, all procedures currently profiled are returned to their original state.

```
unprofile( procedureName );
```

When a procedure is unprofiled, all run-time information for that procedure is lost.

```
> unprofile();
> showprofile();
```

<table>
<thead>
<tr>
<th>function</th>
<th>depth</th>
<th>calls</th>
<th>time</th>
<th>time%</th>
<th>bytes</th>
<th>bytes%</th>
</tr>
</thead>
<tbody>
<tr>
<td>fib2</td>
<td>10</td>
<td>177</td>
<td>0.040</td>
<td>80.00</td>
<td>95300</td>
<td>88.61</td>
</tr>
<tr>
<td>fib1</td>
<td>10</td>
<td>19</td>
<td>0.010</td>
<td>20.00</td>
<td>12244</td>
<td>11.39</td>
</tr>
<tr>
<td>total:</td>
<td>20</td>
<td>196</td>
<td>0.050</td>
<td>100.00</td>
<td>107544</td>
<td>100.00</td>
</tr>
</tbody>
</table>

## 8.5 Managing Resources

Maple provides a number of commands to use for managing the computer’s resources during computation. In particular, timelimit controls the maximum amount of time available for a computation, gc causes garbage collection, and kernelopts provides communication with the Maple kernel.

### Setting a Time Limit on Computations

The timelimit command is used to limit the amount of CPU time for a computation. The timelimit command has the following syntax, where time is the time limit (in seconds) to evaluate expression.
8.5 Managing Resources

• timelimit(time, expression);

If the expression is successfully evaluated within the specified time, timelimit returns the value of the expression. If the time limit is reached before the expression is evaluated, timelimit generates an error message.

> f := proc ()
> local i;
> for i to 100000 do
> 2^i
> end do
> end proc:

> timelimit(0.25, f());
Error, (in f) time expired

Garbage Collection
Garbage collection deletes all objects that are no longer in use by the program and are occupying space in memory. In Maple, garbage collection also clears the remember tables of procedures that use an option system or option builtin by removing entries that have no other references to them.\(^{12}\)

The Maple garbage collection function is gc. The gc command has the following syntax.

gc();

The gc command explicitly invokes a garbage collection process and returns NULL. Otherwise, garbage collections are done automatically by Maple every 1,000,000 words used. To change the frequency of automatic garbage collections, use the kernelopts command.

The kernelopts command can also be used to query other garbage collection information such as the number of bytes returned after the last garbage collection, and the number of times garbage collection has been invoked. The kernelopts command is discussed more in the next section.

\(^{12}\)For more information about procedure options, see page 206 or refer to ?options.
Communicating with the Kernel

The kernelopts command\(^{13}\) is provided as a mechanism of communication between the user and the kernel. You have already seen how to use kernelopts to include assertions in procedures. Specifically, this command is used to set and query variables that affect computations in Maple.\(^{14}\) For example:

- \texttt{kernelopts(gcfreq)} sets or queries the frequency of automatic garbage collections.
- \texttt{kernelopts(gcbytesavailable)} queries the number of bytes available after the last garbage collection.
- \texttt{kernelopts(gctimes)} queries the number of times that garbage collection has been invoked.
- \texttt{kernelopts(memusage)} reports the amount of storage used by objects of different types.
- \texttt{kernelopts(stacklimit)} sets the total amount of stack space, in kilobytes, that Maple can consume.

For more information about kernelopts, refer to \texttt{?kernelopts}.

8.6 Exercises

1. The following procedure tries to compute \(1 - x^{|a|}\).

\begin{verbatim}
> f := proc(a::integer, x::anything)
> if a<0 then
> a := -a
> end if;
> 1-x^~a;
> end proc;
\end{verbatim}

Determine what is wrong with this procedure.

\textit{Hint:} Use the Maple debugger described in 8.1 A Tutorial Example and 8.2 Maple Debugger Commands to isolate the error.

\(^{13}\)For information about the KernelOpts Maplet application, which provides a graphical user interface to the kernel options, refer to \texttt{?Maplets[Examples][KernelOpts]}.

\(^{14}\)For more information about kernelopts, refer to \texttt{?kernelopts}.
2. The following recurrence relation defines the Chebyshev polynomials of the first kind, $T_n(x)$.

$$T_0(x) = 1, \quad T_1(x) = x, \quad T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

The following procedure computes $T_n(x)$ in a loop for any given integer $n$.

```maple
> T := proc(n::integer, x) local t1, tn, t;
>     t1 := 1; tn := x;
>     for i from 2 to n do
>         t := expand(2*x*tn - t1);
>         t1 := tn; tn := t;
>     end do;
>     tn;
> end proc:
```

Warning, ‘i’ is implicitly declared local to procedure ‘T’

This procedure has several errors. Which variables must be declared local? What happens if $n$ is zero or negative? Identify and correct all errors, using the Maple debugger where appropriate. Modify the procedure so that it returns unevaluated if $n$ is a symbolic value.

### 8.7 Conclusion

This chapter surveyed a variety of Maple commands that are available to help you find errors in procedures, and those available to analyze the time and memory consumption involved in running a program. In particular, the Maple debugger was presented as a tool that you can use to find and correct errors.
A Maplet application is a graphical user interface for Maple, which is launched from a Maple session. It allows a Maple software user to combine packages and procedures with interactive windows and dialogs.

This chapter provides an overview of the Maplets package. For detailed information and a tutorial, enter ?Maplets at the Maple prompt.

In This Chapter

- Uses of Maplet applications
- Structure of the Maplets package
- Elements used to construct a Maplet application definition
- Example Maplet applications and authoring tools provided in the package
- Writing and running Maplet applications

9.1 Uses of Maplet Applications

The Maplet user interface customization system can be used to create custom Maple calculators, interfaces to Maple packages and routines, queries, and messages.
Custom Maple Calculators

The Maplet system can be used to create calculators. Students or professionals, who have little or no experience with Maple, can use the calculators.
Interfaces to Maple Packages and Routines

A Maplet application can facilitate changing default values and options in Maple packages and routines. For example, a package may have a list of global settings, which users need or want to modify occasionally. A Maplet application to a Maple package can also be created to help users learn about a package. For example, many routines have a myriad of associated options. Giving users an interface to options facilitates the use of a package. The following Maplet application is a graphical interface to a function in the LinearAlgebra package.\(^1\)\(^2\)

![Maplet Application](image)

\(^1\)For more information on this Maplet application, refer to ?examples/VectorNormMaplet.

\(^2\)For information on using this Maplet application, refer to ?Maplets[Examples][VectorNorm].
Chapter 9: Introduction to the Maplet User Interface Customization System

Queries
The Maplet system can be used to create queries. Users can be prompted for information, from the basic name and student number query, to ‘Is x positive?’ and arguments to a routine.

![Question](image)

Messages
The Maplet system can be used to create various messages, which display information, a warning, or an error. Additionally, users can be informed of their progress in a session.

![Message](image)

9.2 The Maplets Package
The Maplets package contains three subpackages:

- Elements
- Examples
- Tools

and one top-level function.

- Display
9.3 Terminology

Elements
Elements are the individual components used to create a Maplet application, for example, windows, buttons, and check boxes. A variety of different elements are used in the Maplets[Examples] subpackage. Each element has a help page, describing various options.

Examples
Example Maplet applications show how the Maplet system can be used. The more complicated examples have an associated help page and worksheet describing how particular Maplet applications were constructed. The worksheets guide you from simple to complex Maplet applications.

Tools
Tools are aids for Maple software users developing Maplet applications.

Display
The Display function is used to display (run) a Maplet application.

9.3 Terminology

Maplet Application A Maplet application is a collection of elements, such as window, layout, dialog, and command elements. A Maplet application contains windows and dialogs.

Maplet Application Author A programmer who uses Maple code to write a Maplet application.

Maplet Application User Someone who interacts with a Maplet application.

Layout A layout defines how elements within a Maplet application are visually assembled.

Window A window should not be thought of as the Maplet application, but rather as the top-level element within a Maplet application.

Window Element An element that creates a window within a Maplet application. A Maplet application can contain more than one window. Each window can contain many elements that control the layout and function of the window. The Window element is classified under Other Elements.
Window Body Elements  Window body elements is a category of elements that specify viewable elements in a window other than menu bars and toolbars. There are many viewable elements in the window body category, including drop-down boxes, sliders, and tables.

Dialog Element  A dialog element has a predefined layout. For a dialog, a Maplet application author can only specify text. This is different from the Window element, which can contain other elements, for example, buttons and layout elements. The dialog elements in the Maplets package include the color selection and alert dialogs.

9.4 Elements

Each element belongs to one of seven categories:

- Window Body Elements
- Layout Elements
- Menubar Elements
- Toolbar Elements
- Command Elements
- Dialog Elements
- Other Elements

The most significant element, which can be included in a Maplet application, is the Window element. A Window element can contain layout, menu, toolbar, and command elements.

The Elements section of this guide provides:

- Descriptions of individual elements.
- A short section on Reference Options.

Window Body Elements

Button  Defines a button that can appear in a Maplet application window. Each button can be associated with an Action element, which is run when the button is clicked. The text, font, colors, and other properties of the button can be modified.
**CheckBox**  Displays a box in a Maplet application window which, when selected, contains a check mark. Like buttons, each check box can be associated with an Action element, which is run when the check box is selected or cleared.

**ComboBox**  Displays a box in which users can enter a value or select from a predefined list of strings. An initial value can be specified, otherwise it defaults to the first element in the list. The predefined list of strings can be entered either by using Item elements, or simply as a list of strings. When the combo box is queried for its value, the currently selected string is returned.
Chapter 9: Introduction to the Maplet User Interface Customization System

**DropDownBox** The DropDownBox is similar to the ComboBox, except that users cannot enter a value. This is most useful when users must select from a finite set of objects. To view an example of both elements, compare the VectorNorm and MatrixNorm advanced example Maplet applications in the LinearAlgebra subpackage of the Maplets[Examples] package.³

![DropDownBox Example](image)

**Label** Contains either a single line of text or an image. If multiple lines of text are preferred, a TextBox element is more appropriate. A Label element (text or image) in a running Maplet application cannot be highlighted and copied. To create text that can be highlighted and copied within a Maplet application, use the TextField or TextBox element. For more information, see TextField or TextBox element later in this section.

³For more information on the MatrixNorm and VectorNorm Maplet applications, refer to ?Maplets[Examples][MatrixNorm] and ?Maplets[Examples][VectorNorm].
**ListBox**  A list box is similar to a drop-down box, in that the user must select from a list, but more than one selection can be made, and more than one entry appears on the screen. The SHIFT and CONTROL keys are used to make multiple selections. The return value is a comma-delimited (separated) string, which can be converted to a list of strings by using the `ListBoxSplit` function from the `Maplets[Tools]` subpackage.
MathMLEditor Displays, and allows the user to edit and create mathematical expressions. To enter mathematical constructs, use the input expression templates and keyboard. Subexpressions can be deleted, cut, or copied to the clipboard as MathML. From the clipboard, you can paste them into a Maple session.

![MathMLEditor Example](image)

MathMLViewer Displays MathML expressions. The displayed expression cannot be copied and pasted into a Maple session. The expression is displayed by using formatted mathematical notation instead of Maple syntax. For example, the square root symbol is displayed not the Maple function sqrt.

![MathMLViewer Example](image)
Plotter  Displays a 2-D or 3-D static Maple plot in a Maplet application.
RadioButton A radio button is similar to a check box. It appears as a circle, which is filled when selected. Radio buttons should always be grouped together. This is done by using the 'group' option, in combination with the ButtonGroup element. For more information, see ButtonGroup element in Other Elements on page 361.

Slider Allows users to specify an integer value from a range.

Table Allows data to be arranged into columns. The TableHeader, TableItem, and TableRow elements are described in the Other Elements section on page 362.
**TextBox**  A multiple line box for input, output, or labels. The `editable` option can be set to `true` to create input fields and `false` to create output fields or labels. Text boxes have pop-up menus, which can be accessed by right-clicking the field. By default, an `editable=true` text box has the entries **Cut**, **Copy**, **Paste**, **Delete**, and **Select All** in the pop-up menu. An `editable=false` text box has **Copy** and **Select All** in the pop-up menu.

![Maplet](image)

**Note:** Other entries can be appended to the associated pop-up menu, by using the `popupmenu` option.

![Popup Menu](image)
TextField  A single line input or output field depending on whether the ‘editable’ option is set to true or false. Text fields, like text boxes, have pop-up menus, which can be accessed by right-clicking the field. Entries can be appended to the pop-up menu. For more information, see the previous description for TextBox or the Menu Elements section on page 354.

ToggleButton  A toggle button is similar to a check box, except in appearance. The toggle button can contain text or an image on the face of the button.
Layout Elements
The layout of a window describes how various elements in a Maplet application are positioned. Two different forms of layout are provided by the elements BoxLayout and GridLayout. Additionally, nested lists can be used to define box layouts. When writing a Maplet application, text strings, user prompts, or text fields are defined as expressions in lists. A Maplet application window definition includes the main list, which contains the lists of text strings, user prompts, and other elements.

The following elements are classified as layout elements:

- BoxCell
- BoxColumn
- BoxLayout
- BoxRow
- GridCell
- GridLayout
- GridRow

For details about each element, see the help pages.

BoxLayout A relative layout scheme in which you can control where items appear horizontally or vertically relative to other elements. For horizontal control in box layout, use the BoxRow element. For vertical control in box layout, use the BoxColumn element.

GridLayout A square layout where all elements must appear within a square grid. Grid layout is best used for simple layout designs. For complex layout designs, box layout is recommended.

Nested Lists Nested lists (lists of lists) can be used to define box layouts. In Maple, a list is an ordered sequence of comma-delimited expressions that is enclosed in square brackets ([ ]).

For example:

```
> List := [1,5,7];

List := [1, 5, 7]
```

A nested list is an ordered sequence of expressions that is enclosed in square brackets, in which each expression can be a list.
For example:

\[
\text{\texttt{NestedList:=[1, [2,3], [4,[5,6]], 7, 8, [9,10]];}}
\]

\[
\text{\texttt{NestedList := [1, [2, 3], [4, [5, 6]], 7, 8, [9, 10]]}}
\]

**MenuBar Elements**

The following elements are classified as menubar elements:

- **MenuBar** - Must be defined in a `Window` element.
- **Menu** - Must be defined in a `Menu`, `MenuBar`, or `PopupMenu` element.
- **MenuItem** - Must be defined in a `MenuBar` or `PopupMenu` element.
- **CheckBoxMenuItem** - Must be defined in a `MenuBar` or `PopupMenu` element.
- **RadioButtonMenuItem** - Must be defined in a `MenuBar` or `PopupMenu` element.
- **MenuSeparator** - Must be defined in a `MenuBar` or `PopupMenu` element.
- **PopupMenu** - Must be defined in a `TextField` or `TextBox` element.

A menu bar can contain any number of menus. Each menu can contain items, defined by using the `MenuItem`, `CheckBoxMenuItem`, and `RadioButtonMenuItem` elements, and submenus, defined by nesting `Menu` elements. Separators can be used to group the menu into logically distinct groups separated by horizontal bars.
The default pop-up menu contains **Copy** and **Select All**. If the box or field is editable, it also contains the selection items **Paste**, **Delete**, and **Clear**. Other entries can be appended to the pop-up menu.

![Pop-up menu example](image)

**ToolBar Elements**

A toolbar can contain any number of buttons. The buttons can be grouped into logically distinct groups by using a separator, which produces a large space between adjacent buttons. The following elements are classified as toolbar elements:

- ToolBar - Must be defined in a **Window** element.
- ToolBarButton - Must be defined in a **ToolBar** element.
- ToolBarSeparator - Must be defined in a **ToolBar** element.

![Toolbar example](image)
**Command Elements**

A Maplet application can perform actions in response to user actions such as clicking a button or changing the value in a text field. Each command element is performed before the next one is started. These are non-visual elements. The following elements are classified as command elements:

- CloseWindow
- Evaluate
- RunDialog
- RunWindow
- ShutDown
- SetOption

**CloseWindow**  Closes a running window by referencing the window.

**Evaluate**  The Evaluate command element runs a Maple procedure with the given set of arguments args in the underlying Maple session. An Evaluate element can contain Argument elements.

**RunDialog**  Displays a dialog element. The RunDialog element takes only one option 'dialog', which is a reference to the dialog to be run. If the dialog is already running, nothing happens.

**RunWindow**  Displays a Window element. The RunWindow element takes only one option 'window', which is a reference to the window to be run. If the window is already running, nothing happens.

**Note:** There are separate RunDialog and RunWindow command elements because the Maplets[Elements][Window] element is intrinsically different from the Maplets dialog elements. A dialog has a predefined structure. A Maplet application author can specify options for a dialog, but cannot add elements. A window does not have a predefined structure. A Maplet application author specifies its structure by using elements and options. They also behave differently. For example, a window can be minimized.

**SetOption**  Allows values of certain options to be changed while the Maplet application is running. For example, if a user clicks a button, the 'onchange' option of the button can use the SetOption element to initiate a change such as clearing a text field. The SetOption element is used within a Maplet application and the Set routine (see 9.6 Tools) is used within a procedure.
**Shutdown**  Closes a running Maplet application. Optionally, it can send a return value to the Maple session. It can return specific values stored within the Maplet application, for example, the content of a text field, or a fixed value.

**Dialog Elements**

Dialogs are small windows that provide information, such as alert or warning messages, and gather input, such as a filename, from users. Users respond to a dialog by clicking a button. The Maplet application author can modify only specific features in the layout, for example, the text in the title bar, in a caption, or on a button. Dialogs are displayed by using the `RunDialog` element.

**Important**: Dialog element values cannot be queried after the dialog closes. As a Maplet application author, it is important to create a Maplet application that returns relevant information to the Maple session.

**AlertDialog**  Draws attention to a potential issue. Allows users to indicate approval to continue (OK) or to heed the warning (Cancel).
Chapter 9: Introduction to the Maplet User Interface Customization System

ColorDialog  A standard color chooser that provides users with interfaces for choosing a color: Color swatches, RGB palettes, and HSB palettes.

![ColorDialog](image1.png)

ConfirmDialog  Allows users to specify whether an action is performed. A dialog box with a statement like: "Is $x$ greater than 0 ?" is displayed with the options Yes (and exit), No (but exit), and Cancel (do not exit).

![ConfirmDialog](image2.png)
FileDialog  A standard file chooser dialog.

![FileDialog](image)

InputDialog  TheInputDialog element is similar to the AlertDialog element except that it contains a text field in which the user can enter or modify data. An initial value can be included in the text field of the box when it is displayed.

![InputDialog](image)
MessageDialog  Presents users with information and closes the dialog when the OK button is clicked.

QuestionDialog  Poses a question to the user and allows the user to reply, Yes or No.
Other Elements

**Action** Defines an action in a Maplet application. Contains any number of Action or command elements to be executed.

**Argument** Specifies the argument of a Maple function call. It can be used only in an Evaluate element.

**ButtonGroup** A radio button must be associated with a button group so that at any time, only one button in the group can be selected. By using the 'group' option in combination with the ButtonGroup element, radio buttons are grouped, as required.

**Font** Specifies a font for an element. The fonts available are dependent on your operating system. If you specify a family that is not recognized by your system, the Unrecognized Font message dialog displays. To view the Maplet application using the default font, click OK. To determine which font families are recognized, click Valid fonts....

**Image** Specifies a jpeg or gif image in a Maplet application.

**Item** Specifies an entry in a ComboBox, DropDownBox, or ListBox element. The Item element cannot contain other elements.

**Maplet** Contains the elements defining a Maplet application. The top-level Maplet application element.

**Return** Encapsulates values to be returned to the Maple session when the Maplet application closes. It can contain any number of ReturnItem elements.

**ReturnItem** Specifies which values are to be returned to the Maple session when the Maplet application closes. It cannot contain other elements.
**TableHeader**  A **Table** element contains up to one **TableHeader** element, which displays the text in the header of the table, and any number of **TableRow** elements.

**Note:** A table with a header must be specified in a scroll pane, that is, a **BoxCell** or **GridCell** element with the ‘vscroll’=’as_needed’ option. Otherwise, the header is not displayed. If a table without a header is specified in a scroll pane, default header values are used: A,...Z, AA,...ZZ...

**TableWidgetItem**  Specifies an entry in a Maplet application table header or row.

**TableRow**  Specifies a row in a Maplet application table. The individual columns in a row are defined by using the **TableWidgetItem** element.

**Note:** Each **TableRow** must have the same number of **TableWidgetItem** elements. The number of **TableWidgetItem** elements in the **TableHeader**, if specified, must equal the number of **TableWidgetItem** elements in each **TableRow**.
Window  Defines a window in a Maplet application. A Maplet application can contain more than one Window element. Each window can contain many elements that control the layout and function of the window. The MenuBar and ToolBar elements (or a reference to these elements) must be defined within the Window element. When creating a window, the Maplet application author can choose from options that control the height, width, and whether the window is resizable, visible, or contains a title. A complete list of options is listed in the Maplets[Elements][Window] help page.

Reference Options
Most elements can be given an identifying reference by using the 'reference' option, or as a short cut, by placing the reference in an index.

Long form:  TextField('reference' = 'TF1', 'value' = "Hello")

Short form:  TextField[TF1]( 'value' = "Hello")

If references are specified by using both an index and an option, the index reference takes precedence. The reference can be a name or a string. Note that the variable name TF1 and the string "TF1" are interpreted as different references.

9.5 Example Maplet Applications
The Maplets[Examples] subpackage provides examples of how Maplet applications can be used to solve problems. These Maplet applications provide information to a user, query a user, or display an interface to a package or routine. The Examples package contains the following routines and subpackages:
Alert
Confirm
GetColor
GetEquation
GetExpression
GetFile
GetInput
Message
Question
Selection
SignQuery

Advanced Example Maplet Applications:
Integration
KernelOpts
ShowTable
LinearAlgebra subpackage.

Linear Algebra Subpackage
The LinearAlgebra subpackage contains the following example Maplet applications, which are interfaces for some of the LinearAlgebra routines.

- Constructors - BezoutMatrix and HilbertMatrix
- Queries - ConditionNumber
- Solvers - QRDecomposition
- Eigenvalue Problems - SingularValues
- Standard Routines - MatrixNorm and VectorNorm

Using the Example Maplet Applications
As stated in 9.1 Uses of Maplet Applications, the Maplet system can be used to create calculators, interfaces to Maple packages and routines, queries, and messages. The following example Maplet applications demonstrate these four major uses.
9.5 Example Maplet Applications

Creating a Calculator and Interface to a Maple Package  The LinearAlgebra subpackage shows how Maplet applications can be used to support an existing package.

![Singular Values Maplet](image1)

**Creating Queries**  The example GetEquation Maplet application prompts the user for an equation. If the user enters an expression, it is set equal to zero when the user clicks **OK**. The equation is returned by the Maplet application.

![Get Equation Maplet](image2)
Creating Messages  The example Message Maplet application displays a message to the user.

![Message Maplet Application](image)

For a list of all example Maplet applications, refer to ?examples,ExampleMaplets.

9.6 Tools

Maplets tools are aids for Maplet application authors. The Maplets[Tools] subpackage contains routines for manipulating and interacting with Maplet applications and Maplet application elements.

The package can be accessed by using either the long or short form of the function name in the command calling sequence. For example, the short form is with(Maplets[Tools]). For complete details regarding the command calling sequence, see the Maplets[Tools], overview help page.

The following functions are available in the Tools subpackage:

**AddAttribute**  Adds attributes to a previously constructed element. For example, if you have created and run a Maplet application that displays a button, but want to add color to the button, you must add a color attribute (`'color'="#FF00FF"`) to the Maplet application code. Instead of recoding the Maplet application, you can test the look of the Maplet application by using the AddAttribute function.

**AddContent**  Adds content to a previously constructed element.

**Get**  Retrieves the value of a specified element option from a running Maplet application. Must be used in a procedure. Cannot be used in a Maplet application definition.

**ListBoxSplit**  Converts a list box value to a list of strings.
Print  Prints the XML data structure of a Maplet application. Default values are included. This is useful when investigating why a Maplet application does not display as expected.

Set  Cannot be used in a Maplet application definition. Must be used in a procedure. The Set function sets the value of a specified element option in a running Maplet application. The SetOption (a command element) is used in a Maplet application.

StartEngine  Starts the Maplets environment.

StopEngine  Stops the Maplets environment. All running Maplet applications are closed.

Maplet System Global Variables
There are two global variables in the Maplets[Tools] subpackage that are useful tools to Maplet application authors:

lastmaplet  Each time a user runs a Maplet application by using the Display function, the Maplet application is assigned to the global variable, lastmaplet. This Maplet application can be used to debug or display the last Maplet application run.

thismaplet  Each time a procedure is evaluated by using an Evaluate action element, the global variable, thismaplet is assigned the handle for the Maplet application that called the procedure. After evaluation, the variable is reset to its previous value.

9.7  Running a Maplet Application

To run a Maplet application, you must use the top-level Display function. In the following example, the "Hello World" Maplet application is displayed (or run).

> Maplets[Display](Maplet["Hello World",
>            Button( "OK", Shutdown())]);
For more information on running a Maplet application, refer to chapter 9 of the *Maple User Manual*.

### 9.8 Writing a Maplet Application

**Defining a Maplet Application**

Each Maplet application must be defined in a top-level element `Maplet`. For example, a `Button` element is defined in the following `Maplet` element.

```maple
> Maplet(["Hello World", Button( "OK", Shutdown() )]);
```

**Note:** To run this Maplet application, use the top-level `Display` function. For more information, see 9.7 Running a Maplet Application.

**Maplet Application Programming Style Guidelines**

1. Use only one element definition per line.

2. Indent each subsequent nested element or list.

3. Place closing parentheses, brackets of elements, or nested lists at the same level of indentation as the opening line.

4. The `Font` and `Image` elements can be defined anywhere. The Maplet application code, however, is easier to read if these elements are defined first in the Maplet application. These two elements are listed under the category of Other Elements on page 361.

5. Keep attributes separate from the content of an element.

6. Place unevaluation quotes (right single quotes) around any symbols or names used as option names or references. Otherwise, if any of the symbols or names are assigned values in the user’s Maple session, your Maplet application definition will generate errors.
7. In a procedure, use the `use` statement rather than the `with` statement. If another user runs your code, the `with` command affects that user’s environment, while `use` does not.

The following illustrates correct code structure.

```plaintext
> use Maplets[Elements] in
> maplet := Maplet(
>   'onstartup' = RunWindow('W1'),
>   Font['F1']('family' = "times"),
>   Window['W1']("Vector Norm",
>     [BoxCell('left',
>       TextBox('editable' = 'false', 'width' = 50,
>         'font' = 'F1', "A vector norm is a function
>         \|V\| that satisfies the conditions of a norm."
>       ),
>       ),
>       "Norm (Select from list or enter non-negative
>       number):",
>       ComboBox['CoB1'](["infinity", "1", "Euclidean (2)", "Frobenius"])
>     ],
>     ["Vector has real entries: ",
>       CheckBox['ChB1']('true')
>     ],
>     ["Evaluate result: ",
>       CheckBox['ChB2']('true')
>     ],
>     [Button['B1']("OK", Shutdown(['CoB1','ChB1', 'ChB2'])),
>      Button['B2']("Cancel", Shutdown())
>     ])
> );
> end use;
> Maplets[Display](maplet);
```

9.9 After Reading This Chapter

When first reviewing the help pages:
1. Read the short Introduction to Maplets help page (\(?\text{Maplets}\)).

2. Depending on your experience with the Maple software and Maplets package, you can choose from three paths:

- As a Maplet application user, select the Maplet Application User Hints link.

- As a beginning Maplet application author, select the Roadmap to Using the Maplet User Interface Customization System link. The roadmap guides you through the Tutorial: Creating a Maplet Application, Maplet Application Code Style Guide worksheet, Maplet Application Layout Guidelines worksheet, Overviews of the Elements and Tools subpackages, and Example Maplet Applications.

- If you are an experienced Maplet application author, select the Maplets Package Index link. You can navigate to all the Maplets help pages and worksheets from this index.

### 9.10 Conclusion

This book described the basic programming concepts in the Maple language: the construction of expressions and other statements from tokens, data structures, flow control, procedures, I/O, debugging, and efficiency. In this chapter, the Maple Maplets package, which provides the tools necessary to create graphical user interfaces to Maple, was introduced.

For more information about programming topics, refer to the Maple online help system and the Maple Advanced Programming Guide.
Index

!, 27, 51, 78, 89
$; 26, 27, 89, 187
 vs seq, 186
%, 27, 77, 89, 105
%%, 77
%%%, 77
&, 27, 81, 89
&*, 26
', 49
(), 49
*, 8, 26, 71, 89
+, 8, 26, 27, 71, 89
,, 26, 50
-, 8, 26, 27, 71, 89
->, 26
., 26, 27, 71, 74, 89
..., 26, 77, 89
/, 8, 26, 71, 89
:; 7, 48
::, 26
:=, 8, 20, 26
;.; 7, 48
<, 26, 82, 89
<=, 26
<>, 26, 50, 82, 89
=, 20, 26, 89
=>, 82, 89
>, 6, 26, 82, 89
>=, 26, 82, 89
?, 7, 51
@, 26, 76, 89
@@, 26, 76, 89
[], 50
#, 51
%, 25
\, 51
", 7

|, 30, 42
 names, 30
 strings, 42
 vs cat, 32, 33

{}, 50
^, 26, 71, 89
`, 49
2-D, 349
3-D, 349

Action, 361
add, 183
 vs sum, 186
AddAttribute, 366
AddContent, 366
adjacency matrices, 248
Advanced Programming Guide, 2
AlertDialog, 357
and, 26, 85, 89
anything, 107
args, 224, 234
Argument, 356, 361
arithmetic operators, 8, 71
Array
 accessing, 148
 bounds, 148
 data type, 148
 entries, 148
 indexing function, 148
 op, 148
 operands, 148
array
 accessing, 147
 bounds, 147
 data type, 147
 entries, 147

371
indexing function, 147
op, 147
operands, 147
arrow option, 207, 231
ASSERT, 320
assertions
in procedures, 320
return value type, 198
variable type, 202
warnings, 322
assignment, 20
:=, 8
names, 8
vs. equality, 20
assignment statement, 111
assuming, 88, 89
audience, 1

binary operators, 26
binary search, 242
BoxCell, 353
TableHeader, 362
BoxColumn, 353
BoxLayout, 353
BoxRow, 353
break statement, 118, 179
breakpoints
explicit, 298
removing, 298
removing explicit, 300
setting, 297
built-in routines, 6, 13
builtin option, 16, 207
ButtonGroup, 350, 361
RadioButton, 361
ToggleButton, 361
C programming language
and Maple code, 13
code generation, 279
calculator, 340, 365
call_external option, 208
calling
procedures, 13
cat, 29, 41
names, 29
strings, 41
vs ||, 32, 33
Catalan, 35
characters
special, 24
CheckBox, 345
CheckBoxMenuItem, 354
classic worksheet, 1
CloseWindow, 356
codegen, 279
CodeGeneration, 279
color
HSB palettes, 358
RGB palettes, 358
swatches, 358
ColorDialog, 358
columns, 350
ComboBox, 345
Item, 361
comma-delimited expression, 353
comma-delimited string, 347
command calling sequence, 366
command-line interface, 6
command-line version, 1
comments, 47, 226
complex numbers, 68
types of, 68, 69
composition, 76
operators, 26, 76, 89
repeated, 76
concatenation, 29–33, 41–43, 55
names, 29–33
strings, 32–33, 41–43
conditional execution, 167–172
ConfirmDialog, 358
connected graphs, 246
constants
  complex numbers, 37
  floating-point numbers, 37
  fractions, 37
  integers, 37
constants, 35
cont, 305
control key, 347
converting
  data structures, 158
  sequences to sets or lists, 158
Copyright option, 208, 225
creating help pages, 227
customer feedback, 4
data
  exporting, 278
  importing, 274
  reading, 271
  saving, 271
data structures, 89–91, 123–166
  and procedures, 239
  converting, 158
DEBUG, 298
debugger, 284, 296
  breakpoints, 297, 298
  clearing watchpoints, 301
  controlling execution, 305
  explicit breakpoints, 298
  interactive, 284
  invoking, 296
  numbering statements, 296
  removing explicit breakpoints, 300
  tutorial, 284
  watchpoints, 301, 302
debugging
  procedures, 284, 296
default
  pop-up menu, 355
declare external, 208
defining
  Maplet applications, 368
  procedures, 9
definition
  procedure, 12
description, 226
detecting
  errors, 315
Digits, 35
Display, 343, 367
display
  procedure, 12
displaying
  library source code, 229
ditto operators, 25, 77
do loop, 176
double quotes
  `, 7
  and strings, 7
DropDownBox, 346
  Item, 361
delif, 167
defined, 168
declare external, 368
end proc, 9, 193
entering
  input, 6
  statements, 6
environment
  worksheet, 6
equality, 20
testing
  evalb, 20
  vs. assignment, 20
equations, 20
ero statement, 118, 219, 323
error return, 219
errors
  detecting, 315
  syntax, 19
  trapping, 302, 325
eval, 15
evalb, 20
Evaluate, 356
thismaplet, 367
evaluation
delayed, 49
eval, 15
evalb, 20
full, 14
last name, 14
of parameters, 200
evaluation rules
parameters, 199
variables, 203
exceptions, 323
executing
procedures, 10, 13, 194
explicit return, 217
exponent, 66
exporting
data, 278
factorial operator, 78
FAIL, 35
false, 35
FileDialog, 359
files
data, 271
language, 270
reading, 273, 274
worksheet, 270
writing, 271, 278
floating-point numbers
parts, 66
flow control, 167–191
Font, 361
for
classes, 174
loop, 173
default clause values, 174
for..from loop, 172, 173
for..in loop, 173, 176
Fortran programming language
code generation, 279
full evaluation, 14
and chained assignments, 14
and named objects, 14
vs. last name evaluation, 14
functional operators, 231
functions
composition of, 76
gamma, 35
Get, 366
Getting Started Guide, 2
gif, 361
global
variables, 201, 367
global variables, 201
graphical interface
versions, 1
graphical user interface, 6
graphs
connected, 246
GridCell, 353
TableHeader, 362
GridLayout, 353
definition, 353
GridRow, 353
GUI, see graphical user interface
help, 7
?, 7
online, 7
help pages
creating, 227
Introduction to Maplets, 370
Maplets Package Index, 370
Roadmap, 370
User Hints, 370
I, 35
I/O, 257–282
Index

- **disk**, 269
- **file formats**, 270
- **files**
  - data, 271
  - language, 270
  - worksheet, 270
- **keyboard**, 257–269
- **screen**, 257–269
- **if**
  - vs **map**, 185
- **if operator**, 171
- **if statement**, 167
  - general form, 168, 170
  - nested, 169
- **Image**, 361
- **implies**, 26, 85, 89
- **importing**
  - data, 274
- **in**, 89
- **indent**, 368
- **infinity**, 35
- **initially known names**, 35
- **inline option**, 209
- **input**
  - and output, 257–282
  - data, 271
  - entering, 6
  - file, 273, 274
  - interactive, 264
  - prompt (＞), 6
  - terminal, 264, 265
- **InputDialog**, 359
- **integer**
  - subtypes, 52
- **interactive debugger**, 284
- **interactive input**, 264
- **interface**
  - command-line, 6
  - worksheet, 6
- **interface**, 15, 258, 322
- **interrupting**
- **Maple computation**, 16
- **intersect**, 26, 88, 89
- **into**, 305
- **invalid arguments**, 19
- **invoking**
  - debugger, 296
  - procedures, 10, 13
- **Item**, 361

**Java programming language**
- **code generation**, 279
- **jpeg**, 361

**kernel**
- about, 6
- built-in routines, 6, 13
- **C programming language**, 13
- fast computations, 13
- size, 6
- **kernelopts**, 320
- **keywords**, 25

**Label**, 346
- last name evaluation, 14, 229
  - and procedures, 15
  - and tables, 15
  - vs. full evaluation, 15
- **last statement return**, 195
- **lasterror**, 35
- **lastexception**, 324
- **lastmaplet**, 367
- **libname**, 35

**library**
- about, 6
- code
  - viewing, 15
- **packages**, 6
- routines, 6
  - in statements, 9

**library source code**
- displaying, 229

- **line continuation**, 47, 225
LinearAlgebra subpackage, 364
  MatrixNorm, 346
  VectorNorm, 346
  VectorNorm Maplet example application, 341
list, 309
list of lists, 353
ListBox, 347
  Item, 361
ListBoxSplit, 366
lists, 126
  applying functions to, 132
  concatenating, 128
  inserting elements, 129
  manipulating, 128
  replacing elements, 130
  reversing order, 131
  sorting, 131
  testing membership, 128
local variable, 12
  in procedures, 11, 12
local variables, 201
logical operators
  truth tables of, 87
loop invariants, 320
looping, 172–187
  do loop, 176
  for loop, 173
  for..from loop, 172, 173
  for..in loop, 173, 176
  while loop, 177
lprint, 262
mantissa, see significand
manual
  audience, 1
  conventions, 2
  customer feedback, 4
  set, 2
  using, 3
map, 180, 232
vs if, 185
Maple
  clearing memory, 17
  commands, 9
  components, 5
  computation
    interrupting, 16
    stopping, 16
  kernel, 6
  library, 6, 14
  library routines, 9
  memory
    clearing, 17
    modularity of, 6
    parts of, 5
    platforms, 6
    programming, 3
    prompt (>, 6
    restart, 17
    restarting, 17
    statements, 7
    user interface, 6
    vs. traditional languages, 3
    worksheet, 6
Maple Advanced Programming Guide, 2
Maple Debugger, 284, 296
tutorial, 284
Maple Getting Started Guide, 2
Maple session
  returning information, 357
  returning values, 361
Maple User Manual, 2
maplemint, 329
Maplet
  applications, 1
Maplet
  top-level, 361, 367
Maplet application
  alert message, 357
calculator, 340
close, 357
defining, 368
definition, 339, 343
layout, 343
message, 342
programming indent, 368
programming style, 368
running, 343, 367
Shutdown, 357
window, 343
window body elements, 344
window definition, 353
with statement, 369
writing, 368
Maplet application author, 343
tools, 366
Maplet application button, 344
  ButtonGroup, 361
group, 352, 355
  RadioButton, 350
  ToggleButton, 352
Maplet application calculator, 365
Maplet application categories, 344
Maplet application command elements, 356
Maplet application default
table header values, 362
Maplet application default options
changing, 341
Maplet application default values
changing, 341
Maplet application dialog elements, 357
definition, 344
Maplet application dialogs
  AlertDialog, 357
  ColorDialog, 358
  ConfirmDialog, 358
  FileDialog, 359
  InputDialog, 359
  MessageDialog, 360
  QuestionDialog, 360
Maplet application elements
  Action, 361
  AlertDialog, 357
  Argument, 361
  BoxCell, 353
  BoxColumn, 353
  BoxLayout, 353
  BoxRow, 353
  Button, 344
  ButtonGroup, 361
  CheckBox, 345
  CheckBoxMenuItem, 354
  CloseWindow, 356
  ColorDialog, 358
  ComboBox, 345
  command, 356
  ConfirmDialog, 358
dialog, 357
  DropDownBox, 346
  Evaluate, 356
 FileDialog, 359
  Font, 361
  GridCell, 353
  GridLayout, 353
  GridRow, 353
  Image, 361
  InputDialog, 359
  Item, 361
  Label, 346
  layout, 353
  ListBox, 347
  MathMLEditor, 348
  MathMLViewer, 348
  Menu, 354
  MenuBar, 354
  MenuItem, 354
  MenuSeparator, 354
  MessageDialog, 360
non-visual, 356
other, 361
Plotter, 349
PopupMenu, 354
QuestionDialog, 360
RadioButton, 350
RadioButtonMenuItem, 354
Return, 361
ReturnItem, 361
RunDialog, 356
RunWindow, 356
SetOption, 356
Shutdown, 357
Slider, 350
Table, 350
TableHeader, 362
TableItem, 362
TableRow, 362
TextBox, 351
TextField, 346, 352
ToggleButton, 352
ToolBar, 355
ToolBarButton, 355
ToolBarSeparator, 355
Window, 363
window, 344
window body, 344
Maplet application examples, 363, 364
advanced, 364
GetEquation, 365
help page, 366
Message, 366
Maplet application layout
boxlayout, 353
complex, 353
horizontal, 353
nested lists, 353
simple, 353
vertical, 353
Maplet application layout elements, 353
Maplet application menu bar elements, 354
CheckBoxMenuItem, 354
Menu, 354
MenuBar, 354
MenuItem, 354
MenuSeparator, 354
PopupMenu, 354
RadioButtonMenuItem, 354
Maplet application message
alert, 357
warning, 357
Maplet application options
changing, 356
editable, 351, 352
group, 350, 361
height, 363
reference, 363
resizable, 363
title, 363
visible, 363
vscroll, 362
width, 363
Maplet application queries
creating, 342
GetEquation, 365
Maplet application reference options, 363
Maplet application title
Window element, 363
Maplet application toolbar elements, 355
Maplet application tools, 366
AddAttribute, 366
AddContent, 366
Get, 366
global variables, 367
lastmaplet, 367
ListBoxSplit, 366
Index

Print, 366
Set, 367
StartEngine, 367
StopEngine, 367
thismaplet, 367
Maplet application top-level function, 367
Maplet application user, 343
Maplet application vsroll as needed, 362
Maplet application window body elements, 344
Maplet system
Display function, 343, 367
global settings, 341
uses, 339
Maplet system help pages
Introduction to Maplets, 370
Maplets Package Index, 370
Roadmap, 370
User Hints, 370
Maplets, 1
Maplets elements, 343
Maplets examples, 343
Maplets tools, 343
Maplets
Elements subpackage, 344–363
Examples subpackage, 363–366
LinearAlgebra subpackage, 341, 364
constructors, 364
eigenvalue problems, 364
queries, 364
solvers, 364
standard routines, 364
package, 339–370
Tools subpackage, 366–367
Maplets[Elements], 343
Maplets[Examples], 343
Maplets[Tools], 343
MathMLEditor, 348
MathMLViewer, 348
MATLAB programming language
code generation, 279
matrices
adjacency, 248
Matrix, 162
matrix, 162
MatrixNorm Maplet application, 346
memory
clearing the internal, 17
Menu, 354
MenuBar, 354
Window element, 363
MenuItem, 354
MenuSeparator, 354
message, 342
MessageDialog, 360
mint, 329
minus, 26, 88, 89
missing operator, 18
mod, 26, 79, 89
modp, 79
mods, 79, 80
mul, 183
vs product, 186
multiple selections, 347
multiplication
*, 18
missing operator, 18
non-commutative, 26, 74
names
assignment, 8
initially known, 35
procedure, 12
special characters in, 33
nargs, 224, 235
nested
if statements, 169
lists, 353
networks, 246
neutral operators, 81
new, 246
next, 305
next statement, 118, 179
non-commutative multiplication operator, 74
non-visual Maplet application elements, 356
nops, 97
not, 27, 85, 89
NULL, 35
nullary operators, 25
numbering statements, 296

one-level evaluation, 204
online help, 7
op, 97
operands
procedures, 237
operator option, 209, 231
operators
!, 78
%, 77
%%, 77
%%%, 77
., 74
..., 77
<, 82
<=, 82
>=, 82
@, 76
@@, 76
and, 85
arithmetic, 8, 71
assuming, 88
binary, 26
composition, 76
ditto, 77
factorial, 78
if, 171
implies, 85
intersect, 88
logical, 88
minus, 88
mod, 79
modp, 79
mods, 79
neutral, 81
non-commutative multiplication, 74
not, 85
or, 85
precedence rules, 89
range, 77
relational, 82
set, 88
subset, 88
unary, 27
union, 88
xor, 85

options
arrow, 207
builtin, 16, 207
call_external, 208
Copyright, 208, 225
inline, 209
operator, 209, 231
remember, 210
system, 212
trace, 213
or, 26, 85, 89
Order, 35
order
sort, 131
doing evaluation, 200
other Maplet application elements, 361
output
   and input, 257–282
data, 271
file, 271, 278
screen, 257

packages
   library, 6
parameter return, 220
parameters
   actual, 195, 197
evaluating, 200
evaluation rules, 199
formal, 195, 196
type-checking, 197
parse, 268
Pi, 35
plot, 349
Plotter, 349
plotting
   polynomial roots, 243
pop-up menu, 351
   appended entries, 351, 355
default, 355
PopupMenu, 354
precedence rules
   operator, 89
predefined structure, 356
preface, 1
premature end of input, 17
pretty print
   MathMLViewer, 348
Print, 366
print, 15, 260
printf, 262
printlevel, 35, 260, 315
proc, 9, 193
procedure
   type, 236
procedures, 9, 193–256, 366, 367
accessing operands, 237
actual parameters, 195, 197
and colons, 12
and data structures, 239
and semicolons, 12
assertions, 320
body, 215
breakpoints, 297, 298
built-in, 13
C programming language, 13
calling, 13, 194
characteristics of, 12
clearing watchpoints, 301
comments, 226
components, 196
controlling execution, 305
Copyright, 208, 225
debugging, 284, 296
defining, 9, 193
   alternate methods, 231
   functional operators, 231
   unapply, 232
   unnamed, 232
definitions, 11, 12, 193
   components, 196
displaying, 13
   multi-line, 225
   omitting semicolons, 12
   suppressing display, 13
   vs value, 12
description, 214, 226
display, 12
   suppressing, 13
documenting, 225
empty parentheses, 12
end proc, 9, 193
entering, 10
   on multiple lines, 10
   on one line, 10
error return, 219
evaluation rules, 228
exceptions, 323
executing, 10, 194
explicit breakpoints, 298
explicit return, 217
formal parameters, 195, 196
formatting, 225
functional operators, 231
global variables, 201
individual commands, 12
input required, 11
interpretation, 215
introduction, 9
last name evaluation, 15, 229
last semicolon, 12
last statement executed, 10, 13
last statement return, 195
local variables, 11, 12, 201
Maple library, 13
multi-line, 225
name, 11, 12
naming, 194
numbering statements, 296
omitting semicolons, 12
one-level evaluation, 204
op, 237
operands, 237
options, 206–214
output, 12
parameter return, 220
parameters
  actual, 195, 197
  evaluation rules, 199
  formal, 195, 196
  type-checking, 197
proc, 9, 193
read, 228
readability, 10
removing explicit breakpoints, 300
result, 10, 13
returns, 217–224
  error, 219
  explicit, 217
  last statement, 195
  parameter, 220
  unevaluated, 222
reusing code, 13
running, 10
save, 227
simple, 3, 9
simplification, 215
special evaluation rules, 228
statements within, 12
syntax, 329
tracing, 315, 317, 319
undeclared variables, 205
unevaluated return, 222
unnamed, 232
use statement, 369
user-defined, 13
using, 10, 11
value, 12, 13
variables, 201
declarations, 201
evaluation rules, 203
global, 201
local, 11, 12, 201
one-level evaluation, 204
type assertions, 202
undeclared, 205
viewing
  built-in routines, 16
definitions, 15
watchpoints, 301, 302
writing, 3
procname, 224
product, 187
vs mul, 186
programming
  indent, 368
Maple vs. traditional languages, 3
rationale for learning, 3
style, 368
programs, see procedures
prompt (>), 6
punctuation marks, 48
queries
creating, 342
GetEquation Maplet application, 365
QuestionDialog, 360
queues, 161
quit, 305
quit statement, 119
RadioButton, 350
RadioButtonMenuItem, 354
raising exceptions, 323
range operator, 77
read statement, 117, 228
readdata, 277
reading
data, 271
file data, 274
from terminal, 264, 265
variable values, 273
readline, 264, 266, 268
readstat, 265, 268
reference options, 363
long form, 363
short form, 363
relational operators, 82
remember option, 210
remember tables, 210
remove, 181
repetition, 115, 172–187
reserved words, 25
restart, 17
result
computing, 7
displaying, 7
preventing display, 7
Return, 361
return, 305
return statement, 118, 217
return values, 217–224
type assertion, 198
returning
information to Maple session, 357
values to Maple session, 361
ReturnItem, 361
returns
error, 219
explicit, 217
last statement, 195
parameter, 220
unevaluated, 222
right single quotes, 368
routines
built-in, 13
library, 6
reusing code, 13
user-defined, 13
rules
precedence, 89
RunDialog, 356, 357
running
Maplet applications
Display, 343, 367
procedures, 10, 194
RunWindow, 356
save statement, 117, 227
saving
data, 271
variable values, 271
scroll pane, 362
search
binary, 242
select, 181
selection, 167–172
    operation, 152
    statements, 114
selectremove, 182
semicolons
    omitting in procedures, 12
separators, 354
seq, 77, 183
Set, 367
set operators, 88
SetOption, 356
    reference options, 363
    Set routine, 356
sets, 123
    applying functions to, 125
    arithmetic, 124
    manipulating, 124
    testing equality, 125
    testing membership, 125
shift key, 347
showstat, 296, 297
showstop, 312
Shutdown, 357
significand, 66
SingularValues, 365
Slider, 350
software
    Maple, 5
sort
    order, 131
special characters
    >, 6
    *, 8
    +, 8
    -, 8
    /, 8
    :, 7
    :=, 8, 20
    ;, 7
    =, 20
?, 7
", 7
list of, 24
using, 46
square brackets, 353
sscanf, 275
stacks, 159
standard worksheet, 1
start, 367
StartEngine, 367
statements, 7
    arithmetic, 8
    assignment, 8, 111
    break, 118, 179
    displaying strings, 7
    entering, 6, 8
    error, 118
    help, 7
    if, 167
    library routines, 9
    Maple commands, 9
    next, 118, 179
    quit, 119
    read, 117
    repetition, 115
    return, 118, 217
    save, 117
    selection, 114
    try..catch, 219
    use, 119
    step, 305, 306
    stop, 367
    stopat, 297, 306
    StopEngine, 367
    stoperror, 302
    stopping
        Maple computation, 16
    stopwhen, 301
strings, 7, 366
    ", 7
    double quotes, 7
Index

entering, 7
special characters in, 43
subset, 26, 88, 89
subspop, 97
subtype
    integer, 52
sum, 187
    vs add, 186
syntax
    checking procedures, 329
    errors, 19
system
    Maple, 5
system option, 212
Table, 350
    TableHeader, 362
table
    accessing, 139
    data type, 139
    entries, 139
    indexing function, 139
    op, 139
    operands, 139
TableHeader, 350, 362
TableRow, 362
TableItem, 350, 362
    TableHeader, 362
    TableRow, 362
TableRow, 350, 362
tables
    last name evaluation, 15
    remember, 210
TextBox, 346, 351, 352
TextField, 346, 352
thismaplet, 367
ToggleButton, 352
ToolBar, 355
    Window element, 363
ToolBarButton, 355
ToolBarSeparator, 355
top-level
    Maplet, 361, 367
    Maplet function, 342
trace, 315, 317
trace option, 213
tracelast, 315, 319
tracing
    procedures, 315, 317, 319
traperror, 324
trapping
    errors, 325
troubleshooting, 17, 162
    invalid arguments, 19
    missing operator, 18
    premature end of input, 17
    syntax errors, 19
ture, 35
truth tables, 87
try, 302, 323, 325
try..catch statement, 219
type, 97
    1, 55
    complex(float), 69
    complex(integer), 69
    complex(numeric), 68, 69
    complex(rational), 69
    concatenation, 55
    even, 52
    extended_numeric, 62, 66
    float, 65, 66
    fraction, 64, 66
    indexed, 53
    integer, 52, 62–66
    integer subtypes, 52
    name, 53
    negint, 52
    nonnegint, 52
    nonposint, 52
    numeric, 65, 68
    numeric_type, 66
    odd, 52
of complex numbers, 68
posint, 52
prime, 53
procedure, 236
rational, 64
string, 53
symbol, 53

unapply, 232
unary operators, 27
undeclared variables, 205
undefined, 35
unevaluated return, 222
unevaluation quotes, 368
union, 26, 88, 89
unnamed procedures, 232
unstopat, 298, 300
unstopwhen, 301
untrace, 319
use statement, 119, 369
user interface, see graphical user interface

User Manual, 2

uses

calculators, 340

variables, 201, 367
evaluation rules, 203
global, 201
local, 11, 12, 201
one-level evaluation, 204
reading values, 273
saving values, 271
type assertions, 202
undeclared, 205

Vector, 162
vector, 162

VectorNorm

Maplet example application, 341

VectorNorm Maplet application, 346

verboseproc, 15
version
classic worksheet, 1
command-line, 1
standard worksheet, 1

viewing
library code, 15
print, 15
procedure definitions, 15
interface(VerboseProc), 15

Visual Basic programming language
code generation, 279

WARNING, 322
warning message, 357
warnlevel, 322

watchpoints
clearing, 301
setting, 301, 302

where, 309

while loop, 177

Window element, 344, 363
definition, 343
RunWindow, 356
title, 363
toolbar, 363

with statement, 369

worksheet, 6
classic, 1
graphical interface, 1
standard, 1
versions, 1

worksheet interface, 6
writedata, 278, 279
writeline, 278

writing

file data, 278

Maplet applications, 368

wrong number of arguments, 19
wrong type of arguments, 19
XML data structure, 366
xor, 26, 85, 89
zip, 182
Index