

Designing a More Effective Car Radiator

The challenge: To determine the design parameters of a smaller radiator assembly capable of dissipating the same amount of heat as the original assembly.

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Problem Definition

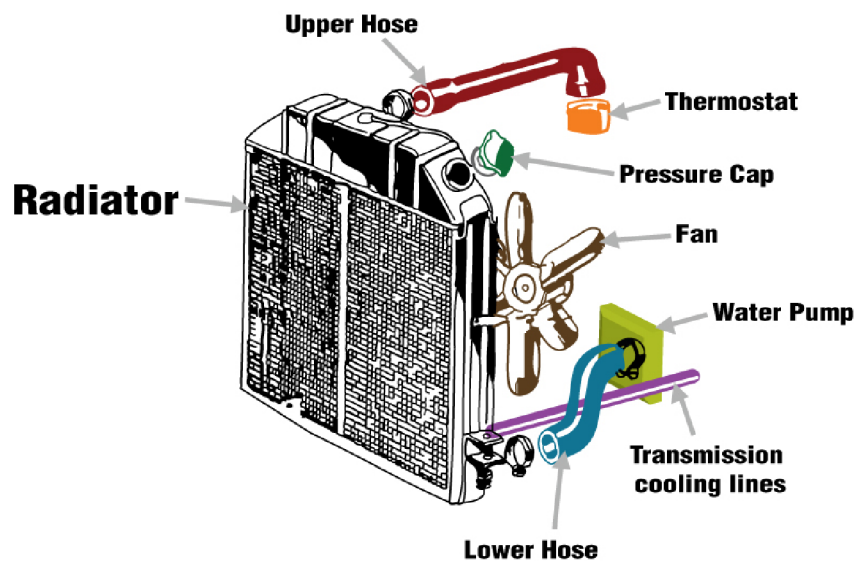
1. Original & Proposed Radiator Dimensions

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▼ Executive Summary

The demand for more powerful engines in smaller hood spaces has created a problem of insufficient rates of heat dissipation in automotive radiators. Upwards of 33% of the energy generated by the engine through combustion is lost in heat. Insufficient heat dissipation can result in the overheating of the engine, which leads to the breakdown of lubricating oil, metal weakening of engine parts, and significant wear between engine parts. To minimize the stress on the engine as a result of heat generation, automotive radiators must be redesigned to be more compact while still maintaining high levels of heat transfer performance.

Most four-cylinder automobiles, depending on their size, have radiator cores that vary from 19" × 11.5" × 0.7" to 27" × 17" × 0.9". We believe that we can greatly reduce the size of automotive radiators while maintaining the current levels of heat transfer performance expected. Moreover, this can be done without significant modification to the existing internal radiator structure. There are several different approaches that one can take to optimize the heat transfer performance of a smaller radiator design. These include: 1) changing the fin design, 2) increasing the core depth, 3) changing the tube type, 4) changing the flow arrangement, 5) changing the fin material, and 6) increasing the surface area to coolant ratio. The latter method was chosen for our proposed design.

To prove this hypothesis, we conducted tests on our current radiator assembly, which measures 24" × 17" × 1", to determine the heat transfer performance under typical operating conditions. We found our current radiator assembly to be capable of dissipating heat at a rate of $4025 \frac{Btu}{minute} \left(70729 \frac{J}{s} \right)$. Next,

using the ϵ -Ntu (effectiveness-Ntu), we calculated the heat transfer performance of our new radiator assembly, which has a radiator length 30% smaller than the length of the current design (18" × 17" × 1"). As expected, the heat transfer performance decreased. However, by increasing the metal-to-air surface area from 384 fins per row to 437 fins per row, we increased the heat transfer performance of our proposed design to the same level as the current design under the same operating conditions.

▼ Introduction

In an automobile, fuel and air produce power within the engine through combustion. Only a portion of the total generated power actually supplies the automobile with power -- the rest is wasted in the form of exhaust and heat. If this excess heat is not removed, the engine temperature becomes too high which results in overheating and viscosity breakdown of the lubricating oil, metal weakening of the overheated engine parts, and stress between engine parts resulting in quicker wear, among other things.

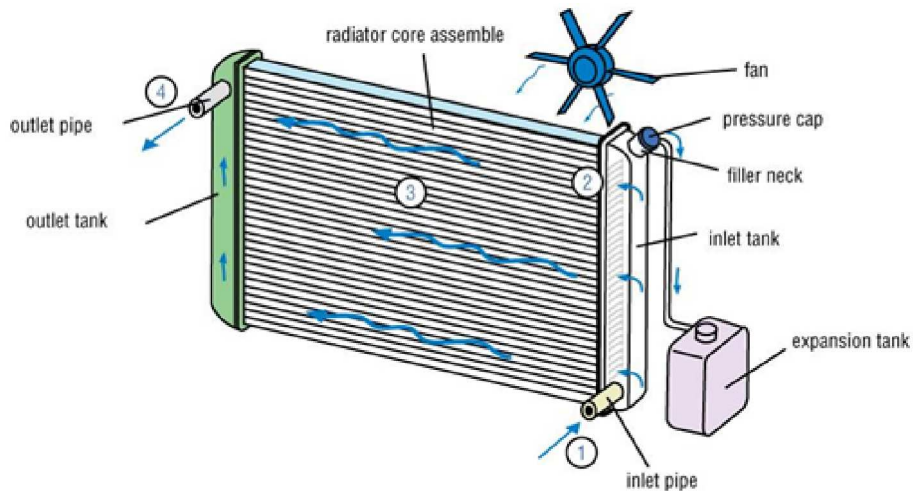


Figure 1: Components within an automotive cooling system

A cooling system is used to remove this excess heat. Most automotive cooling systems consist of the following components: radiator, water pump, electric cooling fan, radiator pressure cap, and thermostat. Of these components, the radiator is the most prominent part of the system because it transfers heat.

As coolant travels through the engine's cylinder block, it accumulates heat. Once the coolant temperature increases above a certain threshold value, the vehicle's thermostat triggers a valve which forces the coolant to flow through the radiator.

As the coolant flows through the tubes of the radiator, heat is transferred through the fins and tube walls to the air by conduction and convection.

▼ Problem Description

From the laws of thermodynamics, we know that heat transfer increases as we increase the surface area of the radiator assembly. That said, the demand for more powerful engines in smaller hood spaces has created a problem of insufficient rates of heat dissipation in automotive radiators. As a result, many radiators must be redesigned to be more compact while still having sufficient cooling power capabilities.

This application proposes a new design for a smaller radiator assembly. The new design is capable of dissipating the same heat as the original, given a set of operating conditions.

▼ 1. Original & Proposed Radiator Dimensions

The dimensions of our original radiator design can be extracted from the SolidWorks® drawing file ([CurrentRadiatorDrawing.SDPR](#)). The drawing is a scaled down version of the full radiator assembly which measures 24" × 17" × 1". For the purpose of our analysis, the dimensions obtained from CAD are scaled up

to reflect the radiator's actual dimensions.

Note: This application uses a SolidWorks design diagram to extract the dimensions of the original radiator. This design file can be found in the zip file this document came in. If you have SolidWorks version 8.0 or above, save the design file, and then click the radio button below to tell Maple™ where to find the file. If you do not have SolidWorks installed on your computer, the values will be pre-populated.

WithoutSolidWorks WithSolidWorks

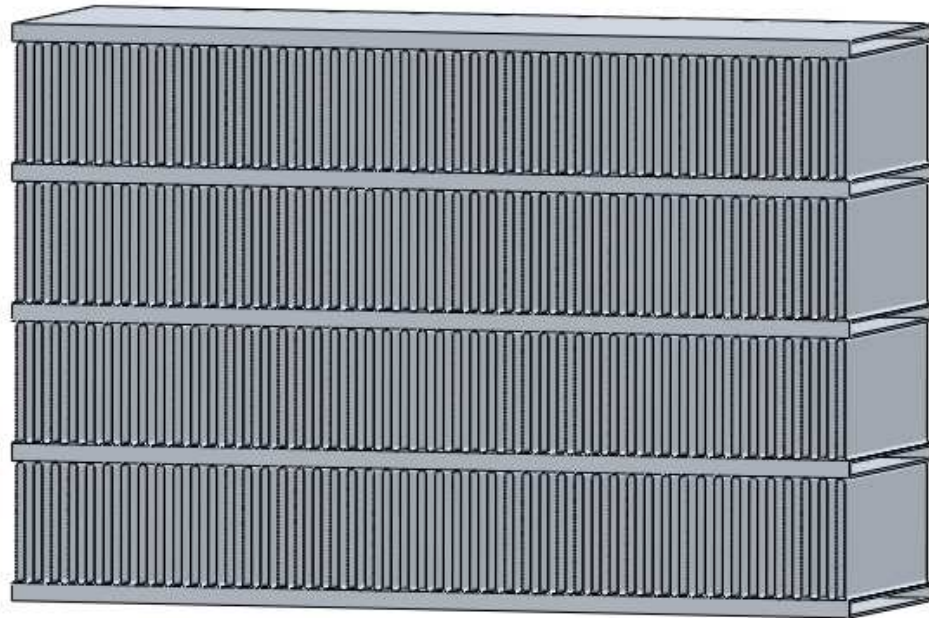


Figure 2: CAD rendering of current Radiator Model

▼ Original Radiator Model Dimensions

The table below summarizes the current radiator dimensions.

Current Radiator Dimensions			
Radiator length (rL_{cur}) :	<input type="text"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.609600"/>	<i>m</i>	<input type="checkbox"/> Use SI value
Radiator width (rW_{cur}) :	<input type="text" value="1.41657"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.431771"/>	<i>m</i>	<input type="checkbox"/> Use SI value

Radiator height (rH_{cur}) :	<input type="text" value="0.0807292"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.0246063"/>	<i>m</i>	<input type="checkbox"/> Use SI value
Tube width (tW_{cur}) :	<input type="text" value="0.0807292"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.0246063"/>	<i>m</i>	<input type="checkbox"/> Use SI value
Tube height (tH_{cur}) :	<input type="text" value="0.00512667"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.00156261"/>	<i>m</i>	<input type="checkbox"/> Use SI value
Fin width (fW_{cur}) :	<input type="text" value="0.0807292"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.0246063"/>	<i>m</i>	<input type="checkbox"/> Use SI value
Fin height (fH_{cur}) :	<input type="text" value="0.0389808"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.0118813"/>	<i>m</i>	<input type="checkbox"/> Use SI value
Fin thickness (fT_{cur}) :	<input type="text" value="0.00008333"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.00002540"/>	<i>m</i>	<input type="checkbox"/> Use SI value
Distance Between Fins (fD_{cur}) :	<input type="text" value="0.00520833"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.00158750"/>	<i>m</i>	<input type="checkbox"/> Use SI value
Number of tubes ($ntube_{cur}$) :	<input type="text" value="33."/>		

Testing this radiator design under different coolant flow and air flow conditions yielded the following graph of heat transfer performance vs. coolant flow rate at different airflow speeds.

A heat transfer performance of $4025 \frac{Btu}{minute}$ was obtained using a coolant volumetric flow, air volumetric flow and air velocity of 30 gpm , $2349 \frac{ft^3}{minute}$, $10 \frac{mi}{h}$, respectively.

These results are summarized in the table below.

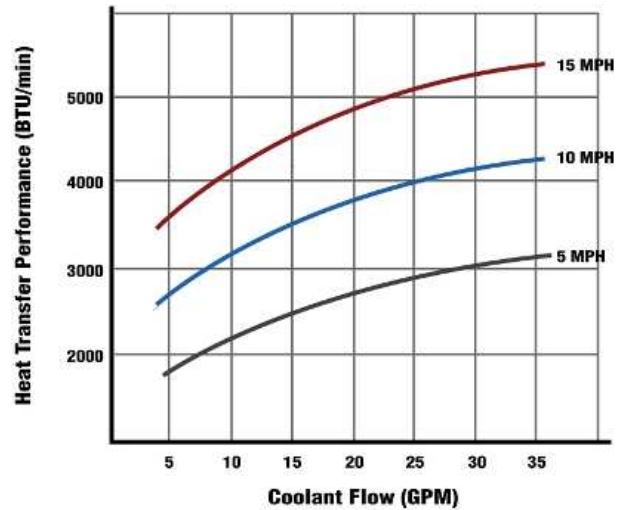


Figure 3: Heat transfer performance vs. coolant flow rate at different airflow speeds

Radiator Operating Conditions			
Coolant Volumetric Flow (v_f_c)	<input type="text" value="30"/>	gpm	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.0018927"/>	$\frac{m^3}{s}$	<input type="checkbox"/> Use SI value
Air Volumetric Flow (v_f_a)	<input type="text" value="2349"/>	$\frac{ft^3}{minute}$	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="1.10860"/>	$\frac{m^3}{s}$	<input type="checkbox"/> Use SI value
Air Velocity (v_a)	<input type="text" value="10"/>	$\frac{mi}{h}$	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="4.4704"/>	$\frac{m}{s}$	<input type="checkbox"/> Use SI value
Heat Transfer Performance (q_{cur})	<input type="text" value="4025"/>	$\frac{Btu}{minute}$	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="70729.3"/>	$\frac{J}{s}$	<input type="checkbox"/> Use SI value

▼ Proposed Radiator Model Dimensions

Our proposed design has a radiator length that is 30% smaller than that of the original model. The dimensions of the radiator core (radiator length, radiator width and radiator height) can be adjusted to any dimension.

The table below summarizes the radiator dimensions for our proposed design.

Proposed Radiator Dimensions			
Radiator length (rL_{new}) :	<input type="text" value="1.50000"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.457200"/>	<i>m</i>	<input type="checkbox"/> Use SI value
Radiator width (rW_{new}) :	<input type="text" value="1.41657"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.431771"/>	<i>m</i>	<input type="checkbox"/> Use SI value
Radiator height (rH_{new}) :	<input type="text" value="0.0807292"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.0246063"/>	<i>m</i>	<input type="checkbox"/> Use SI value
Tube width (tW_{new}) :	<input type="text"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.0246063"/>	<i>m</i>	<input type="checkbox"/> Use SI value
Tube height (tH_{new}) :	<input type="text" value="0.00512667"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.00156261"/>	<i>m</i>	<input type="checkbox"/> Use SI value
Fin width (fW_{new}) :	<input type="text" value="0.0807292"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.0246063"/>	<i>m</i>	<input type="checkbox"/> Use SI value
Fin height (fH_{new}) :	<input type="text" value="0.0389808"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.0118813"/>	<i>m</i>	<input type="checkbox"/> Use SI value

Fin thickness (fT_{new}) :	<input type="text" value="0.00008333"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.00002540"/>	<i>m</i>	<input type="checkbox"/> Use SI value
Distance Between Fins (fD_{new}) :	<input type="text" value="0.00520833"/>	<i>ft</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.00158750"/>	<i>m</i>	<input type="checkbox"/> Use SI value
Number of tubes ($ntube_{new}$) :	<input type="text" value="33."/>		

▼ Coolant and Air Property Tables

The thermal fluid properties for the coolant and air are listed in the following two tables.

Coolant Properties: 50-50 Glycol-Water			
Thermal conductivity (k_c) :	<input type="text" value="0.24"/>	$\frac{Btu}{h \cdot ft \cdot degF}$	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.415098"/>	$\frac{W}{m \cdot K}$	<input type="checkbox"/> Use SI value
Specific Heat (C_c) :	<input type="text" value="0.88"/>	$\frac{Btu}{lb \cdot degF}$	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="3681.92"/>	$\frac{J}{kg \cdot K}$	<input type="checkbox"/> Use SI value
Density (ρ_c) :	<input type="text" value="63.4"/>	$\frac{lb}{ft^3}$	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="1015.57"/>	$\frac{kg}{m^3}$	<input type="checkbox"/> Use SI value
Dynamic Viscosity (μ_c) :	<input type="text" value="0.0005"/>	$\frac{lb}{ft \cdot s}$	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.000744082"/>	<i>Pa·s</i>	<input type="checkbox"/> Use SI value
Coolant Temperature (T_c) :	<input type="text" value="250"/>	<i>degF</i>	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="138.889"/>	<i>K</i>	<input type="checkbox"/> Use SI value

Air Properties:

Thermal conductivity (k_a) :	<input type="text" value="0.0154"/>	$\frac{Btu}{h \cdot ft \cdot degF}$	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.0266355"/>	$\frac{W}{m \cdot K}$	<input type="checkbox"/> Use SI value
Specific Heat (C_a) :	<input type="text" value="0.240"/>	$\frac{Btu}{lb \cdot degF}$	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="1004.16"/>	$\frac{J}{kg \cdot K}$	<input type="checkbox"/> Use SI value
Density (ρ_a) :	<input type="text" value="0.071"/>	$\frac{lb}{ft^3}$	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="1.13731"/>	$\frac{kg}{m^3}$	<input type="checkbox"/> Use SI value
Dynamic Viscosity (μ_a) :	<input type="text" value="0.00001285"/>	$\frac{lb}{ft \cdot s}$	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="0.00001912"/>	$Pa \cdot s$	<input type="checkbox"/> Use SI value
Coolant Temperature (T_a) :	<input type="text" value="150"/>	$degF$	<input checked="" type="checkbox"/> Use FPS value
	<input type="text" value="83.3333"/>	K	<input type="checkbox"/> Use SI value

▼ 2. Heat Transfer Performance of Proposed Radiator Assembly

We expect the heat transfer performance of our proposed radiator assembly to be smaller than that of the original model because we are reducing the surface area to coolant ratio. The question that we answer in this section is "How much smaller is the heat transfer performance?" If the heat transfer performance is only marginally smaller, we can take other approaches to increase the performance, for example, increase the number of fins per row, change the fin material, or change the flow arrangement.

The ϵ -Ntu (effectiveness-Ntu) method is used to predict the heat transfer performance of our new system.

The more common equations that are typically used in heat exchange design are listed below.

Heat Exchange Equations:	Definitions:
$HeatTransferEquation := q = \epsilon \cdot C_{min} \cdot ITD :$	The rate of conductive heat transfer
$UniversalHeatTransferEquation := \frac{1}{UA} = \frac{1}{hc \cdot Ac} + \frac{1}{nfha \cdot Aa} :$	The overall thermal resistance present in the system
$ReynoldsEquation := ReynoldsNum = \left(\frac{\rho \cdot v \cdot D_H}{\mu} \right) :$	A dimensionless modulus that represents fluid flow conditions
$HydraulicDiameter := D_H = \frac{4 \cdot A_{min}}{WP} :$	Parameter used to equate any flow geometry to that of a round pipe
$DittusBoelterEquation := NusseltNum = 0.023 \cdot ReynoldsNum^{0.8} \cdot PrandtlNum^{\frac{1}{3}} :$	An equation used to calculate the surface coefficient of heat transfer for fluids in turbulent flow
$PrandtlEquation := PrandtlNum = \frac{C \cdot \mu}{k} :$	A dimensionless modulus that relates fluid viscosity to the thermal conductivity, a low number indicates high convection
$NusseltEquation := NusseltNum = \frac{hc \cdot D_H}{k} :$	A dimensionless modulus that relates surface convection heat transfer to fluid conduction heat transfer
$NtuEquation := Ntu = \frac{UA}{C_{min}} :$	A dimensionless modulus that defines the number of transferred units
$\epsilon NtuEquation := \epsilon = 1 - e^{-\left(\frac{C_{max}}{C_{min}}\right) \cdot (1 - e^{-Cratio \cdot Ntu})} :$	A mathematical expression of heat exchange effectiveness vs. the number of heat transfer units

$ITDEquation := ITD = CoolantTemperature$
 $- AirTemperature :$

Measure of the initial temperature
 difference

Choose which unit system to perform calc... Perform calculations in FPS units ▼

We must first calculate the overall heat transfer coefficient UA_{new} of the smaller radiator before we can determine it's heat transfer performance, q_{new}

▼ Solve for UA_{new}

The Universal Heat Transfer Equation is defined in **(1)**

UniversalHeatTransferEquation

$$\frac{1}{UA} = \frac{1}{hc Ac} + \frac{1}{nfha Aa} \quad (1)$$

The next several steps will take us through the process for solving for the unknown values of Ac , Aa , hc and $nfha$

▼ Solve for Ac_{new} & Aa_{new}

$$CoolantSurfaceArea := Ac = NumberOfTubes \cdot (2 \cdot (TubeHeight \cdot RadiatorLength) + 2 \cdot (TubeWidth \cdot RadiatorLength)) :$$

$$AirSurfaceArea := Aa = TotalNumberOfAirPassages \cdot (2 \cdot (FinDistance \cdot FinHeight) + 2 \cdot (FinHeight \cdot FinWidth)) :$$

where

$$TotNumAirPassages := TotalNumberOfAirPassages = NumRowsOfFins \cdot \left(\frac{RadiatorLength}{FinDistance} \right) :$$

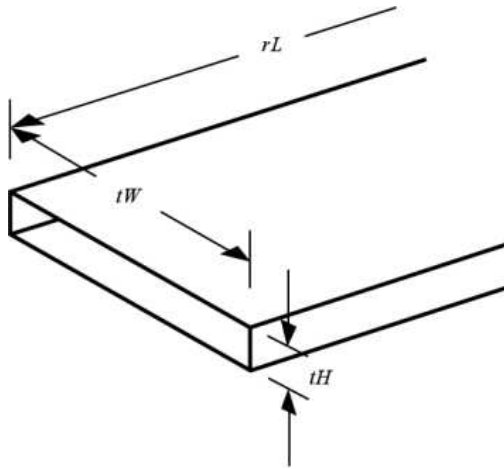


Figure 4: Expanded view of tubes

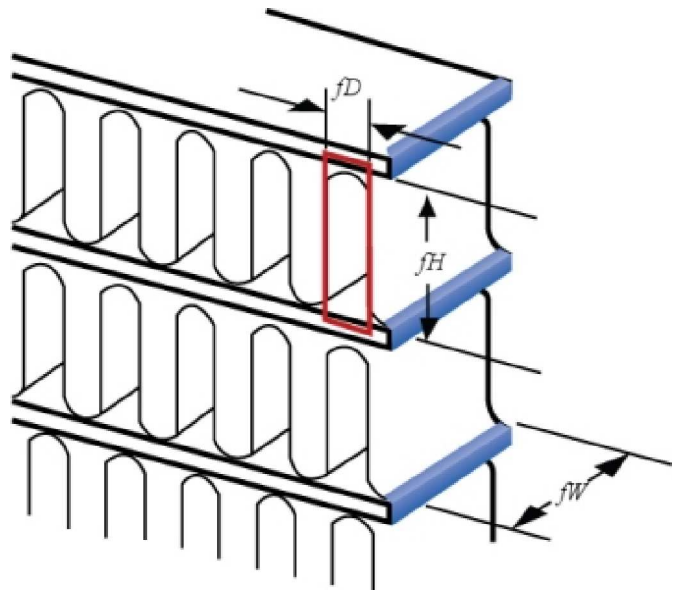


Figure 5: Expanded view of fins

Solving the unknown values leads to the following values for the *TotalNumberOfAirPassages*, *Ac*, and *Aa*.

$$\begin{aligned} TotalNumberOfAirPassages_{new} := & \text{simplify}(\text{solve}(\text{subs}((NumRowsOfFins = (ntube_{new} - 1), \\ & RadiatorLength = rL_{new}, FinDistance = fD_{new}), TotNumAirPassages), \\ & TotalNumberOfAirPassages)) \\ & 9216. \end{aligned} \quad (2)$$

$$\begin{aligned} Ac_{new} := & \text{simplify}(\text{solve}(\text{subs}(TubeHeight = tH_{new}, TubeWidth = tW_{new}, RadiatorLength \\ & = rL_{new}, NumberOfTubes = ntube_{new}, CoolantSurfaceArea), Ac)) \\ & 8.49968 \text{ [ft}^2\text{]} \end{aligned} \quad (3)$$

$$\begin{aligned} Aa_{new} := & \text{simplify}(\text{solve}(\text{subs}((TotalNumberOfAirPassages \\ & = TotalNumberOfAirPassages_{new}, FinDistance = fD_{new}, FinHeight = fH_{new}, FinWidth \\ & = fW_{new}), AirSurfaceArea), Aa)) \\ & 61.7456 \text{ [ft}^2\text{]} \end{aligned} \quad (4)$$

$$\begin{aligned} Atotal_{new} := & \text{simplify}(Ac_{new} + Aa_{new}); \\ & 70.2453 \text{ [ft}^2\text{]} \end{aligned} \quad (5)$$

▼ Solve for hc_{new}

The value of hc depends on the physical and thermal fluid properties, fluid velocity and fluid geometry.

The *ReynoldsEquation* defined below can be used to determine the flow characteristics of the coolant as it passes through the tubes.

ReynoldsEquation

$$ReynoldsNum = \frac{\rho v D_H}{\mu} \quad (6)$$

The value D_H is found from the *HydraulicDiameter* equation:

HydraulicDiameter

$$D_H = \frac{4 Amin}{WP} \quad (7)$$

where

$$Amin_c := simplify(tW_{new} \cdot tH_{new});$$

$$0.000413872 \text{ [ft}^2\text{]} \quad (8)$$

$$WP_c := 2 \cdot simplify(tW_{new} + tH_{new});$$

$$0.171712 \text{ [ft]} \quad (9)$$

$$D_{Hc} := simplify(solve(subs([Amin = Amin_c, WP = WP_c], HydraulicDiameter), D_H));$$

$$0.00964108 \text{ [ft]} \quad (10)$$

The velocity of the coolant as it flows through the tubes is:

$$v_c := simplify\left(\frac{vf_c}{ntube_{new} \cdot Amin_c}\right)$$

$$4.89391 \text{ [ft/s]} \quad (11)$$

$$ReynoldsNum_c := simplify(solve(subs([D_H = D_{Hc}, v = v_c, \rho = \rho_c, \mu = \mu_c], ReynoldsEquation), ReynoldsNum));$$

$$5982.76 \quad (12)$$

For fluids that are in turbulent flow (that is, $ReynoldsNum > 5000$), we can use the *DittusBoelterEquation* to relate the $ReynoldsNum$ with the $NusseltNum$. The $NusseltNum$ is dependent upon the fluid flow conditions and can generally be correlated with the $ReynoldsNum$. Solving for the $NusseltNum$ will enable us to determine the value of hc .

DittusBoelterEquation

$$NusseltNum = 0.023 ReynoldsNum^{0.8} PrandtlNum^{1/3} \quad (13)$$

PrandtlEquation

$$PrandtlNum = \frac{C \mu}{k} \quad (14)$$

NusseltEquation

$$NusseltNum = \frac{hc D_H}{k} \quad (15)$$

$$\begin{aligned} PrandtlNum_c := & \text{simplify}(\text{solve}(\text{subs}([C = C_c, \mu = \mu_c, k = k_c], PrandtlEquation), \\ & PrandtlNum)) \end{aligned}$$

$$6.59999 \quad (16)$$

$$\begin{aligned} NusseltNum_c := & \text{solve}(\text{subs}([ReynoldsNum = ReynoldsNum_c, PrandtlNum = PrandtlNum_c], \\ & DittusBoelterEquation), NusseltNum) \end{aligned}$$

$$45.3346 \quad (17)$$

Knowing the *NusseltNumber* we can now solve for hc_{new}

$$\begin{aligned} hc_{new} := & \text{simplify}(\text{solve}(\text{subs}([k = k_c, D_H = D_{Hc}, NusseltNum = NusseltNum_c], \\ & NusseltEquation), hc)) \end{aligned}$$

$$1128.53 \left[\frac{Btu}{h \cdot ft^2 \cdot degF} \right] \quad (18)$$

▼ Determine $nfha_{new}$

We solve for ha_{new} in a similar manner as we did for hc_{new} (by determining the *ReynoldsNum* for air)

$$\begin{aligned} Amin_a := & \text{simplify}(fH_{new} \cdot fD_{new}) \end{aligned}$$

$$0.000203025 \left[ft^2 \right] \quad (19)$$

$$\begin{aligned} WP_a := & 2 \cdot \text{simplify}(fD_{new} + 8 \cdot fD_{new}) \end{aligned}$$

$$0.0937500 \left[ft \right] \quad (20)$$

$$\begin{aligned} D_{Ha} := & \text{simplify}(\text{solve}(\text{subs}([Amin = Amin_a, WP = WP_a], HydraulicDiameter), D_H)) \end{aligned}$$

$$0.00866240 \left[ft \right] \quad (21)$$

$$\begin{aligned} ReynoldsNum_a := & \text{simplify}(\text{solve}(\text{subs}([D_H = D_{Ha}, v = v_a, \rho = \rho_a, \mu = \mu_a], ReynoldsEquation), \\ & ReynoldsNum)) \end{aligned}$$

$$701.984 \quad (22)$$

The *ReynoldsNum* for air indicates that the air flow is laminar (that is, $ReynoldsNum_a < 2100$ -- *LaminarFlow*). As a result, we cannot use the *DittusBoelterEquation* to relate the *ReynoldsNum* to the *NusseltNum* and hence determine the value for ha_{new} . Another approach to determining the value of ha_{new} is to solve for the value of ha_{cur} since the value of $ha_{new} = ha_{cur}$. In the next section, we will show how the value of ha_{cur} is calculated by first obtaining the heat transfer coefficient for the original radiator (UA_{cur}).

▼ Solve for $nfha_{cur}$

The equation, which relates Number of Transferred Units (Ntu) to Universal Heat Transfer, will be used to determine the Universal Heat Transfer Coefficient (UA_{cur}) of the current model.

NtuEquation

$$Ntu = \frac{UA}{Cmin} \quad (23)$$

$Cmin$ is obtained by comparing the thermal capacity rate (CR) for the coolant and air.

$$ThermalCapacityRate := CR = C \cdot mfr,$$

$$MassFlowRate := mfr = FluidViscosity \cdot \rho;$$

$$CR = C mfr$$

$$mfr = FluidViscosity \rho \quad (24)$$

The Mass Flow Rate for the coolant and air are:

$$mf_a := simplify(solve(subs([FluidViscosity = \nu_{fa}, \rho = \rho_a], MassFlowRate), mfr))$$

$$2.77964 \left[\frac{lb}{s} \right]$$

$$mf_c := simplify(solve(subs([FluidViscosity = \nu_{fc}, \rho = \rho_c], MassFlowRate), mfr))$$

$$4.23765 \left[\frac{lb}{s} \right]$$

(26)

The Thermal Capacity Rates for the coolant and air are:

$$CR_a := simplify(solve(subs([mfr = mf_a, C = C_a], ThermalCapacityRate), CR))$$

$$40.0268 \left[\frac{Btu}{min \ degF} \right]$$

(27)

$$CR_c := simplify(solve(subs([mfr = mf_c, C = C_c], ThermalCapacityRate), CR))$$

$$223.748 \left[\frac{Btu}{min \ degF} \right]$$

(28)

Since $CR_a < CR_c$:

$$Cmin_{cur} := CR_a;$$

$$Cmax_{cur} := CR_c;$$

$$40.0268 \left[\frac{Btu}{min \ degF} \right]$$

$$223.748 \left[\frac{Btu}{min \ degF} \right]$$

(29)

and

$$Cratio_{cur} := \frac{CR_a}{CR_c} \quad 0.178892 \quad (30)$$

Next, we need to calculate the Number of Transfer Units (Ntu) of the original radiator assembly. To do this we for ITD_{cur} , ϵ_{cur} and q_{cur} from the ϵNtu Equation HeatTransferEquation, and ITDEquation, respectively.

ITDEquation

$$ITD = CoolantTemperature - AirTemperature \quad (31)$$

$$ITD_{value} := simplify(evalf(T_c - T_a)) = 150. \quad [degF]$$

ϵNtu Equation

$$\epsilon = 1 - e^{-\frac{Cmax(1 - e^{-Cratio Ntu})}{Cmin}} \quad (32)$$

$$\begin{aligned} \epsilon_{cur} &:= simplify(solve(subs([q = q_{cur}, Cmin = Cmin_{cur}, ITD = ITD_{value}], \\ &\quad HeatTransferEquation), \epsilon)) \\ &= 0.670384 \end{aligned}$$

Using the HeatTransferEquation the value of Ntu_{cur} can be found.

HeatTransferEquation

$$q = \epsilon Cmin ITD \quad (33)$$

$$\begin{aligned} Ntu_{cur} &:= solve(subs([Cmax = Cmax_{cur}, Cmin = Cmin_{cur}, Cratio = Cratio_{cur}, \epsilon = \epsilon_{cur}], \\ &\quad \epsilon Ntu Equation), Ntu) \\ &= 1.23717 \end{aligned} \quad (34)$$

We can finally solve for UA_{cur} by substituting the values of Ntu_{cur} and $Cmin$ in to the Ntu Equation

Ntu Equation

$$Ntu = \frac{UA}{Cmin} \quad (35)$$

$$UA_{cur} := solve(subs([Ntu = Ntu_{cur}, Cmin = Cmin_{cur}, q = q_{cur}], Ntu Equation), UA) = 49.5199 \quad \left[\frac{Btu}{min degF} \right]$$

Now that we have the value of UA_{cur} , we can use the UniversalHeatTransferEquation to

determine the value for $nfha_{cur}$, which is equal to the value of $nfha_{new}$.

Solving for the unknown values of Aa_{cur} and Ac_{cur} yields the following:

$$\begin{aligned} TotalNumberOfAirPassages_{cur} := & \text{simplify}(\text{solve}(\text{subs}((NumRowsOfFins = (ntube_{cur} \\ & - 1), RadiatorLength = rL_{cur}, FinDistance = fD_{cur}), TotNumAirPassages), \\ & TotalNumberOfAirPassages)) \end{aligned}$$

$$12288. \quad (36)$$

$$\begin{aligned} Aa_{cur} := & \text{simplify}(\text{solve}(\text{subs}((TotalNumberOfAirPassages \\ & = TotalNumberOfAirPassages_{cur}, FinDistance = fD_{cur}, FinHeight = fH_{cur}, FinWidth \\ & = fW_{cur}), AirSurfaceArea), Aa)) \end{aligned}$$

$$82.3274 \left[\frac{ft^2}{ft^2} \right] \quad (37)$$

$$\begin{aligned} Ac_{cur} := & \text{simplify}(\text{solve}(\text{subs}(TubeHeight = tH_{cur}, TubeWidth = tW_{cur}, RadiatorLength \\ & = rL_{cur}, NumberOfTubes = ntube_{cur}, CoolantSurfaceArea), Ac)) \end{aligned}$$

$$11.3329 \left[\frac{ft^2}{ft^2} \right] \quad (38)$$

Finally at this point we can solve for the value of $nfha_{cur}$ by substituting the values for Aa_{cur} , Ac_{cur} , and UA_{cur} into the *UniversalHeatTransferEquation*:

$$\begin{aligned} & \text{UniversalHeatTransferEquation} \\ & \frac{1}{UA} = \frac{1}{hc Ac} + \frac{1}{nfha Aa} \end{aligned} \quad (39)$$

$$\begin{aligned} nfha_{cur} := & \text{simplify}(\text{solve}(\text{subs}([UA = UA_{cur}, hc = hc_{new}, Ac = Ac_{cur}, Aa = Aa_{cur}], \\ & \text{UniversalHeatTransferEquation}), nfha)) \end{aligned}$$

$$47.0113 \left[\frac{Btu}{h ft^2 degF} \right] \quad (40)$$

Since the value of nf is the same for both the original and proposed radiator models we can determine the value of $nfha_{new}$ directly from the value of $nfha_{cur}$.

$$\begin{aligned} nfha_{new} := & nfha_{cur} \end{aligned}$$

$$47.0113 \left[\frac{Btu}{h ft^2 degF} \right] \quad (41)$$

▼ Solve for q_{new}

We can determine the heat transfer performance (q_{new}) of the new radiator assembly by using the *HeatTransferEquation* :

HeatTransferEquation

$$q = \epsilon C_{min} ITD \quad (42)$$

We can determine the value of ϵ_{new} from the *εNtuEquation*.

εNtuEquation

$$\epsilon = 1 - e^{-\frac{C_{max}(1 - e^{-Cratio Ntu})}{C_{min}}} \quad (43)$$

The unknown value for Ntu_{new} : can be determined using the *NtuEquation*

NtuEquation

$$Ntu = \frac{UA}{C_{min}} \quad (44)$$

Finally the value of UA_{new} can be determined from the *UniversalHeatTransferEquation*

UniversalHeatTransferEquation

$$\frac{1}{UA} = \frac{1}{hc Ac} + \frac{1}{nfha Aa} \quad (45)$$

Solving for UA_{new} , Ntu_{new} , and ϵ_{new} yields the following:

$$\begin{aligned} UA_{new} &:= \text{simplify}(\text{solve}(\text{subs}([hc = hc_{new}, nfha = nfha_{new}, Aa = Aa_{new}, Ac = Ac_{new}], \\ &\quad \text{UniversalHeatTransferEquation}), UA)) \\ &= 37.1398 \left\| \left\| \frac{Btu}{min degF} \right\| \right\| \end{aligned}$$

$$Ntu_{new} := \text{solve}(\text{subs}([UA = UA_{new}, C_{min} = C_{min}_{cur}], NtuEquation), Ntu) = 0.927873$$

$$\begin{aligned} \epsilon_{new} &:= \text{solve}(\text{subs}([C_{max} = C_{max}_{cur}, C_{min} = C_{min}_{cur}, Ntu = Ntu_{new}, Cratio = Cratio_{cur}], \\ &\quad \epsilon NtuEquation),) \\ &= 0.574696 \end{aligned}$$

The heat transfer performance q_{new} of our smaller radiator design can be found by substituting the value of value of ϵ_{new} and C_{min} into the *HeatTransferEquation* :

$$q_{new} := \text{simplify}(\text{solve}(\text{subs}([\epsilon = \epsilon_{new}, C_{min} = C_{min}_{cur}, ITD = ITD_{value}],$$

HeatTransferEquation), q))

$$3450.48 \left[\frac{\text{Btu}}{\text{min}} \right] \quad (46)$$

As expected the heat transfer performance of our proposed radiator design is smaller than that of the original.

$q_{diff} := \text{simplify}(q_{cur} - q_{new})$

$$574.52 \left[\frac{\text{Btu}}{\text{min}} \right] \quad (47)$$

▼ 3. Adjusting Heat Transfer Performance of Proposed Radiator Design

▼ Effects of Radiator Length on Heat Transfer Performance

The effects of radiator length on heat transfer performance (while keeping all other parameters the same as in the proposed design) can be examined by changing the adjacent dial.

The heat transfer performance values for four different radiator lengths are summarized in the table below.

Radiator Length		Heat Transfer Performance	
0.5 ft	0.152400 m	1560.09 $\frac{Btu}{min}$	27414.7 $\frac{J}{s}$
1.0 ft	0.304800 m	2661.09 $\frac{Btu}{min}$	46762.1 $\frac{J}{s}$
1.5 ft	0.457200 m	3450.50 $\frac{Btu}{min}$	60633.9 $\frac{J}{s}$
2.0 ft	0.609600 m	4025.01 $\frac{Btu}{min}$	70729.5 $\frac{J}{s}$

Radiator Length vs. Heat Transfer Performance



m

ft

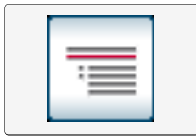
$\frac{J}{s}$

$\frac{Btu}{minute}$

From the table, we can confirm our hypothesis that changing radiator length alone will not be sufficient to generate the desired heat transfer performance. As mentioned in the previous section, there are several methods available to increase the heat transfer performance of a radiator assembly. For our proposed design, we have chosen to increase the metal-to-air surface area by increasing the number of fins per row.

▼ Effects of Surface Area on Heat Transfer Performance

To achieve a heat transfer performance for our proposed design equal to that of the current design (that is, $4025 \left[\frac{\text{Btu}}{\text{minute}} \right] \approx 70729.3 \left[\frac{\text{J}}{\text{s}} \right]$), we must increase the number of fins per row. The procedure called *DetermineNumberOfFins*(q_{cur}), defined within the Code Edit Region, calculates the number of fins per row needed to achieve the desired heat transfer performance for our assembly.



```
# Calculate number of fins per
```

simplify(*DetermineNumberOfFins*(q_{cur})) = *NumFinsPerRow* = 436.056

Thus, the number of fins per row must be increased from 384 to 437 to achieve a heat transfer performance of $4025 \frac{\text{Btu}}{\text{minute}} \approx 70729.3 \frac{\text{J}}{\text{s}}$. The graph in Figure 6 shows the effects of changing the number of fins per row on the heat transfer performance for our smaller radiator design.

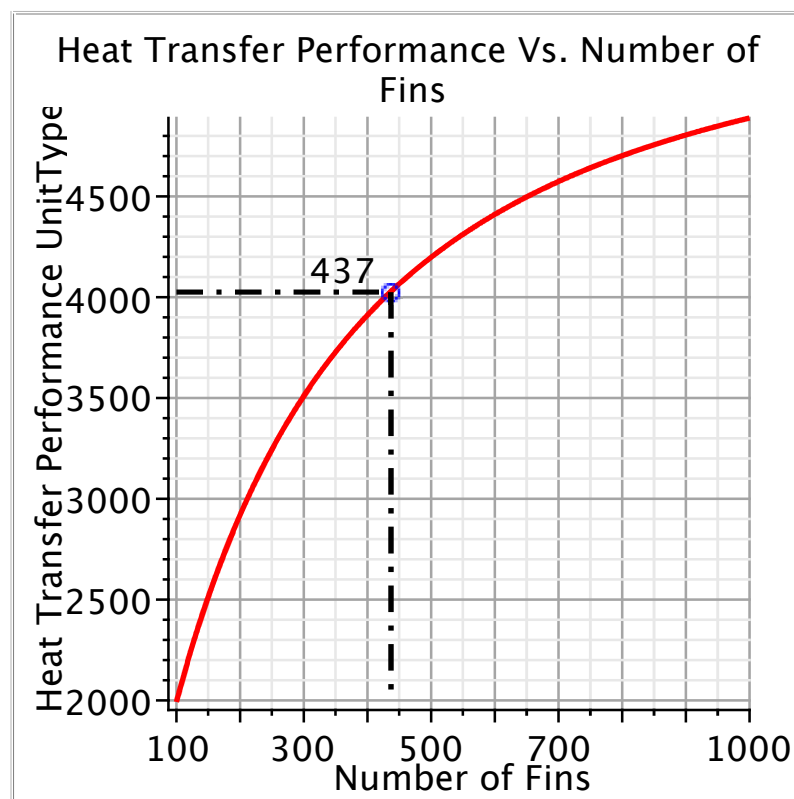


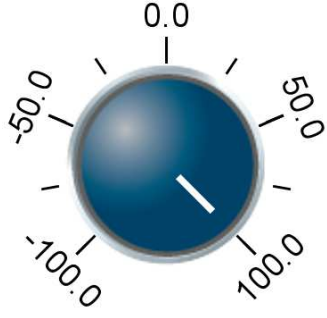
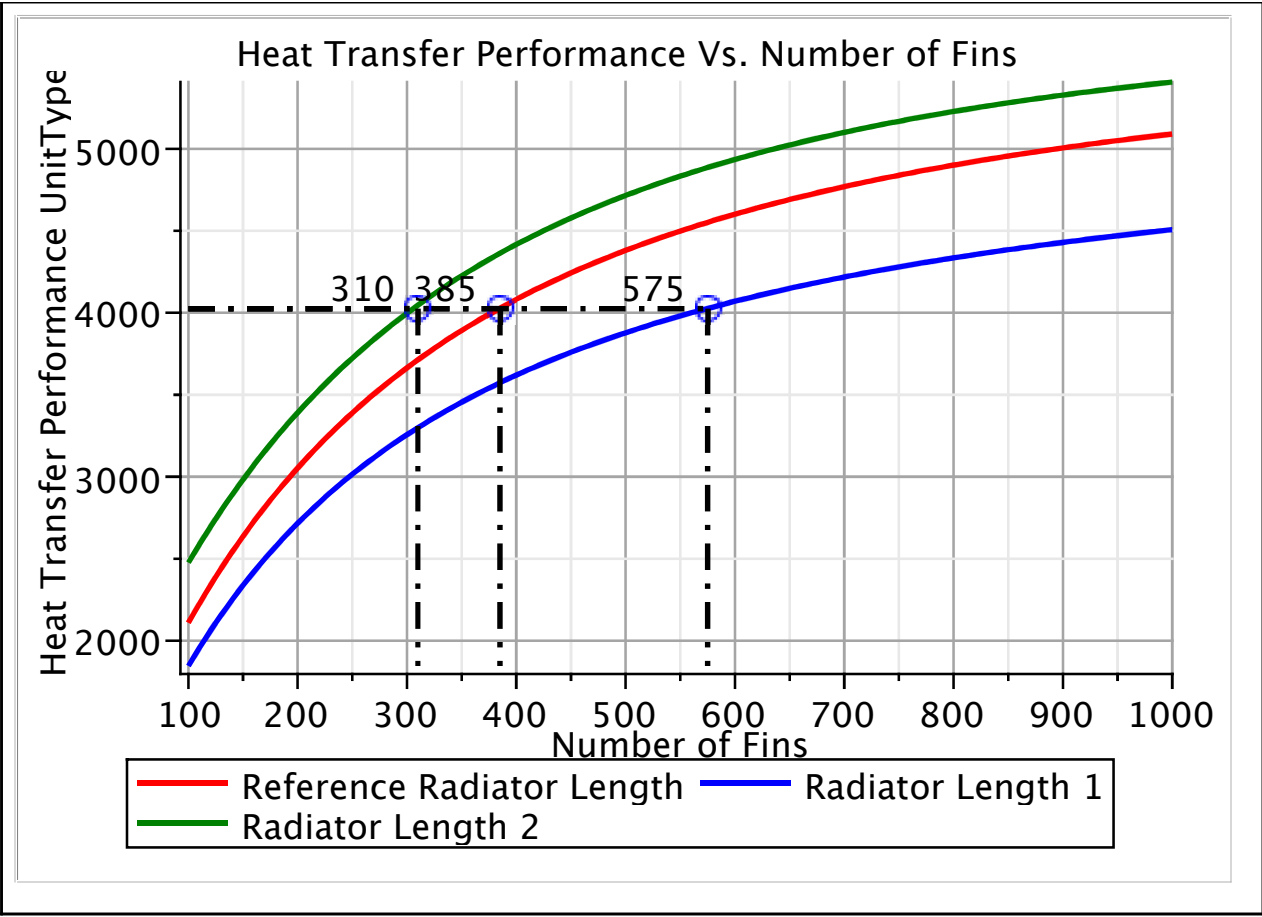


Figure 6: Effects of surface area on heat transfer performance

The application below allows you to compare the effects of changing the number of fins per row on the heat transfer performance for two different radiator lengths based on a given reference radiator length. The two different radiator lengths can be defined in the terms of the percent or absolute change of the reference.

Maple Application -- Effects of heat transfer performance vs. number of fins per row for different radiator lengths

Reference Radiator Length	Radiator Length 1	Radiator Length 2
	 <p>Absolute Chang...</p> <input data-bbox="706 819 966 892" type="text" value="-1.0"/>	 <p>Percent Chan...</p> <input data-bbox="1144 819 1404 892" type="text" value="100.0 %"/>
	<input type="radio"/> Percent Change <input checked="" type="radio"/> Absolute Change	<input checked="" type="radio"/> Percent Change <input type="radio"/> Absolute Change
<input data-bbox="300 1144 495 1207" type="text" value="0.609600"/> m <input data-bbox="300 1218 495 1281" type="text" value="2"/> ft	<input data-bbox="722 1144 917 1207" type="text" value="0.304800"/> m <input data-bbox="722 1218 917 1281" type="text" value="1"/> ft	<input data-bbox="1161 1144 1356 1207" type="text" value="1.21920"/> m <input data-bbox="1161 1218 1356 1281" type="text" value="4"/> ft



Plot Response

▼ 4. Export Optimized Radiator Dimensions to SolidWorks

We can create a CAD rendering of our smaller radiator assembly. The design parameters of our new design are the same as the original, except it is smaller in length and has more fins per row.

The parameters of our new radiator model are listed in the table below. It is important to note that the number of fins per row is actually a measure of the distance between the fins (that is, how the fins are spread out within a row).

Export Radiator Dimensions to SolidWorks			
Number of Fins Per Row	<input type="text" value="437"/>		
Radiator Length ($rL_{SolidWorks}$) :	<input type="text" value="0.457200"/>	ft	<input type="text" value="0.457200"/> m
Distance Between Fins ($fD_{SolidWorks}$) :	<input type="text" value="0.003432"/>	ft	<input type="text" value="0.001046"/> m

Export Dimensions to SolidWorks

Close SolidWorks Connection

* Note: For consistency, we are creating a scaled CAD rendering model of the new optimized radiator assembly similar to that of the original CAD rendering

▼ Results

In this worksheet, we proposed the design of a new smaller radiator assembly that is capable of the same heat dissipation as the current design.

Using the effectiveness-Ntu method, we calculated the heat transfer performance of the proposed design. As expected, decreasing the radiator length by 30% caused the heat transfer performance to decrease; the heat transfer performance decreased by ~14%. That said, by increasing the number of fins per row, from 384 to 437, we increased the heat transfer performance back to its original level of $4025 \frac{Btu}{minute} \left(70729.3 \frac{J}{s} \right)$.

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