Advanced Math Improvements in Maple 2024

Maple 2024 includes many improvements to the math engine.

▼ fsolve for Univariate Polynomials

• The `fsolve` command now uses `RootFinding:-Isolate` for computing roots of univariate polynomials of degree greater than two.

• By default no method is specified. The `Isolate` methods `ABND`, `RS`, `RC`, `HR`, and `PW` are optionally accepted by `fsolve` and passed to `Isolate`.

• The `PW` method is new. For more about that method and the improved underlying `RootFinding:-Isolate` command, see the Faster Univariate Complex Solver section of the Performance Improvements in Maple 2024 help page.

• Supplying the option `NAG` to `fsolve` will force use of the modified Laguerre method followed by iterated root-polishing, which was the prior default.

▼ Multivariate Complex Solver

• The `RootFinding:-Isolate` command can now find complex solutions for multivariate polynomial systems.

```maple
> unassign('a','b','c','d','e');

> cyclic5 := [a+b+c+d+e, a*b+b*c+c*d+d*e+e*a, a*b*c+b*c*d+c*d*e+d*e*a+a*e*a*b,
            a*b*c*d+b*c*d*e+c*d*e*a+d*e*a*b+e*a*b*c, a*b*c*d*e-1];
cyclic5 := [a + b + c + d + e, a*b + e*a + b*c + c*d + d*e, a*b*c + e*a*b + d*e*a + b*c*d + c*d*e + e*a*b*c + a*b*c*d*e - 1]

> map(print, RootFinding:-Isolate(cyclic5, [a,b,c,d,e]));
[a = -2.618033989, b = -0.3819660113, c = 1.000000000, d = 1.000000000, e = 1.000000000]
[a = -2.618033989, b = 1.000000000, c = 1.000000000, d = 1.000000000, e = -0.3819660113]
[a = -0.3819660113, b = -2.618033989, c = 1.000000000, d = 1.000000000, e = 1.000000000]
[a = -0.3819660113, b = 1.000000000, c = 1.000000000, d = 1.000000000, e = -2.618033989]
[a = 1.000000000, b = -2.618033989, c = -0.3819660113, d = 1.000000000, e = 1.000000000]
[a = 1.000000000, b = -0.3819660113, c = -2.618033989, d = 1.000000000, e = 1.000000000]
[a = 1.000000000, b = 1.000000000, c = -0.3819660113, d = -2.618033989, e = 1.000000000]
[a = 1.000000000, b = 1.000000000, c = 1.000000000, d = -2.618033989, e = -0.3819660113]
[a = 1.000000000, b = 1.000000000, c = 1.000000000, d = -0.3819660113, e = -2.618033989]
```
> map(print, RootFinding:-Isolate(cyclic5, [a, b, c, d, e], 'complex')):

\[a = -2.618033989, b = -0.3819660112, c = 1., d = 1., e = 1.\]
\[a = -2.618033989, b = 1., c = 1., d = 1., e = -0.3819660112\]
\[a = -0.8090169944 - 2.489882851 I, b = -0.1180339887 - 0.3632712640 I, c = 0.3090169944 + 0.9510565163 I, d = 0.3090169944 + 0.9510565163 I, e = 0.3090169944 + 0.9510565163 I\]
\[a = -0.8090169944 - 2.489882851 I, b = 0.3090169944 + 0.9510565163 I, c = 0.3090169944 + 0.9510565163 I, d = 0.3090169944 + 0.9510565163 I, e = -0.1180339887 - 0.3632712640 I\]
\[a = -0.8090169944 - 0.5877852523 I, b = -0.8090169944 - 0.5877852523 I, c = -0.8090169944 - 0.5877852523 I, d = 0.3090169944 + 0.2245139883 I, e = 2.118033989 + 1.5388417691 I\]
\[a = -0.8090169944 - 0.5877852523 I, b = -0.8090169944 - 0.5877852523 I, c = -0.8090169944 - 0.5877852523 I, d = 2.118033989 + 1.5388417691 I, e = 0.3090169944 + 0.2245139883 I\]
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\[a = -0.8090169944 - 0.5877852523 I, b = -0.8090169944 - 0.5877852523 I, c = -0.8090169944 - 0.5877852523 I, d = 0.3090169944 + 0.2245139883 I, e = -0.8090169944 - 0.5877852523 I\]
\[a = -0.8090169944 - 0.5877852523 I, b = -0.8090169944 + 0.5877852523 I, c = -0.8090169944 + 0.5877852523 I, d = 0.3090169944 - 0.9510565163 I, e = 0.3090169944 - 0.9510565163 I\]
\[a = -0.8090169944 - 0.5877852523 I, b = -0.8090169944 + 0.5877852523 I, c = -0.8090169944 + 0.5877852523 I, d = 0.3090169944 - 0.9510565163 I, e = -0.8090169944 + 0.5877852523 I\]
\[a = -0.8090169944 - 0.5877852523 I, b = -0.8090169944 + 0.5877852523 I, c = -0.8090169944 + 0.5877852523 I, d = -0.8090169944 + 0.5877852523 I, e = 2.118033989 + 1.5388417691 I\]
\[a = -0.8090169944 - 0.5877852523 I, b = -0.8090169944 + 0.5877852523 I, c = -0.8090169944 + 0.5877852523 I, d = -0.8090169944 + 0.5877852523 I, e = -0.8090169944 + 0.5877852523 I\]
\[a = -0.8090169944 - 0.5877852523 I, b = 2.118033989 + 1.5388417691 I, c = 0.3090169944 + 0.2245139883 I, d = -0.8090169944 - 0.5877852523 I, e = -0.8090169944 - 0.5877852523 I\]
\[a = -0.8090169944 - 0.5877852523 I, b = 2.118033989 + 1.5388417691 I, c = 0.3090169944 + 0.2245139883 I, d = -0.8090169944 - 0.5877852523 I, e = 2.118033989 + 1.5388417691 I\]
\[a = -0.8090169944 - 0.5877852523 I, b = -0.8090169944 + 0.5877852523 I, c = -0.8090169944 + 0.5877852523 I, d = 0.3090169944 + 0.9510565163 I, e = 0.3090169944 + 0.9510565163 I\]
\[a = -0.8090169944 - 0.5877852523 I, b = -0.8090169944 + 0.5877852523 I, c = -0.8090169944 + 0.5877852523 I, d = 0.3090169944 + 0.9510565163 I, e = -0.8090169944 - 0.5877852523 I\]
\[a = -0.8090169944 - 0.5877852523 I, b = -0.8090169944 + 0.5877852523 I, c = -0.8090169944 - 0.5877852523 I, d = 0.3090169944 - 0.9510565163 I, e = 0.3090169944 - 0.9510565163 I\]
\[a = -0.8090169944 - 0.5877852523 I, b = -0.8090169944 + 0.5877852523 I, c = -0.8090169944 + 0.5877852523 I, d = 0.3090169944 - 0.9510565163 I, e = -0.8090169944 + 0.5877852523 I\]
\[ a = -0.8090169944 + 0.5877852523 I, b = 0.3090169944 - 0.9510565163 I, c = 0.3090169944 + 0.9510565163 I, d = -0.8090169944 - 0.5877852523 I, e = 1. \]

\[ a = -0.8090169944 + 0.5877852523 I, b = 0.3090169944 - 0.2245139883 I, c = 2.118033989 - 1.538841769 I, d = -0.8090169944 + 0.5877852523 I, e = -0.8090169944 + 0.5877852523 I \]

\[ a = -0.8090169944 + 0.5877852523 I, b = 0.3090169944 + 0.9510565163 I, c = 1., d = 0.3090169944 - 0.9510565163 I, e = 0.3090169944 - 0.9510565163 I] \]

\[ a = -0.8090169944 + 2.4898982851 I, b = -0.1180339887 + 0.3632712640 I, c = 0.3090169944 - 0.9510565163 I, d = 0.3090169944 - 0.9510565163 I, e = 0.3090169944 - 0.9510565163 I \]

\[ a = -0.9510565163 I, d = 0.3090169944 - 0.2245139883 I, e = -0.8090169944 + 0.5877852523 I \]

\[ a = -0.8090169944 + 0.5877852523 I, b = 0.3090169944 + 0.5877852523 I, c = 0.3090169944 + 0.9510565163 I, d = -0.8090169944 + 0.5877852523 I, e = -0.3819660112, b = 1., c = 1., d = 1., e = 1. \]

\[ a = -0.8090169944 - 0.9510565163 I, d = 0.3090169944 - 0.9510565163 I, e = 0.3090169944 + 0.9510565163 I \]

\[ a = -0.8090169944 + 0.5877852523 I, b = 0.3090169944 + 0.9510565163 I, c = 0.3090169944 - 0.9510565163 I, d = 0.3090169944 + 0.9510565163 I, e = -0.8090169944 + 0.5877852523 I \]

\[ a = -0.8090169944 - 0.9510565163 I, d = 0.3090169944 - 0.9510565163 I, e = 0.3090169944 + 0.9510565163 I \]

\[ a = -0.8090169944 + 0.5877852523 I, b = 0.3090169944 + 0.9510565163 I, c = 1., d = 0.3090169944 + 0.9510565163 I, e = 0.3090169944 - 0.9510565163 I \]

\[ a = -0.3819660112, b = 1., c = 1., d = 1., e = -0.3819660112 \]

\[ a = -0.8090169944 - 0.9510565163 I, d = 0.3090169944 - 0.9510565163 I, e = 0.3090169944 + 0.9510565163 I \]

\[ a = -0.3632712640 I, b = -0.8090169944 - 0.9510565163 I, c = 0.3090169944 + 0.9510565163 I, d = 0.3090169944 + 0.9510565163 I, e = -0.8090169944 + 0.5877852523 I \]

\[ a = -0.3632712640 I, b = 0.8090169944 + 0.9510565163 I, c = 0.3090169944 - 0.9510565163 I, d = 0.3090169944 + 0.9510565163 I, e = -0.8090169944 - 2.4898982851 I \]

\[ a = -0.1180339887 + 0.3632712640 I, b = 0.3090169944 + 0.9510565163 I, c = 0.3090169944 + 0.9510565163 I, d = 0.3090169944 + 0.9510565163 I, e = -0.8090169944 - 2.4898982851 I \]

\[ a = -0.1180339887 - 0.3632712640 I, b = 0.3090169944 + 0.9510565163 I, c = 0.3090169944 + 0.9510565163 I, d = 0.3090169944 + 0.9510565163 I, e = -0.8090169944 + 2.4898982851 I \]

\[ a = -0.3632712640 I, b = -0.8090169944 + 0.9510565163 I, c = 0.3090169944 + 0.9510565163 I, d = 0.3090169944 + 0.9510565163 I, e = -0.8090169944 - 2.4898982851 I \]

\[ a = -0.3632712640 I, b = 0.8090169944 - 0.9510565163 I, c = 0.3090169944 - 0.9510565163 I, d = 0.3090169944 + 0.9510565163 I, e = -0.8090169944 + 2.4898982851 I \]

\[ a = -0.9510565163 I, d = 0.3090169944 + 0.9510565163 I, e = 0.3090169944 + 0.9510565163 I \]

\[ a = -0.9510565163 I, d = 0.3090169944 + 0.9510565163 I, e = 0.3090169944 - 0.9510565163 I \]

\[ a = -0.9510565163 I, d = 0.3090169944 + 0.9510565163 I, e = 0.3090169944 - 0.9510565163 I \]

\[ a = -0.9510565163 I, d = 0.3090169944 + 0.9510565163 I, e = 0.3090169944 - 0.9510565163 I \]

\[ a = -0.9510565163 I, d = 0.3090169944 + 0.9510565163 I, e = 0.3090169944 - 0.9510565163 I \]
\[a = 0.3090169944 - 0.9510565163 I, b = 0.3090169944 - 0.9510565163 I, c = 0.3090169944 - 0.9510565163 I, d = -0.8090169944 + 2.489898285 I, e = -0.1180339887 + 0.3632712640 I\]
\[a = 0.3090169944 - 0.9510565163 I, b = 0.3090169944 - 0.9510565163 I, c = 0.3090169944 - 0.9510565163 I, d = -0.1180339887 + 0.3632712640 I, e = -0.8090169944 + 2.489898285 I\]
\[a = 0.3090169944 - 0.9510565163 I, b = 0.3090169944 + 0.9510565163 I, c = -0.8090169944 - 0.5877852523 I, d = 1., e = -0.8090169944 + 0.5877852523 I\]
\[a = 0.3090169944 - 0.9510565163 I, b = 1., c = 0.3090169944 + 0.9510565163 I, d = -0.8090169944 + 0.5877852523 I, e = -0.8090169944 - 0.5877852523 I\]
\[a = 0.3090169944 - 0.2245139883 I, b = -0.8090169944 + 0.5877852523 I, c = -0.8090169944 + 0.5877852523 I, d = -0.8090169944 + 0.5877852523 I, e = 2.118033989 - 1.538417691 I\]
\[a = 0.3090169944 - 0.2245139883 I, b = 2.118033989 - 1.538417691 I, c = -0.8090169944 + 0.5877852523 I, d = -0.8090169944 + 0.5877852523 I, e = -0.8090169944 + 0.5877852523 I\]
\[a = 0.3090169944 + 0.9510565163 I, b = -0.8090169944 - 2.489898285 I, c = -0.1180339887 - 0.3632712640 I, d = 0.3090169944 + 0.9510565163 I, e = 0.3090169944 + 0.9510565163 I\]
\[a = 0.3090169944 + 0.9510565163 I, b = -0.8090169944 - 0.5877852523 I, c = 1., d = -0.8090169944 + 0.5877852523 I, e = 0.3090169944 - 0.9510565163 I\]
\[a = 0.3090169944 + 0.9510565163 I, b = -0.8090169944 + 0.5877852523 I, c = -0.8090169944 - 0.9510565163 I, d = 0.3090169944 - 0.9510565163 I, e = 1.\]
\[a = 0.3090169944 + 0.9510565163 I, b = -0.1180339887 - 0.3632712640 I, c = -0.8090169944 - 2.489898285 I, d = 0.3090169944 + 0.9510565163 I, e = 0.3090169944 + 0.9510565163 I\]
\[a = 0.3090169944 + 0.9510565163 I, b = 0.3090169944 - 0.9510565163 I, c = -0.8090169944 + 0.5877852523 I, d = 1., e = -0.8090169944 - 0.5877852523 I\]
\[a = 0.3090169944 + 0.9510565163 I, b = 0.3090169944 + 0.9510565163 I, c = -0.8090169944 - 0.9510565163 I, d = -0.1180339887 - 0.3632712640 I, e = 0.3090169944 + 0.9510565163 I\]
\[a = 0.3090169944 + 0.9510565163 I, b = 0.3090169944 + 0.9510565163 I, c = -0.1180339887 - 0.3632712640 I, d = -0.8090169944 - 2.489898285 I, e = 0.3090169944 + 0.9510565163 I\]
\[a = 0.3090169944 + 0.9510565163 I, b = 0.3090169944 + 0.9510565163 I, c = -0.8090169944 - 0.5877852523 I, d = 1., e = -0.8090169944 - 0.5877852523 I\]
\[a = 0.3090169944 + 0.9510565163 I, b = 1., c = 0.3090169944 - 0.9510565163 I, d = -0.8090169944 - 0.5877852523 I, e = -0.8090169944 + 0.5877852523 I\]
\[a = 1., b = -2.618033989, c = -0.3819660112, d = 1., e = 1.\]
Better Handling of Parameters in Solve Solutions

- By default, *solve* gives a representative solution for the following problems:

  > solve( cos(x)*sin(x)=0, x );

  \[
  \frac{\pi}{2}, 0
  \]

  > solve( cos(x)=1/2, x );

  \[
  \frac{\pi}{3}
  \]

- The *allsolutions* option can be used to get a more complete solution, using \(_Z\) to represent any integer.

  > solve( cos(x)=1/2, x, 'allsolutions' );

  \[
  \frac{1}{3} \pi + 2 \pi _{Z}, -\frac{1}{3} \pi + 2 \pi _{Z}
  \]

- If preferred, the new command *SolveTools:-DisplaySolutions* can be used to format the solution in a more readable way using standard notation.
\[ \text{soll := solve}(x \cdot \sin(x)^2 = -x, \{x\}, \text{allsolutions}); \]

\[
\begin{align*}
soll & = \left[ \{x = 0\}, \{x = 2\pi \_Z2\_ ~ + 1\ln(1 + \sqrt{2})\}, \{x = -1\ln(1 + \sqrt{2}) + \pi + 2\pi \_Z2\_ ~ - 1\ln(1 + \sqrt{2})\}, \{x = 1\ln(1 + \sqrt{2}) + \pi + 2\pi \_Z3\_ \} \right] \\
\text{SolveTools:-DisplaySolutions(soll);} \\
&= \begin{cases} 
  x = 0 \\
  x = 2\pi n_1 + 1\ln(1 + \sqrt{2}) \\
  x = -1\ln(1 + \sqrt{2}) + \pi + 2\pi n_1, \quad n_1 \in \mathbb{Z} \\
  x = 2\pi n_2 - 1\ln(1 + \sqrt{2}), \quad n_2 \in \mathbb{Z} \\
  x = 1\ln(1 + \sqrt{2}) + \pi + 2\pi n_2 
\end{cases}
\]

- When called with the option `allsolutions=true` in Maple 2023 and earlier, solve may return a solution in terms of parameters starting with `_B`, `_NN`, and `_Z`. In Maple 2024, solutions will no longer include the `_B` variables but instead solve will expand those automatically into multiple solutions.

- The `_NN` and `_Z` parameters typically stand for all natural numbers or integers, respectively, so cannot be expanded, but the new command `SolveTools:-DisplaySolutions` can be used to format them in an easier to read format.

### Pattern Matching for Definite Summation

- Maple 2024 includes a new pattern matching method for definite summation problems, accessible through the `sum` command. This method uses a database of known definite summation problems with their closed forms. By matching a given definite sum to all entries of the database, Maple can now return closed forms for several definite sums which earlier versions were unable to compute.

- The `sum` command now recognizes power series expansions of some non-hypergeometric functions.

\[
\begin{align*}
&\text{sum}(z^k*(-k)^{(k-1)}/k!, \quad k=1..\infty) \quad \text{assuming abs}(z) < \exp(-1); \quad \text{LambertW}(z) \\
&\text{sum}(\text{bernoulli}(2*n)*(-4)^n*(1-4^n)*x^((2*n-1))/(2*n)!, \quad n=1..\infty) \quad \text{assuming abs}(x) < \pi/2; \quad \text{tan}(x) \\
&\text{sum}(\text{Stirling2}(n,k)*z^k, \quad k=0..n) \quad \text{assuming n::nonnegint}; \quad \text{BellB}(n,z)
\end{align*}
\]

- The database also contains some definite sums involving the `floor` function.
> sum(binomial(n,k)*binomial(n-k,n-2*k)/4^k, k=0..floor(n/2)) assuming n::nonnegint;

\[ 2^{-n} \binom{2n}{n} \]

- Option `formal` can be used instead of assumptions.

> sum(z^k*(-k)^(k-1)/k!, k=1..infinity);

\[ \sum_{k=1}^{\infty} \frac{z^k(-k)^{k-1}}{k!} \]

> sum(z^k*(-k)^(k-1)/k!, k=1..infinity, 'formal');

LambertW(z)

### Improvements to the Collocation Method for Solving Integral Equations

- The `intsolve` command now supports special nodes, custom nodes, and custom basis functions for the collocation method. For example, consider the following integral equation:

\[ a := 0; \]

\[ a := 0 \]

\[ b := 2 \pi; \]

\[ b := 2\pi \]

\[ ie := \sin(x) - 3 * f(x) - 2 * \int_{a}^{b} y * f(y) \, dy; \]

\[ ie := \sin(x) - 3f(x) - 2 \left( \int_{0}^{2\pi} yf(y) \, dy \right) \]

- We can find the exact solution, and use it to compare with two collocation approximations:

\[ p := \text{rhs( intsolve( ie, f(x) ) )}; \]

\[ p := \frac{\sin(x)}{3} + \frac{4\pi}{3 \left( 4\pi^2 + 3 \right)} \]

- For our first approximation, we use Chebyshev nodes and polynomial degree of 10:

\[ q := \text{rhs( intsolve( ie, f(x), 'method' = 'collocation', 'nodes' = 'chebyshev', 'order' = 10 ) )}; \]
\[ q := 0.0986117180199666 + 0.333260721136436x + 0.000459774186936986x^2 - 0.0566851981501486x^3 + 0.00141350936988733x^4 + 0.00175692703461348x^5 + 0.000451378463967157x^6 - 0.00018998544324564x^7 + 0.0000202277956674285x^8 - 7.15411949464578 \times 10^{-7}x^9 + 1.39733837106331 \times 10^{-15}x^{10} \]

\[ > \quad \text{plot}( \ abs( p - q ), x = a .. b, 'color' = 'blue', 'adaptive' = 'false', 'numpoints' = 1000, 'size' = [0.4,0.4], 'title' = "Error of Collocation Solution Using Chebyshev Nodes" ); \]

Error of Collocation Solution Using Chebyshev Nodes

\[ r := \text{rhs}( \ \text{intsolve}( \ ie, f(x), 'method' = 'collocation', 'basis' = [1,x,sin(x),cos(x)], 'numeric' = 'false' ) ); \]

\[ r := \frac{\sin(x)}{3} + \frac{4\pi}{3(4\pi^2 + 3)} \]

• For our second approximation, we specify a custom basis, which gives the exact solution here:
> is( p = r );
          true

> a, b, q, r := 'a, b, q, r':

\textbf{Improvements to is, coulditbe, argument, and signum}

\begin{itemize}
  \item Maple 2024 includes improvements to assumption handling, leading to better results from the \texttt{is}, \texttt{coulditbe}, \texttt{argument}, and \texttt{signum} commands.
  \item \texttt{is} and \texttt{coulditbe} are now careful when dealing with inequalities with nonreal operands:
    \begin{verbatim}
    > is(I*(x-1)^20+1)+I<0);
    false
    \end{verbatim}
  \item A subalgorithm used by \texttt{is} and \texttt{coulditbe} which assumed that variables were real has been corrected, leading to these improved answers:
    \begin{verbatim}
    > conditions := [w^2 = abs(w)^2, w = conjugate(w), cos(z)/abs(cos(z)) = abs(cos(z))/cos(z)]:
    > map(is, conditions);
    [false, false, false]
    > map(is, conditions) assuming real;
    [true, true, true]
    \end{verbatim}
  \item Some improvements in normalization of properties (see \texttt{assume}) have been fixed, leading to the following improvements for top-level calls.
  \end{itemize}

This call to \texttt{coulditbe} now returns immediately:

\begin{verbatim}
> coulditbe(y^4*(y^8+6*y^4+1)^2*(y-1)^6*(y+1)^6*(y^2+1)^6, 0) assuming y::real;
true
\end{verbatim}

This square root previously simplified only by replacing \( z/x \) by a new variable. Now that's not necessary:

\begin{verbatim}
> sqrt(sin(z/x)^2) assuming 0<z/x, z/x<Pi/2;
\sin\left(\frac{z}{x}\right)
\end{verbatim}

These results no longer return \text{FAIL}:

\begin{verbatim}
> coulditbe(y^2*(y^4+1)/(y^2+1)=0) assuming y::real;
true
\end{verbatim}

\begin{verbatim}
> is(1/x=0) assuming x=0;
false
\end{verbatim}
\[
> \text{coulditbe}(-N^{(1/2)}/\Pi, \text{integer}) \ \text{assuming} \ (N^{(1/2)}/\Pi)\text{::Not(integer)};
\]
\[
\text{false}
\]
- \text{coulditbe} is better at dealing with the conjunction of multiple conditions which depend on the same subexpression:

\[
> \text{coulditbe}(\text{And}(k::\text{integer}, k \leq 20, 0 \leq k));
\]
\[
\text{true}
\]
- Maple 2023 included some new improved functionality for recognizing difference of squares patterns in \text{is} and \text{coulditbe}:

\[
> \text{is}(0 \leq \sqrt{p^3 + q^2} + q) \ \text{assuming} \ (0 < p, q::\text{real});
\]
\[
\text{true}
\]
This feature has been improved again for Maple 2024. For example, this problem:

\[
> \text{is}((a*u+b)^{(1/2)}/b^{(1/2)}-1>0) \ \text{assuming} \ a>0, b>0, u>0;
\]
\[
\text{true}
\]
is now solved by recognizing that it can be transformed into \text{is}((a*u+b)/b-1 > 0), i.e. \text{is}(a* u/b > 0).
- Maple 2023 included some improvements in \text{coulditbe}/\text{is} when there are assumptions involving \text{Re} or \text{Im}:

\[
> \text{is}(b, \text{real}) \ \text{assuming} \ 0 < \text{Re}(b);
\]
\[
\text{false}
\]
\[
> \text{is}(r + i*I, \text{real}) \ \text{assuming} \ r < 0, i::\text{real};
\]
\[
\text{false}
\]
- These improvements have been extended for Maple 2024:

\[
> \text{coulditbe}(x<0) \ \text{assuming} \ \text{Im}(x)>0;
\]
\[
\text{false}
\]
\[
> \text{coulditbe}(x<0) \ \text{assuming} \ \text{Re}(x)>0;
\]
\[
\text{false}
\]
\[
> \text{piecewise}(a<0, 0, a<=0, 1, \text{Im}(a)<0, 2) \ \text{assuming} \ \text{Im}(a)<0;
\]
\[
2
\]
- This answer used to be \text{true} which was incorrect:

\[
> \text{coulditbe}((a+I*b+1)/2, \text{integer}) \ \text{assuming} \ \text{abs}(a)<1, a::\text{real}, \ b::\text{real};
\]
\[
\text{FAIL}
\]
- There are also some improved results from \text{is} when there are multiple related inequality assumptions:

\[
> \text{is}(\text{exp}(I*y)^2, \text{real}) \ \text{assuming} \ 0<y, y<t, 0<t;
\]
\[
\text{false}
\]
• An improvement in **AndProp** led to this new result in **coulditbe**:

\[ \text{coulditbe}(a+I*b, \text{imaginary}) \text{ assuming } b::\text{real}, a>0; \]

\[ \text{false} \]

• In Maple 2023, some improvements were made to handle nested **signum**:

\[ \text{signum}(a - \text{signum}(a)) \text{ assuming } (1 < a^2); \]

\[ \text{signum}(a) \]

• For Maple 2024, **argument** and **signum** now recognize determining assumptions such as the following:

\[ (\text{argument}, \text{signum})(x) \text{ assuming } x/(I+1) > 0; \]

\[ \frac{\pi}{4}, \frac{\sqrt{2}}{2} + \frac{1\sqrt{2}}{2} \]

### Manipulating Trigonometric Expressions

• Before Maple 2023, **simplify** in general converted trig functions to **sincos** form. Now it uses any of the 6 trig functions (\(\sin, \cos, \tan, \sec, \csc, \cot\)) and the corresponding hyperbolic trig functions (\(\sinh, \cosh, \text{etc.}\)) to express the simplified answer. As a result, many examples can be returned in a more compact form:

\[ \text{simplify}((\sin(x)+\cos(x))/\sin(x)); \]

\[ 1 + \cot(x) \]

Along the same lines, \(1/\tan\) and \(1/\cot\) simplify to \(\cot\) and \(\tan\) respectively as of Maple 2024:

\[ \text{map}(\text{simplify}, [1/\tan(x), 1/\cot(x)]); \]

\[ [\cot(x), \tan(x)] \]

• Trig simplification now includes combining trig functions by default:

\[ \text{simplify}((\sin(8) - 2*\sin(4))/(1 + \cos(8) - 2*\cos(4))); \]

\[ \tan(4) \]

\[ \text{simplify}(2*c[1]*\tan(\sqrt{3}*x/4)/(1 + \tan(\sqrt{3}*x/4)^2) + c[2]*(1 - \tan(\sqrt{3}*x/4)^2)/(1 + \tan(\sqrt{3}*x/4)^2)); \]

\[ c_2 \cos\left(\frac{\sqrt{3} x}{2}\right) + c_1 \sin\left(\frac{\sqrt{3} x}{2}\right) \]

• Conversion of **tan(x/2)** to **sincos** now just uses \(\tan=\sin/\cos\), which is consistent with what happens with arbitrary arguments:

\[ \text{convert~}([\tan(x/2), \tan(f(x)/2), \tan(x)], \text{sincos}); \]

\[
\begin{bmatrix}
\sin\left(\frac{x}{2}\right), & \sin\left(\frac{f(x)}{2}\right), & \sin(x) \\
\cos\left(\frac{x}{2}\right), & \cos\left(\frac{f(x)}{2}\right), & \cos(x)
\end{bmatrix}
\]
• **simplify** more consistently uses the inverse trig identities $\arccos + \arcsin = \arccsc + \arcsec = \frac{\pi}{2}$ to convert expressions to a more compact form if possible:

```maple
> simplify([\pi/2 - \arcsin(\sin(x)), \pi/2 - \arccos(\cos(x)), \pi/2 - \arccsc(\csc(x)), \pi/2 - \arcsec(\sec(x))]);
```

- $\arccos(\sin(x))$, $\arcsin(\cos(x))$, $\arccsc(\csc(x))$, $\arcsec(\sec(x))$

• Combining with respect to *trig* no longer expands unnecessarily. This result is now more compact:

```maple
> combine(-7+(4*cos(1/7*Pi)^3+I*cos(1/14*Pi)-3*cos(1/7*Pi)))*(-7^(5/7)*O*(I*cos(5/14*Pi)+cos(1/7*Pi)))^(1/2), trig);
```

\[-7 + \left(\cos\left(\frac{3\pi}{7}\right) + \cos\left(\frac{\pi}{14}\right)\right)\sqrt{-7^{5/7}O\left(\cos\left(\frac{5\pi}{14}\right) + \cos\left(\frac{\pi}{7}\right)\right)}\]

• **simplify** now converts this expression to a simpler one involving radicals:

```maple
> simplify(-1/7-(-1/7*I*7^(5/7)*exp(2/7*I*Pi)*\sin(1/7*Pi)-1/7*cos(1/7*Pi)*7^5)*O*(I*cos(5/14*Pi)+cos(1/7*Pi)))^(7/2));
```

- $1 - \left(-1\right)^{2/7}\frac{\sqrt{-1}}{7}$

• **simplify** in Maple 2023 included an improved ability to remove terms in trig arguments which are integer multiples of $\frac{\pi}{2}$:

```maple
> simplify(sin(Pi*(x + z))) assuming z::integer;
```

- $\sin(\pi x \cdot -1)^2$

• This feature has been improved for Maple 2024. For example this previously simplified only *without* the assumption:

```maple
> simplify(cos((x+1)*Pi/2)) assuming x::integer;
```

- $\sin \left(\frac{\pi x}{2}\right)$

• In previous versions of Maple, to get the following simplified answer, **simplify** had to be called twice:

```maple
> simplify((-2*F+2*_y1*sin(y)*cos(y))/(-1+cos(2*y)), trig);
```

- $F \csc(y)^2 - _y1 \cot(y)$

• Due to improved trig simplifications this integral is now solved:

```maple
> int(sqrt((1-cos(t))^2+sin(t)^2)*(t-sin(t)),t=0..Pi);
```

- $\frac{16}{3}$

and this answer is smaller:
\[
\int \frac{1}{\sqrt{\cos(x) - \cos(x_0)}} \, dx = 2 \frac{\text{InverseJacobiAM} \left( \frac{x_0}{2}, \csc \left( \frac{x_0}{2} \right) \right)}{\sin \left( \frac{x_0}{2} \right)}
\]

Similarly this result from \texttt{dsolve} is more compact:

\[
\text{dsolve}(\text{diff}(y(t), t, t) - \sin(t)*(\text{diff}(y(t), t)) - y(t));
\]

\[
y(t) = c_1 \text{HeunC} \left( 2, -\frac{1}{2}, -\frac{1}{2}, -1, \frac{15}{8}, \cos(t) + \frac{1}{2} \right) + c_2 \cos \left( \frac{t}{2} \right) \text{HeunC} \left( 2, -\frac{1}{2}, -\frac{1}{2}, -1, \frac{15}{8}, \cos(t) + \frac{1}{2} \right)
\]

\[
\text{After various improvements in Maple 2023 and Maple 2024, the length of this result from simplify has decreased by more than 10 times:\n}
\]

\[
\text{simplify}(-8/Pi^2/cosh(3/2*Pi)*(1/2*exp(1/2*Pi*x-3/2*Pi)-1/2*exp(-1/2*Pi*x+3/2*Pi))\cdot8/9/Pi^2/cosh(9/2*Pi)*\cdot1/2*exp(3/2*Pi*x-9/2*Pi)-8/25/Pi^2/cosh(15/2*Pi)*\cdot1/2*exp(5/2*Pi*x-15/2*Pi)-1/2*exp(-5/2*Pi*x+15/2*Pi))\cdot8/49/Pi^2/cosh(21/2*Pi)*\cdot1/2*exp(7/2*Pi*x-21/2*Pi)-1/2*exp(-7/2*Pi*x+21/2*Pi))\cdot8/81/Pi^2/cosh(27/2*Pi)*\cdot1/2*exp(9/2*Pi*x-27/2*Pi)-1/2*exp(-9/2*Pi*x+27/2*Pi)), \text{trig, size=false});
\]

\[
\frac{1}{\pi^2} \left[ \frac{\pi (-3 + x)}{2} + \frac{\pi (3 - x)}{2} \right] \text{sech} \left( \frac{3 \pi}{2} \right) + \left( \frac{3 \pi (-3 + x)}{2} \right) \text{sech} \left( \frac{9 \pi}{2} \right) + \left( \frac{3 \pi (3 - x)}{2} \right) \text{sech} \left( \frac{15 \pi}{2} \right) + \left( \frac{7 \pi (-3 + x)}{2} \right) \text{sech} \left( \frac{21 \pi}{2} \right) + \left( \frac{7 \pi (3 - x)}{2} \right) \text{sech} \left( \frac{27 \pi}{2} \right)
\]

\[\text{Sum, products, integrals, limits: simplification and evaluation}\]

\[\text{For Maple 2023, there were a number of improvements in evaluating integrals using transformations involving the integration variable:}\]

\[
\text{eval(Int(f(x, a), a, a = x));}
\]

\[
\int_x^\infty f(x, \_a) \, d\_a
\]

\[
\text{eval(Intat(f(x, y), x = g(x)), y=x);}
\]

\[
\text{eval(\text{Intat}(f(x, y), x = g(x)), y=x);}
\]
Also as of Maple 2023, simplify began to follow the semantics of sum in applying limit instead of eval when simplifying a Sum at a singular point:

```maple
> tmp := Sum(sin(k)/k, k = 0 .. 0):
> (simplify = value)(tmp);
1 = 1
```

and improved its ability to pull out provably non-zero factors from potentially infinite sums, integrals, limits, products, etc.: 

```maple
> simplify(Sum(-sin(n*Pi*x)*(exp(Pi) - exp(-Pi))*(-1 + (-1)^n)/n, n = 2 .. infinity));
```

```maple
-\left( \sum_{n=2}^{\infty} \frac{\sin(n \pi x)}{n} \left( -1 + (-1)^n \right) \right) \left( e^{\pi} - e^{-\pi} \right)
```

```maple
> simplify(Int(2*Pi*n*f(x), x = 0 .. infinity));
```

```maple
2\pi \left( \int_{0}^{\infty} f(x) \, dx \right)
```

In Maple 2024, evaluation of a sum or product involving \( t^k \) at \( t=0 \) now correctly recognizes that the \( k=0 \) term should actually be 1:

```maple
> eval(sum(t^k*f(k), k=0..infinity), t=0);
```

```maple
f(0)
```

```maple
> eval(Product(f(t^k), k=0..infinity), t=0);
```

```maple
\left( \prod_{k=0}^{0} f(1) \right) \left( \prod_{k=1}^{\infty} f(0) \right)
```

Also, simplify now recognizes when sums or products can absorb extra terms or factors to simplify the whole expression:

```maple
> simplify(f(n)+Sum(f(k), k=1..n-1)) assuming integer;
\sum_{k=1}^{n} f(k)
```

```maple
> simplify(f(n)*Product(f(k), k=1..n-1)) assuming integer;
\prod_{k=1}^{n} f(k)
```
Argument processing, options and core algorithm improvements

- As of Maple 2023, `simplify` included the following improvements to its option processing and core algorithms:

  It raises an exception when there is an unexpected argument passed to it:

  ```maple
  > simplify(sin(x + Pi/4), foo);
  Error, (in `simplify/do`) unexpected argument: foo
  ```

  The `symbolic` option is now effective even when given not as the last argument:

  ```maple
  > simplify(sqrt(x^2), symbolic, sqrt);
  x
  ```

  When multiple particular simplification procedures are requested via extra arguments to `simplify`, they will each be retried as many times as needed to obtain the full simplification. For example, the extra arguments `exp` and `RootOf` can now be given in either order to achieve the full simplification:

  ```maple
  > e := RootOf(_Z^2 - exp(2*RootOf(_Z^2+1,index=1)*x) - 1):
  > simplify( diff(e-subs(RootOf(_Z^2+1,index=1) = I, e), x), RootOf, exp);
  0
  ```

  It uses `expand` more extensively to simplify functions:

  ```maple
  > simplify(polar(sqrt(13), arctan(3/2)) + polar(sqrt(65), -arctan(7/4)));
  6 - 41
  ```

  It avoids normalizing pure rational polynomials when there is no advantage in doing so:

  ```maple
  > simplify((x + 1)^5/a + 1);
  \frac{(x + 1)^5}{a} + 1
  ```

  ... or when the benefits are outweighed by the drawbacks. In this case, the input expression, although non-polynomial, is nevertheless considered simpler than the 307-term polynomial which results from dividing through by the denominator:

  ```maple
  > simplify((x^307 - 1)/(x - 1));
  \frac{x^{307} - 1}{x - 1}
  ```

  The latter can of course still be obtained via `normal` or `factor`:
As of Maple 2024, simplify cancels factors whose ratio is the imaginary unit:

\[ \text{simplify}((I*x+1)/(x-I)) \]

\[ I \]

### Elliptic functions

Maple now has improved simplification and normalization for various Elliptic functions.

- Symmetries in \texttt{EllipticPi} are now used to automatically factor out or remove minus signs from its arguments:

\[ \text{EllipticPi}(1, \text{nu}, -k), \text{EllipticPi}(\text{nu}, -k), \text{EllipticPi}(-z, \text{nu}, -k) \]

\[ \text{EllipticPi}(v, k), \text{EllipticPi}(v, k), -\text{EllipticPi}(z, v, k) \]

- A symmetry in \texttt{EllipticF} is now used to simplify its arguments:

\[ \text{simplify}(\text{EllipticF}(z*k, 1/k)) \]

\[ \text{EllipticF}(z, k) k \]

\[ \text{simplify}([\text{EllipticPi}((-1)^n, \text{nu}, k), \text{EllipticF}((-1)^n, k)]) \]

\[ \text{assuming } n::\text{integer}; \]

\[ [(-1)^n \text{EllipticPi}(v, k), (-1)^n \text{EllipticK}(k)] \]

\[ \text{simplify}(\text{EllipticPi}((z^2)^{(1/2)}, \text{nu}, k), \text{EllipticPi}); \]

\[ \sqrt{z^2} \text{EllipticPi}(z, v, k) \]

\[ \frac{z}{z} \]

\[ \text{tmp} := [\text{EllipticE}, \text{EllipticK}](k*I/sqrt(1-k^2)) \]

\[ \text{simplify}(\text{tmp}); \]

\[ \left[ \text{EllipticE}\left(\frac{k}{\sqrt{k^2 - 1}}\right), \text{EllipticK}\left(\frac{k}{\sqrt{k^2 - 1}}\right) \right] \]

\[ \text{simplify}(\text{tmp}) \text{ assuming } k^2 < 1; \]

\[ \left[ \frac{\text{EllipticE}(k)}{\sqrt{-k^2 + 1}}, \text{EllipticK}(k) \sqrt{-k^2 + 1} \right] \]

\[ \text{simplify}((-2*k^2 + 2)*\text{EllipticE}(k*I/sqrt(-k^2 + 1)) - \text{EllipticK}(k*I/sqrt(-k^2 + 1)) + (-2*\text{EllipticE}(k) + \text{EllipticK}(k))*\text{sqrt}(-k^2 + 1)) \text{ assuming } 0<k, k<1; \]

\[ 0 \]

- Limits of Elliptic functions at certain infinite values are now more accurate. This example previously returned a complex infinity:
Simplification of \texttt{binomial} was introduced in Maple 2023 and has been improved for Maple 2024:

\texttt{\textcolor{red}{> simplify}(4^n*\texttt{binomial}(n - 1/2, n) - \texttt{binomial}(2*n, n));}

0

Conversion and simplification of \texttt{Beta} has been introduced for Maple 2024:

\texttt{\textcolor{red}{> convert(\texttt{Beta}(a,1-a),\texttt{elementary});}}
\[
\frac{\pi}{\sin(\pi a)}
\]

\texttt{\textcolor{red}{> simplify(-1/2*\texttt{Beta}(1/6,1/2)+1/3*\Pi^2*2^{2/3}/\texttt{GAMMA}(2/3)^3*3^{1/2}})}
\[
0
\]

This integral now returns (unevaluated) quickly due to improved simplification of \texttt{GAMMA}:

\texttt{\textcolor{red}{> \texttt{int}}((1/2)^{2061+2*z}/z/(1+2*z)*\texttt{GAMMA(3/2+z)}^2/\texttt{GAMMA(z)}^2, z);}
\[
\int \frac{\left(\frac{1}{2}\right)^{2061+2z} \Gamma\left(\frac{3}{2} + z\right)^2}{z(1 + 2z) \Gamma(z)^2} dz
\]

\textbf{\texttt{piecewise}}

In Maple 2023, \texttt{simplify} became better at recognizing when function calls can be factored out of piecewise functions:

\texttt{\textcolor{red}{> simplify(\texttt{piecewise}(x < 0, \texttt{f(y)}, \texttt{f(z)}));}}
\[
f\begin{cases}
y & x < 0 \\
z & 0 \leq x
\end{cases}
\]

In Maple 2024, \texttt{simplify} and \texttt{combine} are both now better at recognizing when two piecewise branch values can be merged into one:

\texttt{\{\texttt{simplify, combine}\}(\texttt{piecewise}(x <= 0,2*\texttt{ln(1-x)},0 < x,\texttt{ln((x-1)^2))}})
\texttt{assuming x,\texttt{real};}
\[
\{\texttt{ln((x - 1)^2)}\}
\]

and when multiple piecewise functions can be merged into one:

\texttt{\{\texttt{simplify, combine}\}(\texttt{piecewise}(a<0, \texttt{f1(a)}, a>0, \texttt{f2(a)}, \texttt{Or(a=0, \texttt{Im(a)}<>0)}, \texttt{f3(a)} + \texttt{piecewise}(a<0, \texttt{g1(a)}, a>0, \texttt{g2(a)}, \texttt{Or(a=0, \texttt{Im(a)}<>0)}, \texttt{g3(a))});}
As a result the following integral can now be solved in terms of limits:

\[
\int \text{piecewise}(t \leq 1, -(\cos(\alpha t) - 1)/\alpha, 1 < t, (\cos(\alpha t) + \sin(\alpha t) \cdot \sin(\alpha) - \cos(\alpha t))/\alpha) \cdot \sin(\alpha x), \alpha = 0..\infty);
\]

\[
\begin{align*}
\lim_{\alpha \to \infty} & \left( \text{Si}(\alpha x) - \frac{\text{Si}(\alpha (t + x))}{2} + \frac{\text{Si}(\alpha (t - x))}{2} \right) & t < 1 \\
\lim_{\alpha \to \infty} & \left( \frac{\text{Si}(\alpha (t - x))}{2} - \frac{\text{Si}(\alpha (t + x))}{2} - \frac{\text{Si}(\alpha (x + t - 1))}{2} + \frac{\text{Si}(\alpha (x + t - 1))}{2} \right) & 1 \leq t
\end{align*}
\]

\[\text{\textbullet} \text{ logarithm and dilogarithm} \]

\[\text{\textbullet} \text{ Simplification and numeric evaluation of expressions containing } \ln \text{ with large integer components is now handled more carefully:}\]

\[
\text{tmp} := \text{simplify}(-127/6*\ln(2)-1/2*\ln(3)+7/6*\ln(233512967+14798283*249^(1/2))+\ln(11014713425-698029101*249^(1/2)));
\]

\[
\text{evalf}(\text{tmp});
\]

\[-1.8847449302970215778\]

\[\text{\textbullet} \text{ Simplification of dilog is now more careful to avoid simplifications which are branch dependent. For example, this call to simplify now correctly returns the input unchanged:}\]

\[
\text{simplify(dilog(s + 2) + dilog(1/(s + 2))) assuming s < -2;}
\]

\[\text{dilog}(s + 2) + \text{dilog}\left(\frac{1}{s + 2}\right)\]

\[\text{\textbullet} \text{ power and radical} \]

\[\text{\textbullet} \text{ In Maple 2023, there were some improvements in the simplification of radicals which led to the following improved results:}\]
> simplify(sqrt(x^2), sqrt) assuming x::real;
   |x|
> simplify(-1/(x*(-y + sqrt(-2/x))*(y + sqrt(-2/x))));
   \[ \frac{1}{y^2x + 2} \]
> simplify((A + B*sqrt(x))^a*a^2*B^2/(4*x*(A + B*sqrt(x))^2) - (A + B*sqrt(x))^a*a*B^2/(4*x^\((3/2)\) - pochhammer(a - 1, 2)*B^2*(A + B*sqrt(x))^\(a - 2\)/(4*x)));
0
> simplify(-B*(A^2*sqrt(x) + 2*A*B*x + B^2*x^\((3/2)\))*(a - 1)*(A + B*sqrt(x))^(a - 2) + (2*A*B*sqrt(x) + B^2*x + A^2)*(A + B*sqrt(x))^(a - 1) - (-B*(a - 2)*sqrt(x) + A)*(A + B*sqrt(x))^a);
0

- For Maple 2024, `simplify(...)`, `power` no longer expands unnecessarily. For this example the new result is now more compact:

> simplify( (Pi^k*2^(2*k+2)*5^(2*k+1)/(-1+2*k)*hypergeom([-2*k-1, -k+1/2], [3/2-k], 1/100)/GAMMA(2*k+2)+25*Pi^(k+1)*(2*k+1)*(3+2*k)/GAMMA(k+5/2)^2*sec(Pi*k))*(-1)^(3*k), power);
\[
\begin{align*}
20\pi^k4^k25^k & \hypergeom\left(\begin{array}{c}-2k-1, -k+\frac{1}{2} \\
\frac{3}{2}-k, \frac{1}{100}\end{array}\right) \\
& \left((-1+2k)\Gamma(2k+2)\right) \\
& + \frac{25\pi^{k+1}(2k+1)(3+2k)\sec(\pi k)}{\Gamma\left(k+\frac{5}{2}\right)^2} \right) (-1)^{3k}
\end{align*}
\]
- Also, `simplify` no longer pulls polynomial factors out of a `radical` without good reason:

> simplify(I/(cos(x)-1)^\((3/2)\), radical) assuming 0 < x, x < 1/2*Pi;
\[ -\frac{1}{\left(-\cos(x) + 1\right)^{3/2}} \]

\[\text{\textbullet}\ \text{min/max}\]
- As of Maple 2023, `simplify` included better normalization for `min` and `max`:
\[
\begin{bmatrix}
\min(y,z) + x \\
3 \max(x,y) \\
-\min(x,y) \\
0 \\
2 \min(x,-y) \\
2 \min(x,-x) \\
\vdots
\end{bmatrix}
\]

\[
> \text{simplify(Vector([min(x + y, x + z), max(3*x, 3*y), max(-x, -y), min(x, -y) + max(y, -x), min(x, -y) - max(y, -x), min(x, -x) - max(x, -x), min(x, -x) + max(x, -x)]))};
\]

\[
\text{min}(y, z) + x \\
3 \text{max}(x, y) \\
-\text{min}(x, y) \\
0 \\
2 \text{min}(x, -y) \\
2 \text{min}(x, -x) \\
\vdots
\]

\[
> \text{simplify(max(z*x, z*y)) assuming (z < 0));}
\]

\[
z \text{min}(x, y)
\]

- For Maple 2024, \text{min} and \text{max} are now better at recognizing whether inputs are real. For example, this is a real number in disguise:

\[
> \text{alias(r= -1/2*(-cos(RootOf(sin(_Z)^2*_Z^2+sin(_Z)^2+6*_Z*sin(_Z)+8, -0.-1.931327404*I))-3)*csc(RootOf(sin(_Z)^2*_Z^2+sin(_Z)^2+6*_Z*sin(_Z)+8, 3.761757270)))*sin(1/2*(-cos(RootOf(sin(_Z)^2*_Z^2+sin(_Z)^2+6*_Z*sin(_Z)+8, 3.761757270)))*sin(1/2*(-cos(RootOf(sin(_Z)^2*_Z^2+sin(_Z)^2+6*_Z*sin(_Z)+8, 3.761757270)))*sin(1/2*(-cos(RootOf(sin(_Z)^2*_Z^2+sin(_Z)^2+6*_Z*sin(_Z)+8, 3.761757270)))*sin(1/2*(-cos(RootOf(sin(_Z)^2*_Z^2+sin(_Z)^2+6*_Z*sin(_Z)+8, 3.761757270))))))):}
\]

\[
> \text{evalf(r)};
\]

\[-7.786927399 + 0.1\]

\[
> \text{Im(r)};
\]

\[0\]

\text{min} no longer complains that the number is not real:

\[
> \text{min(2, r)};
\]

\[\text{min}(2, r)\]

\[
> \text{alias(:-r=:-r)}:
\]

\textbf{IntegrationTools:-Change}

- In Maple 2023, \texttt{IntegrationTools:-Change} became better at restricting the change of variables to the subinterval where it is valid. For example, in this case the transformation is not valid on \(x=\pi/4..\pi/2\) so it is only applied to the subinterval \(x=0..\pi/4\):
> IntegrationTools:-Change(Int(sin(x)/(1 + sin(x)*cos(x))^(1/2), x = 0 .. 1/2*Pi), x = arcsin(y)/2);

\[
\begin{align*}
\int_{0}^{1} \frac{\sin\left(\frac{\arcsin(y)}{2}\right)}{\sqrt{-y^2 + 1}} \, dy + \int_{\pi/4}^{\pi/2} \frac{\sin(x)}{\sqrt{1 + \sin(x) \cos(x)}} \, dx
\end{align*}
\]

> After some more improvements for Maple 2024, this change of variables can now be done:

> tmp := convert(-EllipticF(u*2^(1/2)*(1/(cos(y0)+1))^(1/2),cos(1/2*y0))*2^(1/2)/(1/(cos(y0)+1))^(1/2)/(2*cos(y0)+2), Int);

\[
tmp := -\sqrt{\pi} \left( \left( \int_{0}^{1} e^{\sqrt{2} (\sqrt{(y_{0} + \pi) / 4} - \sqrt{1 - \sqrt{2} \sqrt{y_{0} + 1}})} \right) \, \frac{\sinh(\sqrt{2} y_{0} + \pi)}{2} \right) + 1
\]

> IntegrationTools:-Change(tmp, _alpha1=a*T, a);

\[
\begin{align*}
\int_{0}^{1} \frac{\sinh(\sqrt{2} y_{0} + \pi)}{2} \, \frac{\sinh(\sqrt{2} y_{0} + \pi)}{2} \left( e^{\sqrt{1 - \sqrt{2} \sqrt{y_{0} + 1}} - \sqrt{1 - \sqrt{2} \sqrt{y_{0} + 1}}} \right) \, \sinh(\sqrt{2} y_{0} + \pi) \, dy_{0}
\end{align*}
\]
• Also, this request to change variables now returns the given integral unchanged because the change of variables requested does not involve the existing integration variable:

\[
\text{IntegrationTools:-Change(Int(cos(t),z), \{x = r*cos(t)\}, [r])};
\]

\[
\int \cos(t) \, dz
\]

• Finally, these calls to IntegrationTools:-Change now give more useful error messages:

\[
\text{IntegrationTools:-Change(Int(f(x),x=a..b), x=-x)};
\]

Error, (in IntegrationTools:-Change) transformation equations must depend on at least two variables, the old integration variable and the new one

\[
\text{IntegrationTools:-Change(Int(f(k*z+v*t,v*z+k*t),z = 0 .. 1),\{t = (k* y-x*v)/(-v^2+k^2), z = (-v*y+k*x)/(-v^2+k^2)\},[x, y])};
\]

Error, (in IntegrationTools:-Change) only 1 transformation equation(s) \{z = (k*x-v*y)/(k^2-v^2)\} was applicable to the integral(s) in the input; the number of new integration variables specified, [x, y], should be the same

\[
\text{IntegrationTools:-Change(tmp, \{x = u^2+U, y = v^2+V\})};
\]

Error, (in IntegrationTools:-Change) ambiguous change of variables; please specify (using the third argument) which 2 of \{U, V, u, v\} are the new variables

\[\text{Verification}\]

Verification of two expressions up to sign differences has undergone some significant improvements recently.

• As of Maple 2023, verify, sign now recurses into non-algebraic expressions:

\[
\text{verify([x/(1-x)], [-x/(x-1)], sign)};
\]

true

Signs can now be factored out of both even and odd powers as part of the equivalence:

\[
\text{verify((1-x)^2/(1-y)^3, -(x-1)^2/(y-1)^3, sign)};
\]

true

Also two new options oddfuncs = ... and evenfuncs = ... were added, allowing use of odd or even function symmetries in the equivalence:
> verify(-Int(sin(1 - x), x), Int(sin(x - 1), x), 'sign'(oddfuncs = 
{Int, sin}));

true

> verify(a = b, -a = -b, 'sign');

true

> verify(a <> b, -a <> -b, 'sign');

true

> verify(a < b, -b < -a, 'sign');

true

> verify(a <= b, -b <= -a, 'sign');

true

> verify(a < b, -a < -b, 'sign');

false

> verify(a <= b, -a <= -b, 'sign');

false

Miscellaneous Improvements in Advanced Math

• The results of converting between mathematical functions are now better simplified:

> convert(-I-1/z*Im(z)+I*abs(1,z)/signum(z),signum);

\[
\frac{1}{2} \left(1 - \frac{1}{\text{signum}(z)^2}\right)
\]

• There was an improvement in int by expressing exponentials in the integrand as integer powers of a common base exponential:

For this answer the length has decreased by more than 10x:

> int(arctanh(exp(2*x)) * (exp(10*x) - 3*exp(6*x) - exp(-2*x) + 3*exp(2*x)) / 
(exp(4*x)-1) ^2, x);

\[
\frac{\arctanh((e^x)^2)}{2(e^x)^2} + \frac{\arctanh((e^x)^2)}{2(e^x)^2} - \ln(e^x) + \frac{\ln(e^x + 1)}{2} + \frac{\ln((e^x)^2 + 1)}{2} + \frac{\ln(e^x - 1)}{2}
\]

For this answer, the length has decreased by 7.5 times:

> dsolve(-diff(y(x), x)^2/y(x)^2 + diff(y(x), x, x)/y(x) + 2*coth(2*x)
*diff(y(x), x)/y(x) = 2);

\[
y(x) = e^{\frac{\text{arctanh}(e^{2x})}{2}} \sqrt{e^{2x} + 1} \sqrt{e^x + 1} \sqrt{e^x - 1}
\]
• A problem in \texttt{dsolve} was fixed, leading to this now correct (and much more compact) answer:

\begin{verbatim}
> ode := x^4*diff(y(x),x$2)+(a*sin(lambda/x)+b)*y(x)=0:
> ans := dsolve(ode);
ans := y(x) = c_1 x MathieuC\left(\frac{4 b}{\lambda^2}, -\frac{2 a}{\lambda^2}, \arccos\left(\sin\left(\frac{\pi x + 2 \lambda}{4 x}\right)\right)\right) + c_2 x MathieuS\left(\frac{4 b}{\lambda^2}, -\frac{2 a}{\lambda^2}, \arccos\left(\sin\left(\frac{\pi x + 2 \lambda}{4 x}\right)\right)\right)
> odetest(ans, ode);
0
\end{verbatim}

• \texttt{FunctionAdvisor} is now more careful when generating identities:

\begin{verbatim}
> FunctionAdvisor(arccoth, identities, quiet);
\end{verbatim}

\begin{align*}
\coth(\text{arccoth}(z)) &= z, \\
\left[I \coth(\text{arccoth}(-y) + \text{arccoth}(z) + 1\pi) = \frac{yz - 1}{Iz - 1y}\right]
\end{align*}

• \texttt{lcm} and \texttt{gcd} now treat single-argument input consistently with that of multiple argument inputs and just leave it alone rather than potentially returning an error:

\begin{verbatim}
> lcm(sin(2*x));
sin(2x)
> gcd(1+I+z);
1 + I + z
\end{verbatim}

• A case where \texttt{factor} was not idempotent has been fixed:

\begin{verbatim}
> alias(c=-(-x^2*y^n + 2*g(x)*y^n + 5*x*y^n - 4*y^n + 2*y) *(g(x)*x*y^n + x^2*y^n - 10*x*y^n + y*x + 24*y^n)/(y^n*(x - 3)*(x + 4)^2*(g(x)*y^n + x*y^n - 2*y^n + y))):
> alias(d=-(-g(x)*x^3*(y^n)^2 - x^4*(y^n)^2 + 2*g(x)^2*x*(y^n)^2 + 7*g(x)*x^2*(y^n)^2 + 15*x^3*(y^n)^2 - y*x^3*y^n - 24*g(x)*x*(y^n)^2 - 78*x^2*(y^n)^2 + 4*y*g(x)*x*y^n + 7*y*x^2*y^n + 48*g(x)*(y^n)^2 + 160*x*(y^n)^2 - 24*y*x^2*y^n + 2*x*y^2 - 96*(y^n)^2 + 48*y*y^n)/(y^n*(x - 3)*(x + 4)^2*(g(x)*x*y^n + x*y^n - 2*y^n + y))):
> factor(d^(5/4)-c^(5/4));
0
> alias(:-c=:-c, :-d=:-d):
\end{verbatim}

• A formula relating \texttt{InverseJacobiSD} to \texttt{InverseJacobiCN} has been corrected:

\begin{verbatim}
> tmp := InverseJacobiSD(z, k):
> tmp = convert(tmp, InverseJacobiCN);
\end{verbatim}
\[ \text{InverseJacobiSD}(z, k) = -\text{InverseJacobiCN}\left(\sqrt{-k^2 + 1} z, k\right) + \text{EllipticK}(k) \]

This call to \textit{product} no longer gives an unexpected error:

\[
\prod_{q=2}^{\infty} \frac{q \sinh\left(\pi \sqrt{q^2 + 1}\right)}{\sinh(\pi q) \sqrt{q^2 + 1}}
\]

Numeric evaluation of certain trig expressions is now more careful:

\[
\text{evalf}(\cot(-1/8 - 10^{10}\text{I})) = -4.259906707 \times 10^{-8685889639} + \text{I}
\]

\[
\text{evalf}(\Psi(-1/8 - 10^{10}\text{I})) = 23.02585093 - 1.570796327\text{I}
\]

The relative difference between the absolute values of the radical and non-radical part of a large power of the golden ratio is very small, making it difficult to distinguish numerically whether the expanded power is positive or negative:

\[
\phi := (\sqrt{5} - 1)/2:
\]

\[
\phi_{381} := \text{expand}(\phi^{381});
\]

\[
\phi_{381} = -21050428445219715212283513071197708058110642480418964396226164263063778766736638 + 941403779180129823143326315560312689907196353550379906525232657258068886564353 \frac{\sqrt{5}}{5}
\]

\[
\text{evalf([op(\phi_{381})])};
\]

\[
\left[ -2.105042845 \times 10^{79}, 2.105042844 \times 10^{79} \right]
\]

As such, some extra care is now taken by \text{abs} to determine that this expression is positive:

\[
\text{abs}(\phi_{381});
\]

\[
-21050428445219715212283513071197708058110642480418964396226164263063778766736638 + 941403779180129823143326315560312689907196353550379906525232657258068886564353 \frac{\sqrt{5}}{5}
\]

\[
\phi := '\phi':\phi_{381} := '\phi_{381}':
\]

Some improvements to the \text{inttrans} code led to these new results:

\[
\text{inttrans:-fourier}(\sqrt{t^2 + 1} \cdot (\text{Heaviside}(t + 1) - \text{Heaviside}(t - 1)), t, w);
\]

\[
\frac{\pi \text{BesselJ}(1, w)}{w}
\]

This result is new as of Maple 2023:
\[
\frac{1}{2} \left( 2 \int_0^\infty f(y) e^{-\frac{1}{y}\frac{1-s}{\mu}} \text{Ei}_1\left(\frac{-1/s}{y}\right) \, dy + \mu^{-1} \left( -2 \text{Ei}_1\left(\frac{-1/s}{\mu}\right) - 2 \ln\left(\frac{-1/s}{\mu}\right) + 2 \ln(s) + \ln\left(\frac{-1}{\mu}\right) - \ln(1/\mu) \right) \right)
\]

- `expand` was not being called recursively on some examples involving integrals. Now it is:

```maple
> expand(Int(f(x)*(b*x+c),x));
```

- `evalc` no longer assumes that two-argument `GAMMA` must be real if it has real arguments:

```maple
> evalc(Im(GAMMA(2/3, -1/3)));
```

- Expansion of Bessel and Hankel functions has been made much more efficient. This example now runs over 250 times faster than it did in Maple2023:

```maple
> time(expand(BesselJ(750,x)));
0.075
```

As well, expansion of `BesselK` in particular now takes advantage of the symmetry `BesselK(-v,x)=BesselK(v,x)` to achieve a new expanded normal form in terms of `BesselK(v_i,k)` with \( v_i \) between 0 and 1:

```maple
> expand(BesselK(4/3, x));
```

```maple
> expand(2/3*BesselK(1/3, x)/x+BesselK(2/3, x)-BesselK(4/3, x));
```

- `collect` no longer gets confused by the letter `O` (which is used to express series results):

```maple
> collect(O, O, F);
```

\( F(1) \)