

# VARINT

## VARINT FOR MAPLE computer algebra aided design of variational integrators

It is common to create numerical integration algorithms for dynamical systems by discretizing the equations of motion, the Euler-Lagrange equations. That means that all differential operators are to be replaced by their discretized analogues. Many of these numerical algorithms are, however, not geometric. Here, ‘geometry’ refers to the differential geometric (‘symplectic’) structure of the phase space.

For simulations over relatively short time spans as compared to the intrinsic time scales of the dynamical system at hand, these non-geometric integrators are often advantageous, as they include adaptive and multistep methods, which can be both accurate and fast for generic systems. Computations over larger time scales require a different approach, as non-geometric methods tend to generate or dissipate energy artificially, which means that at some point the errors dominate. The reason for this is that the fundamental geometric structure, the symplectic two-form to be precise, is not conserved. The conservation of this mathematical object is crucial in long simulations, as the geometric structure encodes both the dynamics and all conserved quantities. Geometric methods neither generate nor dissipate energy artificially. In fact, the energy errors are often bounded.

Unfortunately, the design of geometric numerical integration algorithms is quite often limited by the knowledge of classes of non-geometric numerical integrators, from which specific instances can be derived that *are* geometric. The actual calculations for higher-order algorithms can be quite daunting an exercise. Enter variational integrators.

Variational integrators reverse the basic approach to obtaining numerical integration methods. Instead of deriving the equations of motion from the action functional, the Euler-Lagrange equations, which are then discretized, the action functional itself is discretized using a quadrature formula. After that, the discrete action, as it is called, is varied to give the discrete Euler-Lagrange equations. These equations preserve the differential geometric structure *automatically*. So, in order to create variational integrators one merely needs a quadrature rule for the action! Because of Maple’s symbolic capabilities, the action can be either a generic expression in order to suit many different dynamical systems with the same mathematical structure, or a specific one, so that the resulting variational integrator can be optimized for performance.

### LICENSE

You are entirely free to use, modify, and redistribute VarInt for non-commercial research purposes, provided that due credit is given to the author. The author is not liable for any damages arising in any way out of the use of VarInt, be they direct, indirect, incidental, special, exemplary, or consequential.

## PREREQUISITES

In order to run VarInt you have to have a computer with Maple installed on it. All releases from Maple 11 on are compatible. VarInt is platform-independent, as it only uses standard Maple procedures.

## OVERVIEW

Here's an overview of the files that come with VarInt:

- VarInt.lib *library file*
- VarInt.ind *index file*
- VarInt.hdb *help database file*
- VarInt.src *source file*

The source file is provided for users who wish to study the inner workings of VarInt and/or modify/extend VarInt themselves. In addition, it can be read directly in Maple with the **read** command.

## INSTALLATION (LINUX)

The installation of VarInt is particularly easy:

- 1) Create a folder that Maple can access, preferably in your home folder:

```
mkdir $HOME/VarInt
```

- 2) Edit/create your initialization file (\$HOME/.mapleinit):

```
userlibname:="/home/username/VarInt":
```

```
libname:=userlibname, libname:
```

- 3) Start Maple.
- 4) Execute **with(VarInt)**; and you should be able to see the four modules that come up VarInt.

## INSTALLATION (WINDOWS/MAC)

The installation procedure is similar for other operating systems supported by Maple. The location of the initialization file may depend on your version of Maple. Please consult the help files for more details.

## NOTE

Please do not install VarInt in your standard Maple library, as it might corrupt Maple. In an active Maple session, you can find your standard Maple library by executing `libname`.

## DE-INSTALLATION

In case you wish to uninstall VarInt, simply remove the package files from your system (\*.lib, \*.ind, and \*.hdb).

## INFORMATION

More information can be found in the author's dissertation: *Numerics of Spacecraft Dynamics*, University of Turku, Finland (2011).

## CONTACT

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## ACKNOWLEDGEMENT

The development of VarInt has been funded by the European Commission's Marie Curie FP6 Astrodynamics Network.