Example 1: (McClave "Statistics for Business and Economics" 4th edition page 1112):

A promoter wants to promote a musical concert. If the concert is held in an outdoor stadium, the promoter will profit $350,000. However, if it rains, the concert will be cancelled, and the promoter will lose $40,000.
The promoter has a second option which is to hold the concert indoors. The rent for the indoor auditorium will be $150,000.
Past experience leads the promoter to predict no rain will occur with a probability of 2/3.

The promoter also has an option to purchase a long-range weather forecast for the concert night. The track record for the people doing the forecast is 85% of the rainy days were correctly predicted, and 70% of the nice days were correctly predicted. The cost for the forecast is $15,000.

Should the promoter rely on his own prediction or should he buy the forecast prediction?

	$\text{Rain} = \frac{1}{3}$	$\overline{\text{Rain}} = \frac{2}{3}$
Rent Stadium	$-\$40,000$	$\$350,000$
$\overline{\text{Rent Stadium}}$	$\$150,000$	$\$150,000$

Payoff table

		Rain	$\overline{\text{Rain}}$
Forecast	Rain	85%	30%
Forecast	$\overline{\text{Rain}}$	15%	70%

Conditional table Note: the columns add up to 100%.

	$\text{Rain} = \frac{1}{3}$	$\overline{\text{Rain}} = \frac{2}{3}$
Rent Stadium	$\$190,000$	$\$0$
$\overline{\text{Rent Stadium}}$	$\$0$	$\$200,000$

Opportunity loss table. Some work with this instead of the payoff table.

SampleSize:=[1,1]
Fixed_CostofSampling:=15000
CostofSampling_per_sample:=0
prior:=[1/3,2/3]*100   Note: the row adds up to 100%.
PE:=[0]   There is no preposterior probability for this example.
Conditional:=[85,30,15,70]   Note: the Conditional matrix has any number of rows but has the same number of columns as the prior matrix.
payoff_set:=[-40000, 350000, 150000, 150000]   Note: Because Conditional and not PE is used, the payoff matrix=[Row=prior column Column=prior column], i.e. 2x2 matrix.
distribution:=1   Note: it's assumed this is a binomial distribution. Some distributions are binomial, some are Poisson and some are hypergeometric. This program uses binomial and Poisson.

Answers:
EPNS (Expected Payoff with No Sampling) = $200,000. This is the payoff using the promoter's prior probabilities before any sampling.
EVPI (Expected Value of Perfect Information) = $63,333. This is the maximum value one should be willing to pay for perfect sampling information. Rarely is perfect information available.
EPS (Expected Payoff for Sampling) = $233833
EVSI (Expected Value of Sampling Information) = EPS-EPNS = $13833.   Be reluctant to exceed this cost for sampling information.
ENGS (Expected Net Gain from Sampling) = EPS-EPNS = $-1167. Since ENGS < 0, don't pay (statistically) for sampling.

The promoter is better off going with his own weather prediction rather than the forecaster's and not paying the $15,000. This is assuming the promoter is comfortable with his own forecast. The prior probability that it will not rain (2/3) increased to posterior probability that it will not rain (0.9032) given the forecast it will not rain.
However, the cost (statistically) of the forecast is too high.

Prior probabilities in % and MUST add up to 100%
Prior probabilities has only one row.

PE (preposterior probabilities) in % and the row need NOT add up to 100%.
PE has only one row and the same number of columns as prior.
If PE is not used, then enter zero: PE=[0]

If Conditional probabilities are NOT used, then enter zero: Conditional=[0].
The Conditional probabilities are in % and each column adds up to 100%.
If Conditional is used, then the Conditional matrix columns must equal the payoff columns with any number of rows.
i.e., Conditional=Matrix[row is any number, columns=prior columns]

If PE is used, then the payoff set matrix is: Rows=SampleSize[2]+1 Columns=prior columns.
If Conditional used, then the payoff matrix is: Rows= prior columns Columns= prior columns.
i.e., payoff set=Matrix[prior columns, prior columns]
Add zeros as necessary to make the square matrix.

Distribution: 1= binomial 2=Poisson

posterior =

		Rain	$\overline{\text{Rain}}$
Forecast	Rain	0.5862	0.4138
Forecast	$\overline{\text{Rain}}$	0.09677	0.9032

Conditional =

		Rain	$\overline{\text{Rain}}$
Forecast	Rain	0.85	0.30
Forecast	$\overline{\text{Rain}}$	0.15	0.70

joint =

		Rain	$\overline{\text{Rain}}$
Forecast	Rain	0.2833	0.2000
Forecast	$\overline{\text{Rain}}$	0.0500	0.4667

		Rain	$\overline{Rain}$	SPI
Forecast	$Rain$	a	b	$\sum row = R_1$
Forecast	$\overline{Rain}$	e	f	$\sum row = R_2$
	$prior$	$\sum col_1 = C_1$	$\sum col_2 = C_2$	1.000

Where $\begin{bmatrix} a & b \\ e & f \end{bmatrix}$ are the joint probabilities.

$$prior = \left[C_1 \quad C_2\right] \qquad PE = \begin{bmatrix} \frac{e}{C_1} & \frac{f}{C_2} \end{bmatrix} \quad Conditional = \begin{array}{|c|c|} \hline \frac{a}{C_1} & \frac{b}{C_2} \\ \hline \frac{e}{C_1} & \frac{f}{C_2} \\ \hline \end{array} \qquad Post = \begin{array}{|c|c|} \hline \frac{a}{R_1} & \frac{b}{R_1} \\ \hline \frac{e}{R_2} & \frac{f}{R_2} \\ \hline \end{array}$$

$P(Rain \mid Rain) = \dfrac{P\left(Rain \cap Rain\right)}{P\left(Rain\right)} = \dfrac{a}{R_1} = \dfrac{0.2833}{0.4833} = 0.5862 =$ the probability that it will rain given the forecast that it will rain.

$P(\overline{Rain} \mid Rain) = \dfrac{P\left(\overline{Rain} \cap Rain\right)}{Rain} = \dfrac{b}{R_1} = \dfrac{0.2000}{0.4833} = 0.4138 =$ the probability that it will not rain given the forecast that it will rain.

$P(Rain \mid \overline{Rain}) = \dfrac{P\left(Rain \cap \overline{Rain}\right)}{\overline{Rain}} = \dfrac{e}{R_2} = \dfrac{0.0500}{0.5167} = 0.0968 =$ the probability that it will rain given the forecast that it will not rain.

$P(\overline{Rain} \mid \overline{Rain}) = \dfrac{P\left(\overline{Rain} \cap \overline{Rain}\right)}{\overline{Rain}} = \dfrac{f}{R_2} = \dfrac{0.4667}{0.5167} = 0.9032 =$ the probability that it will not rain given the forecast that it will not rain

The above is valid only for binomial distributions. It doesn't hold for Poisson distributions.

Example 2: : (McClave "Statistics for Business and Economics" 4th edition page 1120):
A computer company has the capacity to produce seven computers per year. Its assessment of the prior probabilities distribution of next year's demand and the payoff distribution (in millions of dollars) constructed by the company's accounting department are shown in the tables.

prior probabilities =

0	0.05
1	0.15
2	0.22
3	0.22
4	0.16
5	0.10
6	0.05
7	0.02
> 7	0.03

payoff table =

Number of computers demanded →	0	1	2	3	4	5	6	7	> 7
0	0	–1	–2	–3	–4	–5	–6	–7	–7
1	–2	1	0	–1	–2	–3	–4	–5	–5
2	–0.40	0.80	2	1	0	–1	–2	–3	–3
3	–0.60	0.60	1.8	3	2	1	0	–1	–1
4	–0.80	0.40	1.6	2.8	4	3	2	1	1
5	–1	0.20	1.4	2.6	3.8	5	4	3	3
6	–1.2	0	1.2	2.4	3.6	4.8	6	5	5
7	–1.4	–0.20	1	2.2	3.4	4.6	5.8	7	7
> 7	0	0	0	0	0	0	0	0	0

The computer company is considering hiring a market forecaster to predict the demand for the computers in the coming year as shown in the conditional probabilities table. The market forecaster will provide the computer company with a demand forecast for $10,000. Should the computer company spend the $10,000 for the forecast?

Conditional =

Actual Demand →										
		0	1	2	3	4	5	6	7	> 7
	0	0.70	0.30	.015	0	0	0	0	0	0
	1	0.20	0.50	0.20	0.10	0.05	0	0	0	0
Forcast Demand	2	0.10	0.10	0.40	0.30	0.20	0.20	0	0	0
	3	0	0.05	0.15	0.40	0.50	0.40	0.05	0	0
	4	0	0.05	0.05	0.15	0.20	0.30	0.10	0.05	0
	5	0	0	0.05	0.05	0.05	0.10	0.20	0.05	0
	6	0	0	0	0	0	0	0.40	0.10	0
	7	0	0	0	0	0	0	0.20	0.50	0.10
	> 7	0	0	0	0	0	0	0.05	0.30	0.90

SampleSize:=[1,1]
Fixed_CostofSampling:=10000
CostofSampling_per_sample:=0
prior:=[5,15,22,22,16,10,5,2,3] Note: the row adds up to 100%.
PE:=[0]   There is no preposterior probability for this example.
Conditional:=[70,30,15,0,0,0,……0,0,5,30,90] Note: the Conditional matrix has any number of rows but has the same number of columns as the prior matrix.
payoff_set:=[0,-1,-2,-3,-4,…..3.4,4.6,5.8,7,7,…..0,0,0]*1e6
distribution:=1 Note: it's assumed this is a binomial distribution.

Answer:
EPNS = 2,338,000 EVPI =702,000 EPS =2,697,500 EVSI = 359,500 ENGS = 349,500

Because ENGS (349,500) > 10,000, the company should spend the money for the forecast.

Example 3: (McClave "Statistics for Business and Economics" 4th edition page 1121):
A large hospital is considering purchasing 100 new color television sets under one of two different purchase agreements. Under one agreement the TV sets would cost $460 each and all sets that are seriously defective would be replaced at no cost. Under the other agreement, the TV sets would cost $400 each and any seriously defective sets would have to be replaced by the hospital at $400 each. (Assume all replacement sets are nondefective.) The hospital's purchasing agent believes the probabilities shown in the table appropriately characterize the proportion of seriously defective sets in a shipment of 100 sets from the manufacturer.

Use the expected payoff criterion (EPNS) to determine which purchase agreement the hospital should choose.

Suppose the hospital was able to randomly sample one TV set from the incoming shipment of 100 sets before deciding which purchase agreement to choose. According to the expected payoff criterion (EP), which agreement should they select if the sampled set was defective?

On the basis of its prior information (EVPI), what is it worth to the hospital to be able to test one of the 100 television sets before selecting a purchase agreement?

Proportion defective	probability
0.00	0.40
0.05	0.30
0.10	0.10
0.15	0.10
0.20	0.10

payoff=

$-460(100) = -46000$	$-460(100) = -46000$	-46000	46000	-46000
$-400(100) - 100(0.0)(400) = -40000$	$-400(100) - 100(.05)(400) = -42000$	-44000	-46000	-48000

SampleSize:=[100,1]
Fixed_CostofSampling:=0
CostofSampling_per_sample:=0
prior:=[40,30,10,10,10] Note: the row adds up to 100%.
PE:=[0,5,10,15,20]
Conditional:=[0]   There are no conditional probabilities for this example.
payoff_set:=[-46000,-46000,-46000,-46000,-46000,-40000,-42000,-44000,-46000,-48000]
distribution:=1   Note: it's assumed this is a binomial distribution.

Answer:

EPNS = -42400 EVPI =200   EPS =-16468.66   EVSI = 25931.34   ENGS = 25931.34

$$EP = \left[\begin{array}{|c|c|} \hline -46000 & -40010 \\ \hline -46000 & -42010 \\ \hline \end{array}\right]$$

The best payoff= $\begin{bmatrix}2 & 2\end{bmatrix} = \left[\left[0\ \left(\text{1st row}\right)\ \text{defects} = \text{2nd column} = \text{2nd payoff row}\right]\ \ \left[1\ \left(\text{2nd row}\right)\ \text{defect} = \text{2nd column} = \text{2nd payoff row}\right]\right]$

The best payoff for zero defects is the maximum of the first row. This is -40010 which is in the second column and corresponds to the second row of the payoff.

The best payoff for one defect is the maximum of the second row. This is -42010 which is in the second column and corresponds to the second row of the payoff.

EVPI is $200 which is less than the $400 for the replacement of one defective TV. Therefore, don’t pay for a sample of one TV set at $400.

Example 4: (Devore "Probability and Statistics for Engineering and the Sciences" 5th edition page 84):
At a certain gas station, 40% of the customers use regular gas, 35% use extra and 25% use premium. Of those customers using regular, 30% fill their tanks, 60% of those using extra and 50% of those using premium.

What is the probability that the next customer will fill their tank **and** use extra?
i.e. probability of both extra and tank is filled = $P(extra \cap fill)$
What is the probability that the next customer fills their tank? $P(\text{fills tank}) = SPI_2$

If the next customer fills the tank, what is the probability that regular is used? Extra? Premium?

Probability that regular is used given that tank filled = $P(\text{regular} \mid fill) = \dfrac{P(\text{regular} \cap fill)}{P(fill)}$

Probability that extra is used given that tank filled = $P(extra \mid fill) = \dfrac{P(extra \cap fill)}{P(fill)}$

Probability that premium is used given that tank filled = $P(premium \mid fill) = \dfrac{P(premium \cap fill)}{P(fill)}$

SampleSize:=[1,1] = [one customer fills tank]
Fixed_CostofSampling:=0
CostofSampling_per_sample:=0
prior:=[40,35,25] Note: the row adds up to 100%.
PE:=[30,60,50]
Conditional:=[0] There are no conditional probabilities for this example.
payoff_set:=[1,1,1,0,0,0] There is no payoff for this example. Therefore, use any 3x3 matrix.
distribution:=1 Note: it's assumed this is a binomial distribution.

	Regular	Extra	Premium	*SPI*
$\overline{\text{Fill}}$	0.28	0.14	0.125	0.545
Fill	0.12	0.21	0.125	0.455
prior	0.40	0.35	0.250	$\sum$1.000

This chart shows the calculated joint probabilities.

$PE =$

	Regular	Extra	Premium
Fill	0.30	0.60	0.50

$\text{Fill}_{2,1} = prior_1\left(PE_1\right) = 0.40\left(0.30\right) = 0.12$ $\quad \text{Fill}_{2,2} = prior_2\left(PE_2\right) = 0.35\left(0.60\right) = 0.21$ $\quad \text{Fill}_{2,3} = prior_3\left(PE_3\right) = 0.25\left(0.50\right) = 0.125$

Answers:

$P\left(\text{extra} \cap \text{fill}\right) = \text{joint of extra and fill} = 0.21$

$P\left(\text{fill}\right) = SPI_2 = 0.455$

$$P(regular \mid fill) = \frac{P(regular \cap fill)}{P\left(fill\right)} = \frac{0.12}{0.455} = 0.264 = post_{2,1}$$

$$P(extra \mid fill) = \frac{P(extra \cap fill)}{P\left(fill\right)} = \frac{0.21}{0.455} = 0.462 = post_{2,2}$$

$$P(premium \mid fill) = \frac{P(premium \cap fill)}{P\left(fill\right)} = \frac{0.125}{0.455} = 0.275 = post_{2,3}$$

Example 5 : (McClave "Statistics for Business and Economics" 4th edition page 1094):
A replacement parts inventory manager for a computer manufacturer believes the demand for a certain component is distributed as a Poisson random variable with a mean monthly demand of two, three, or four units. Suppose experience suggests that a mean monthly demand for three units is twice as likely as a mean monthly demand of two units, and that a mean monthly demand of two units is as likely as a mean monthly demand of four units.

1. If the demand for this component last month was four units, find the posterior distribution for the mean monthly demand for next month.
2. If the demand in the following month was two units, find the posterior distribution for the mean monthly demand.

To find the prior probabilities, solve: $x + 2x + x = 1$. Therefore, $x = .25$ and $prior = [.25 \quad .50 \quad .25]$

$PE = [2 \quad 3 \quad 4]$ $\text{SampleSize} = [4 \quad 4]$ for problem 1.

SampleSize:=[4,4]
Fixed_CostofSampling:=0
CostofSampling_per_sample:=0
prior:=[25,50,25]
PE:=[2,3,4]
Conditional:=[0] There are no conditional probabilities for this example.
payoff_set:=[5,5,5,4,4,4,3,3,3,2,2,2,1,1,1] There is no payoff given for this example. Therefore, use any 5x3 matrix.
distribution:=2 Poisson distribution

$$posterior = \begin{bmatrix} 0.5344 & 0.3932 & 0.07233 \\ 0.4212 & 0.4648 & 0.1140 \\ 0.3128 & 0.5178 & 0.1693 \\ 0.2190 & 0.5439 & 0.2371 \\ 0.1451 & 0.5406 & 0.3143 \end{bmatrix} = \begin{bmatrix} \text{0 sample unit} \\ \text{1 sample unit} \\ \text{2 sample unit} \\ \text{3 sample unit} \\ \text{4 sample unit} \end{bmatrix}$$

Answer for part 1 is $posterior_5 = [0.1451 \quad 0.5406 \quad 0.3143]$ because the problem requires the 4^{th} sample unit.
Now, for part 2 have the prior = $posterior_5$, and the sample size=[2,2]. The $posterior_5$ is used because the problem requires the *following month*.

SampleSize:=[2,2]
Fixed_CostofSampling:=0
CostofSampling_per_sample:=0
prior:=[14.51,54.06,31.43]
PE:=[2,3,4]
Conditional:=[0] There are no conditional probabilities for this example.
payoff_set:=[5,5,5,4,4,4,3,3,3,2,2,2,1,1,1] There is no payoff given for this example. Therefore, use any 5x3 matrix.
distribution:=2 Poisson distribution

$$posterior = \left[\begin{array}{|c|c|c|}\hline 0.3754 & 0.5145 & 0.1101 \\ \hline 0.2746 & 0.5645 & 0.1610 \\ \hline 0.1902 & 0.5867 & 0.2231 \\ \hline \end{array}\right] = \begin{bmatrix} \text{0 sample unit} \\ \text{1 sample unit} \\ \text{2 sample unit} \end{bmatrix}$$

Answer for part2 is $posterior_3 = [.01902 \quad 0.5867 \quad 0.2231]$ because the problem requires the 2nd sample unit.