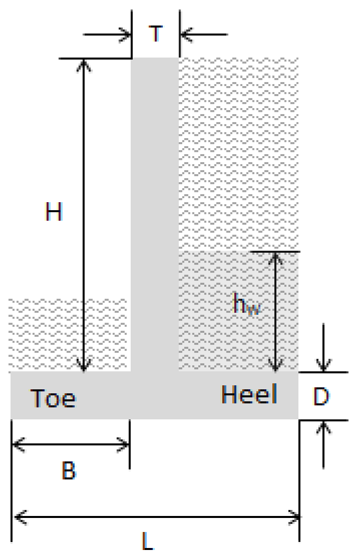
Concrete Retaining Wall

This document analyzes the design of a concrete retaining wall, employing the Strength Design Method as outlined in ACI 318.

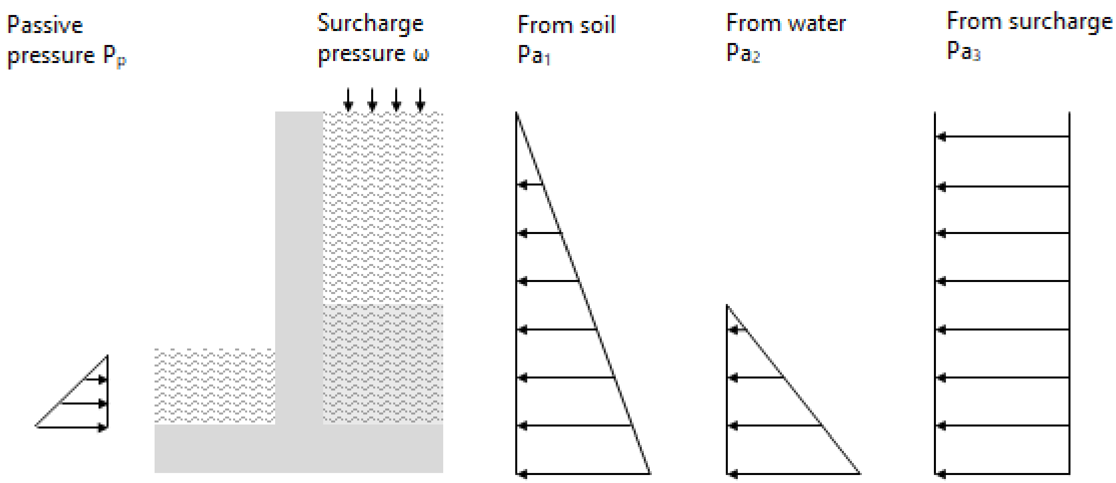


1. Parameters

Dead load (unfactored)	$DL := 0\ kN$
Live load (unfactored)	$LL := 0\ kN$
External moment dead load (unfactored)	$EMDL := 0 kN$
External moment live load (unfactored)	$EMLL := 0\ kN$
Surcharge	$w := 12\ kPa$
Compressive strength of concrete	$fc := 32\ MPa$
Rebar yield stress (<60 ksi, ACI 318. 11.5.2)	$fy := 460 MPa$
Soil bearing capacity	$q_{all} := 100\ kPa$
Unit weight of soil	$Y_{soil} := 18\ kN \cdot m^{-3}$

Unit weight of water	$Y_{water} := 10\ kN \cdot m^{-3}$
Unit weight of concrete	$Y_{concrete} := 25\ kN \cdot m^{-3}$
Coefficient of soil friction	$\phi := 30\ deg$
Height of wall	$H := 2\ m$
Height of soil	$hs := 2\ m$
Height of soil on toe	$h_{soil\_toe} := 0.6\ m$
Height of water	$hw := 1\ m$
Height of surcharge	$h_{surcharge} := 0.8\ m$
Base width of footing	$L_f := 1500\ mm$
Thickness of footing	$D_f := 250\ mm$
Thickness of wall	$T := 250\ mm$
Concrete cover	$cc := 75\ mm$
Diameter of bars	$d_b := 12\ mm$
Load factor dead load	$LF_{dead} := 1.2$
Load factor live load	$LF_{live} := 1.6$
Load factor lateral load	$LF_{lateral} := 1.6$
Strip length of wall to analyze	$b := 1\ m$
Width of toe	$B := \left(L_f - T\right)/2 = 625\ mm$
Strength reduction factor for flexure (ACI 318 9.3.2.1)	$\phi_{flexure} := 0.9$
Strength reduction factor for shear (ACI 313 9.3.2.3)	$\phi_{shear} := 0.75$

2. Forces on Retaining Wall



Soil pressure coefficients

$$ka := \frac{1 - \sin(\phi)}{1 + \sin(\phi)} = 0.333$$

$$kp := \frac{1 + \sin(\phi)}{1 - \sin(\phi)} = 3$$

Lateral force from soil

$$Pa1 := \frac{1}{2} \cdot Y_{soil} \cdot ka \cdot hs^2 \cdot b = 12.000 \text{ kN}$$

Level arm from base

$$L_{base\_soil} := \frac{1}{3} \cdot hs = 0.667 \text{ m}$$

Force from water

$$Pa2 := \frac{1}{2} \cdot Y_{water} \cdot hw^2 \cdot b = 5 \text{ kN}$$

Level arm from base

$$L_{base\_water} := \frac{1}{3} \cdot hw = 0.333 \text{ m}$$

Force from surcharge

$$Pa3 := w \cdot ka \cdot h_{surcharge} \cdot b = 3.200 \text{ kN}$$

Level arm from surcharge

$$L_{base\_surcharge} := \frac{1}{2} \cdot h_{surcharge} = 0.400 \text{ m}$$

Passive pressure

$$Pp := \frac{1}{2} \cdot Y_{soil} \cdot kp \cdot {h_{soil\_toe}}^2 \cdot b = 9.720 \text{ kN}$$

3. Check Wall for Shear

Nominal shear (equal to lateral forces on wall, and ignoring passive pressure)

$S_n := Pa1 + Pa2 + Pa3 = 20.200\ \mathrm{kN}$

Ultimate shear

$S_u := LF_{lateral} \cdot S_n = 32.320\ \mathrm{kN}$

Allowable shear according to ACI 318

$\phi Vc := \phi_{shear} \cdot 0.17 \cdot 1000\ \mathrm{m}^{\frac{-1}{2}} \cdot \mathrm{kg}^{\frac{-1}{2}} \cdot \mathrm{s} \cdot \mathrm{N} \cdot \sqrt{fc} \cdot \left((T - cc) - 0.5 \cdot d_b \right)$

$\phi Vc = 121.891\ \mathrm{kN}$

$\begin{cases} \text{"pass"} & S_u < \phi Vc \\ \text{"fail"} & \text{otherwise} \end{cases} = \text{"pass"}$

4. Design the Wall Stem for Flexure

Nominal moment

$M_n := Pa1 \cdot L_{base\_soil} + Pa2 \cdot L_{base\_water} + Pa3 \cdot L_{base\_surcharge} = 10.947\ \mathrm{kN\ m}$

Ultimate moment

$M_u := LF_{lateral} \cdot Pa1 \cdot L_{base\_soil} + LF_{lateral} \cdot Pa2 \cdot L_{base\_water} + LF_{live} \cdot Pa3 \cdot L_{base\_surcharge} = 17.515\ \mathrm{kN\ m}$

Reinforcement ratio

$\omega := 0.85 \cdot \left(1 - \left(\left(1 - \frac{2}{0.85} \cdot \mathrm{m}^{-1} \cdot \left(\frac{M_u}{\phi_{flexure} \cdot fc \cdot \left((T - cc) - 0.5 \cdot d_b \right)^2} \right) \right)^{0.5} \right) \right)$

$\omega = 0.022$

$\rho := \omega \cdot \frac{fc}{fy} = 1.50 \times 10^{-3}$

Minimum required reinforcement ratio

$\rho_{min} := 0.002$

$As := \rho \cdot \left(T - cc - 0.5 \cdot d_b \right) \cdot b = 253.548\ \mathrm{mm}^2$

$As_{min} := \rho_{min} \cdot b \cdot T = 500.000\ \mathrm{mm}^2$

Required main vertical bars.
Try 10-200 m spacing both sides

$$As_{act\_vert} := \frac{2 \cdot \pi \cdot (10\ mm)^2}{4 \cdot 200} \cdot 1000 = 785.398\ mm^2$$

Required horizontal bars.
Try 10-250 mm spacing both sides

$$As_{act\_horiz} := \frac{2 \cdot \pi \cdot (10\ mm)^2}{4 \cdot 250} \cdot 1000 = 628.319\ mm^2$$

$$\begin{cases} \text{"OK"} & As_{min} < As_{act\_vert} \textbf{ and } \max(As, As_{min}) < As_{act\_horiz} \\ \text{"Increase bar dia or reduce spacing"} & \text{otherwise} \end{cases} = \text{"OK"}$$

5. Stability Check

Check that the overturning moment and sliding forces are within reasonable engineering limits.

Weight due to soil on toe

$$W_{soil\_toe} := Y_{soil} \cdot h_{soil\_toe} \cdot \frac{L_f - T}{2} \cdot b = 6.750 \times 10^3\ N$$

Weight due to footing

$$W_{footing} := 0.9 \cdot Y_{concrete} \cdot L_f \cdot D_f \cdot b = 8.438 \times 10^3\ N$$

Weight of wall

$$W_{wall} := 0.9 \cdot Y_{concrete} \cdot T \cdot H \cdot b = 1.125 \times 10^4\ N$$

Weight of soil on heel

$$W_{soil\_heel} := Y_{soil} \cdot hs \cdot \frac{L_f - T}{2} \cdot b = 22500\ N$$

Weight of water

$$W_{water} := Y_{water} \cdot \frac{L_f - T}{2} \cdot hw \cdot b = 6250\ N$$

Weight of surcharge

$$W_{surcharge} := w \cdot \frac{L_f - T}{2} \cdot b = 7500\ N$$

Total weight

$$W_{total} := W_{soil\_toe} + W_{footing} + W_{wall} + W_{soil\_heel} + W_{water} + W_{surcharge}$$

$$W_{total} = 62.688\ kN$$

Check for Overturning

Overturning moment

$$M_{overturning} := Pa1 \cdot L_{base\_soil} + Pa2 \cdot L_{base\_water} + Pa3 \cdot L_{base\_surcharge}$$

$$M_{overturning} = 10.947\ \text{kN m}$$

Righting moment (for a conservative design, ignore M_{soil_toe})

From soil on toe $$M_{soil\_toe} := W_{soil\_toe} \cdot \frac{L_f - T}{4} = 2.109\ \text{kN m}$$

From wall $$M_{wall} := W_{wall} \cdot \frac{L_f}{2} = 8.438\ \text{kN m}$$

From footing $$M_{footing} := W_{footing} \cdot \frac{L_f}{2} = 6.328\ \text{kN m}$$

From soil $$M_{soil\_heel} := W_{soil\_heel} \cdot \left(\frac{L_f - T}{2} + T + \frac{L_f - T}{4} \right) = 26.719\ \text{kN m}$$

From water $$M_{water} := W_{water} \cdot \left(\frac{L_f - T}{2} + T + \frac{L_f - T}{4} \right) = 7.422\ \text{kN m}$$

From surcharge $$M_{surcharge} := W_{surcharge} \cdot \left(\frac{L_f - T}{2} + T + \frac{L_f - T}{4} \right) = 8.906\ \text{kN m}$$

Total righting moment $$M_{total} := M_{wall} + M_{footing} + M_{soil\_heel} + M_{water} + M_{surcharge}$$

$$M_{total} = 57.813\ \text{kN m}$$

$$\frac{M_{total}}{M_{overturning}} = 5.281$$

Factor of safety $$\begin{cases} \text{"OK for overturning"} & \dfrac{M_{total}}{M_{overturning}} > 2 \\ \text{"Increase sizes"} & \text{otherwise} \end{cases} = \text{"OK for overturning"}$$

Check for Sliding

For stability against sliding ensure that $\dfrac{\text{resisting force}}{\text{sliding force}} > 1.5$

Sliding force $$F_{sliding} := Pa1 + Pa2 + Pa3 = 2.020 \times 10^{4}\ \text{N}$$

Resisting force (passive pressure W_{soil_toe} is ignored for a conservative design)

$$F_{total} := W_{wall} + W_{footing} + W_{soil\_heel} + W_{water} + W_{surcharge}$$

$$F_{total} = 55.938\ \mathbf{kN}$$

$$\frac{F_{total}}{F_{sliding}} = 2.769$$

Factor of safety

$$\begin{cases} \text{"OK for sliding"} & \dfrac{F_{total}}{F_{sliding}} > 1.5 \\ \text{"Increase sizes"} & \text{otherwise} \end{cases} = \text{"OK for sliding"}$$

6. Check Required Length of Base

Maximum soil pressure

$$q_{max} = \frac{P}{A} + \frac{6 \cdot M}{b \cdot d^2} \le q_{all}$$

$$q_{max} := \frac{DL + LL + F_{total}}{b \cdot L_f} + \frac{6 \cdot \left(M_n + EMDL + EMLL\right)}{b \cdot {L_f}^2}$$

$$q_{max} = 66.483\ \mathbf{kPa}$$

$$\begin{cases} \text{"OK for soil bearing"} & q_{max} < q_{all} \\ \text{"Increase wall footing"} & \text{otherwise} \end{cases} = \text{"OK for soil bearing"}$$

Maximum ultimate soil pressure (considering 1 m strip)

$$qu_{max} := \frac{LF_{dead} \cdot DL + LF_{live} \cdot LL + LF_{dead} \cdot \left(W_{wall} + W_{footing}\right) + LF_{live} \cdot W_{surcharge} + LF_{lateral} \cdot \left(W_{soil\_heel} + W_{water}\right)}{b \cdot L_f} + \frac{6 \cdot \left(LF_{dead} \cdot EMDL + LF_{live} \cdot EMLL + M_u\right)}{b \cdot {L_f}^2}$$

$$qu_{max} = 101.122\ \mathbf{kPa}$$

$$qu_{min} := \frac{LF_{dead} \cdot DL + LF_{live} \cdot LL + LF_{dead} \cdot \left(W_{wall} + W_{footing}\right) + LF_{live} \cdot W_{surcharge} + LF_{lateral} \cdot \left(W_{soil\_heel} + W_{water}\right)}{b \cdot L_f} - \frac{6 \cdot \left(LF_{dead} \cdot EMDL + LF_{live} \cdot EMLL + M_u\right)}{b \cdot {L_f}^2}$$

$$qu_{min} = 7.711 \times 10^3\ \mathbf{Pa}$$

If qu_{min} is in tension (+) solve for the required length. Ignore when qu_{min} is in compression (-).

Eccentricity

$$e := \frac{LF_{dead} \cdot EMDL + LF_{live} \cdot EMLL + M_u}{LF_{dead} \cdot DL + LF_{live} \cdot LL + LF_{dead} \cdot \left(W_{wall} + W_{footing}\right) + LF_{live} \cdot W_{surcharge} + LF_{lateral} \cdot \left(W_{soil\_heel} + W_{water}\right)}$$

$e = 0.215\ \mathrm{m}$

Given $a = \dfrac{2 \cdot P}{qu_{max} \cdot b}$

$$a := \frac{2 \cdot \left(LF_{dead} \cdot DL + LF_{live} \cdot LL + LF_{dead} \cdot \left(W_{wall} + W_{footing}\right) + LF_{live} \cdot W_{surcharge} + LF_{lateral} \cdot \left(W_{soil\_heel} + W_{water}\right)\right)}{qu_{max} \cdot b}$$

$a = 1.614\ \mathrm{m}$

$L := 2 \cdot \left(e + \dfrac{a}{3}\right) = 1.505\ \mathrm{m}$ $\qquad$ $L := \mathrm{ceil}(10 \cdot L) \cdot 0.1 = 1.600\ \mathrm{m}$

7. Adequacy of Footing Thickness for Wide Beam Shear

When qu__min is in compression

$q_c = qu_{min} + y$

Solving for y by similar triangles

$$y := \left(\frac{L_f - T}{2} + T + \left(D_f - cc - 0.5 \cdot d_b\right)\right) \cdot \frac{qu_{max} - qu_{min}}{L_f}$$

$y = 6.501 \times 10^4\ \mathrm{Pa}$

$q_c := qu_{min} + y = 72.725\ \mathrm{kPa}$

$$Ld := \frac{L_f - T}{2} - \left(D_f - cc - 0.5 \cdot d_b\right) = 456.000\ \mathrm{mm}$$

When qu__min is in tension

$q_c = y$

$$V_{u\_compression} := \frac{1}{2} \cdot \left(q_c + qu_{max}\right) \cdot Ld \cdot b = 39.637\ \mathrm{kN}$$

Solving for y from similar triangles

$$y := \left(a - \left(\frac{L - T}{2} - \left(D_f - cc - 0.5 \cdot d_b\right)\right)\right) \cdot \frac{qu_{max}}{a}$$

$$y = 6.943 \times 10^4 \ \frac{kg}{m \cdot s^2}$$

$$q_c := y$$

$$Ld := \frac{L - T}{2} - \left(D_f - cc - 0.5 \cdot d_b\right) = 0.506 \ m$$

$$V_{u\_tension} := \frac{1}{2} \cdot \left(q_c + qu_{max}\right) \cdot Ld \cdot b = 43.149 \ kN$$

$$V_{u\_govern} := \begin{cases} V_{u\_compression} & qu_{min} > 0 \\ V_{u\_tension} & \text{otherwise} \end{cases} = 3.964 \times 10^4 \ N$$

Allowable shear

$$\phi Vc := \phi_{shear} \cdot 0.17 kg^{\frac{1}{2}} \cdot m^{\frac{1}{2}} \cdot s^{-1} \cdot \sqrt{fc} \cdot \left(D_f - cc - 0.5 \cdot d_b\right) \cdot 1000$$

$$\phi Vc = 1.219 \times 10^5 \ N$$

$$\begin{cases} \text{"OK for shear"} & V_{u\_govern} < \phi Vc \\ \text{"Increase thickness of wall"} & \text{otherwise} \end{cases} = \text{"OK for shear"}$$

8. Check Wall Thickness for Flexure

When qu_min is in compression

$q_c = q_{min} + y$

Solving for y from similar triangles

$$y := \left(qu_{max} - qu_{min}\right) \cdot \left(\frac{L_f - T}{2} + T\right) \cdot \frac{1}{L_f} = 5.449 \times 10^4 \ Pa$$

$$q_c := qu_{min} + y = 62.201 \ kPa$$

$$M_{u\_compression} := q_c \cdot \frac{L_f - T}{2} \cdot 0.5 m \cdot \frac{L_f - T}{2} + 0.5 \cdot \left(qu_{max} - q_c\right) \cdot \frac{L_f - T}{2} \cdot \frac{2}{3} m \cdot \frac{L_f - T}{2} = 17.217 \ kN \ m$$

When qu_min is in tension

$q_c = y$

Solving for y from similar triangles

$$q_c := \frac{qu_{max}\cdot\left(a - \frac{L-T}{2}\right)}{a} = 58.841\ \text{kPa}$$

$$M_{u\_tension} := q_c\cdot\frac{L-T}{2}\cdot\frac{1}{2}\text{m}\cdot\frac{L-T}{2} + \frac{1}{2}\cdot\left(qu_{max} - q_c\right)\cdot\frac{L-T}{2}\cdot\frac{2}{3}\text{m}\cdot\frac{L-T}{2}$$

$$M_{u\_tension} = 19.826\ \text{kN m}$$

$$M_{u\_govern} := \begin{cases} M_{u\_compression} & qu_{min} > 0 \\ M_{u\_tension} & \text{otherwise} \end{cases}$$

$$M_{u\_govern} = 17.217\ \text{kN m}$$

$$\text{fsolve}\left(M_{u\_govern} = \phi_{flexure}\cdot fc\cdot b\cdot\left(D_f - cc - 0.5\cdot d_b\right)^2\cdot\omega 1\cdot\left(1 - 0.59\cdot\omega 1\right), \omega 1\right)[1] = 0.021$$

Theoretical reinforcement ratio required for flexure

$$\omega := 0.85\cdot\left(1 - \left(\left(1 - \frac{2}{0.85}\cdot\text{m}^{-1}\cdot\left(\frac{M_{u\_govern}}{\phi_{flexure}\cdot fc\cdot\left(D_f - cc - 0.5\cdot d_b\right)^2}\right)\right)^{\frac{1}{2}}\right)\right)$$

$$\omega = 0.021$$

$$\rho := \omega\cdot\frac{fc}{fy} = 0.001$$

$$\rho_{min} := 0.002$$

$$As := \rho\cdot 1\text{m}\cdot\left(D_f - cc - 0.5\cdot d_b\right) = 2.49\times 10^2\ \text{mm}^2$$

$$As_{min} := \rho_{min}\cdot 1\text{m}\cdot D_f = 5.00\times 10^2\ \text{mm}^2$$

Required Main Bars

Try 10-200mm spacing both sides As act

$$As_{act\_main\_bars} := \frac{2\cdot\pi\cdot(10\ \text{mm})^2\cdot 1000}{4\cdot 200} = 785.398\ \text{mm}^2$$

Required Secondary Bars

$$As_{act\_secondary\_bars} := \frac{2\cdot\pi\cdot(10\text{mm})^2\cdot 1000}{4\cdot 250} = 628.319\ \text{mm}^2$$

Try 10-250 mm spacing both sides As act

$$reinforcement := \begin{cases} \text{"OK"} & \max\left(As, As_{min}\right) < As_{act\_main\_bars} \textbf{ and } As_{min} < As_{act\_secondary\_bars} \\ \text{"Increase bar diameter or reduce spacing"} & \text{otherwise} \end{cases}$$

reinforcement = "OK"