Major Advances in Linear Recurrence Solving

Maple 2026 significantly expands its ability to solve linear recurrence relations. Maple can now fully solve over **94% of the 55,979 entries in the Online Encyclopedia of Integer Sequences (OEIS)** that that can be shown to satisfy a linear recurrence relation.

The OEIS is a widely used research database of integer sequences arising in combinatorics, number theory, special functions, and applied mathematics. Achieving this level of coverage represents a major step forward in symbolic recurrence solving.

These advances reflect sustained, cutting-edge research in linear difference equations and their algorithmic implementation in Maple. Through major extensions to the LREtools package and substantial enhancements to rsolve, Maple now solves broad new classes of homogeneous and inhomogeneous recurrences, including many second-, third-, and fourth-order cases that were previously beyond reach.

Maple’s recurrence-solving capability now spans a substantially broader class of linear recurrences than other available systems. While some systems can solve isolated third- or fourth-order cases, Maple provides systematic coverage of higher-order recurrences at a level not available elsewhere.

New Capabilities in LREtools

Maple 2026 introduces several new commands in the LREtools package for working with linear difference operators. These tools provide deeper structural analysis of recurrence operators and underpin the expanded solving power of rsolve.

New commands include:

- IntegralBasis: Compute an integral basis for a linear difference operator
- Homomorphisms: Compute homomorphisms between two solution spaces
- NormalForm: Compute a normal form irreducible second-order operators
- SearchTable: Search tables of solutions for operators in NormalForm
- AbsoluteFactorization: Compute the absolute factorization of a difference operator
- IsIrreducible: Check if an operator is irreducible
- ReduceToOrderTwo: Attempt reduction of irreducible third- or fourth-order operators to order two
- SymmetricProduct and SymmetricPower: Compute symmetric products and symmetric powers of difference operators
- SolveLRE: Compute solutions of linear homogeneous recurrence relations

These additions significantly expand the operator-theoretic and algorithmic framework available for recurrence analysis.

Expanded Solving Power in rsolve

Through use of SolveLRE , rsolve can now obtain solutions for broad classes of linear inhomogeneous recurrences that could not be solved in prior releases. This includes many third- and fourth-order recurrences, as well as notable enhancements for second-order recurrences.

For example, this simple-looking recurrence could not previously be solved using rsolve:

```
> rec := f(n) = n*f(n-1) + f(n-2);
```

$$rec := f(n) = n f(n-1) + f(n-2)$$

```
> soln := rsolve(rec,f);
```

$$soln := \left(\frac{\mathrm{BesselK}(1,-2)\,(\mathrm{BesselI}(0,2)\,f(0) + f(1)\,\mathrm{BesselI}(1,2) - \mathrm{BesselI}(1,2)\,f(0))}{\mathrm{BesselI}(1,2)\,(\mathrm{BesselK}(1,-2)\,\mathrm{BesselI}(0,2) - \mathrm{BesselK}(0,-2)\,\mathrm{BesselI}(1,2))} - \frac{f(0)}{\mathrm{BesselI}(1,2)} \right) (-1)^n \,(n\,\mathrm{BesselI}(n,2) - \mathrm{BesselI}(n-1,2)) - \left((\mathrm{BesselI}(0,2)\,f(0) + f(1)\,\mathrm{BesselI}(1,2) - \mathrm{BesselI}(1,2)\,f(0))\,(-1)^n\,(n\,\mathrm{BesselK}(n,-2) - \mathrm{BesselK}(n-1,-2)) \right) \Big/ (\mathrm{BesselK}(1,-2)\,\mathrm{BesselI}(0,2) - \mathrm{BesselK}(0,-2)\,\mathrm{BesselI}(1,2))$$

The solution is now given in terms of special functions (in this case, Bessel functions). Evaluating the solution at small values of n confirms agreement with the recurrence:

```
> seq(simplify(eval(soln,n=i)),i=0..3);
```

$$f(0), f(1), 2 f(1) + f(0), 7 f(1) + 3 f(0)$$

This 3rd order recurrence could not previously be solved by **rsolve** (this comes form the OEIS database, 151292). The sequence of integers is:

```
> intseq := [1, 2, 7, 23, 85, 314, 1207, 4682, 18493, 73688, 296671,
  1202849, 4910689, 20158436, 83169871, 344628527, 1433631973,
  5984532728, 25060514887, 105240685511, 443102517025,
  1870054761632, 7909539602647, 33521289826778, 142330494633985,
  605375433105734, 2578988979186127, 11003364185437517];
```

$$intseq := [1, 2, 7, 23, 85, 314, 1207, 4682, 18493, 73688, 296671, 1202849, 4910689, 20158436,$$

$83169871, 344628527, 1433631973, 5984532728, 25060514887, 105240685511, 443102517025, 1870054761632, 7909539602647, 33521289826778, 142330494633985, 605375433105734, 2578988979186127, 11003364185437517]$

From this sequence, the underlying recurrence can be obtained and solved:

```
> rec := LREtools:-GuessRecurrence(intseq, q(n));
```

$$rec := (n+4)\,q(n+3) + (-7\,n-25)\,q(n+2) + (-n+14)\,q(n+1) + (55\,n+55)\,q(n) = 0$$

```
> soln := rsolve(rec,q);
```

$$soln := 5^n \left(q(0) + \sum_{n0=0}^{n-1} \left(\frac{1}{7260\,(n0+2)} \Big(\big(-40\sqrt{11}\,q(1) + 125\sqrt{11}\,q(0) + 3\sqrt{11}\,q(2) + 108\,q(1)\,\pi - 90\,q(0)\,\pi - 18\,q(2)\,\pi + 108\,\mathrm{I}\ln\big(\mathrm{I}\sqrt{11}+5\big)\,q(1) - 90\,\mathrm{I}\ln\big(\mathrm{I}\sqrt{11}+5\big)\,q(0) - 18\,\mathrm{I}\ln\big(\mathrm{I}\sqrt{11}+5\big)\,q(2) - 108\,\mathrm{I}\,q(1)\ln(6) + 90\,\mathrm{I}\,q(0)\ln(6) + 18\,\mathrm{I}\,q(2)\ln(6) \big) \sqrt{11} \left(-\frac{\mathrm{I}}{5}\sqrt{11}\right)^{n0} \left(\mathrm{I}\sqrt{11}\,\mathrm{LegendreP}\left(n0+1, \frac{\mathrm{I}}{11}\sqrt{11}\right) - 11\,\mathrm{LegendreP}\left(n0, \frac{\mathrm{I}}{11}\sqrt{11}\right) \right) \Big) - \frac{1}{605\,(n0+2)} \left(3\,\mathrm{I}\,(6\,q(1) - 5\,q(0) - q(2))\sqrt{11} \left(-\frac{\mathrm{I}}{5}\sqrt{11}\right)^{n0} \left(\mathrm{I}\sqrt{11}\,\mathrm{LegendreQ}\left(n0+1, \frac{\mathrm{I}}{11}\sqrt{11}\right) - 11\,\mathrm{LegendreQ}\left(n0, \frac{\mathrm{I}}{11}\sqrt{11}\right) \right) \right) \right) \right)$$

```
> seq(simplify(value(eval(soln,n=i))),i=0..5);
```

$$q(0), q(1), q(2), -\frac{55\,q(0)}{4} - \frac{7\,q(1)}{2} + \frac{25\,q(2)}{4}, -88\,q(0) - \frac{222\,q(1)}{5} + \frac{187\,q(2)}{5}, -\frac{1089\,q(0)}{2} - \frac{1408\,q(1)}{5} + \frac{2031\,q(2)}{10}$$