

Essential Maple 7

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Chapter 2: Useful one-word commands

Section 2.1: Simplification

Session 2.1.2: collect

> **p := 1 + x + 3 + 5*x + 6*y + 17*y^2 + 35*x + 52*x^2 + 99*x*y + (x+y)^3;**

$$p := 4 + 41x + 6y + 17y^2 + 52x^2 + 99xy + (x+y)^3$$

> **collect(p, x);**

$$x^3 + (52 + 3y)x^2 + (41 + 99y + 3y^2)x + 4 + 6y + 17y^2 + y^3$$

> **collect(p, y);**

$$y^3 + (17 + 3x)y^2 + (3x^2 + 99x + 6)y + 4 + 41x + x^3 + 52x^2$$

> **collect(p, [x,y]);**

$$x^3 + (52 + 3y)x^2 + (41 + 99y + 3y^2)x + 4 + 6y + 17y^2 + y^3$$

> **collect(p, [x,y], distributed);**

$$x^3 + 52x^2 + 3yx^2 + 41x + 99xy + 3y^2x + 4 + 6y + 17y^2 + y^3$$

> **restart;**

> **Logistic := x -> diff(x,t) - x*(1-x);**

$$\text{Logistic} := x \rightarrow \left(\frac{\partial}{\partial t} x \right) - x(1-x)$$

Think of v as x_n .

> **alias(w = v + h*v*(1-v));**

w

The following is the cubic Hermite interpolant between v and w , which is the value of the Euler step from v .

> **interpolant := v + t*v*(1-v) + (v*(1-v) - w*(1-w))/h^2*t^2*(h-t);**

$$\text{interpolant} := v + t v (1-v) + \frac{(v(1-v) - w(1-w - h v(1-v))) t^2 (h-t)}{h^2}$$

> **eval(interpolant, t=0);**

v

> **eval(interpolant, t=h);**

w

> **eval(diff(interpolant, t), t=0);**

$v(1-v)$

> **eval(diff(interpolant, t), t=h);**

$$w(1-v-hv(1-v))$$

> **defect := Logistic(interpolant);**

$$\text{defect} := v(1-v) + \frac{2(v(1-v) - w(1-v-hv(1-v)))t(h-t)}{h^2} - \frac{(v(1-v) - w(1-v-hv(1-v)))t^2}{h^2}$$

$$- \left(v + tv(1-v) + \frac{(v(1-v) - w(1-v-hv(1-v)))t^2(h-t)}{h^2} \right) \left(1 - v - tv(1-v) - \frac{(v(1-v) - w(1-v-hv(1-v)))t^2(h-t)}{h^2} \right)$$

> **simpler := collect(eval(defect,t=theta*h), h, factor);**

$$\text{simpler} := v^4(v-1)^4\theta^4(-1+\theta)^2h^6 - 2v^3(2v-1)(v-1)^3\theta^4(-1+\theta)^2h^5$$

$$+ v^2\theta^3(-1+\theta)(v-1)^2(4v^2\theta^2 - 4v^2\theta + 2v^2 - 4\theta^2v + 4v\theta - 2v + \theta^2 - \theta)h^4 - v^2\theta^2(2\theta+1)(-1+\theta)(2v-1)(v-1)^2h^3$$

$$+ v\theta(-1+\theta)(v-1)(4v^2\theta - 2v^2 - 4v\theta + 2v + \theta)h^2 + 3v\theta(2v-1)(v-1)(-1+\theta)h$$

> **series(simpler, h, 3);**

$$3v\theta(2v-1)(v-1)(-1+\theta)h + v\theta(-1+\theta)(v-1)(4v^2\theta - 2v^2 - 4v\theta + 2v + \theta)h^2 + O(h^3)$$

> **restart;**

> **currentdir("C:/books/ess/programs");**

> **read "veil.mpl";**

> **K := table();**

$$K := \text{table}([])$$

> **VK := LEM(K);**

Warning, label K is assigned a value already.

Save its contents and issue the command `unassign(K);`

There is no need to repeat the call to LEM.

> **unassign(K):**

> **VC := LEM(C):**

> **p1 := sort(randpoly([x,y,z], dense, degree=7));**

$$\begin{aligned}
 p1 := & -85x^7 - 55x^6y - 37x^6z + 97x^5y^2 + 50x^5yz + 56x^5z^2 + 57x^4y^3 - 59x^4y^2z - 8x^4yz^2 + 43x^4z^3 + 54x^3y^4 - 5x^3y^3z \\
 & - 61x^3y^2z^2 - 18x^3yz^3 + x^3z^4 + 41x^2y^5 - 58x^2y^4z + 53x^2y^3z^2 + 83x^2y^2z^3 + 19x^2yz^4 + 49x^2z^5 - 86xy^6 + 30xy^5z + 72xy^4z^2 \\
 & - 91xy^3z^3 + 68xy^2z^4 - 66xyz^5 - 34xz^6 - 61y^7 - 60y^6z + 29y^5z^2 + 78y^4z^3 - 17y^3z^4 + 5y^2z^5 - 59yz^6 + 40z^7 - 35x^6 + 79x^5y \\
 & + 49x^5z + 45x^4y^2 - 93x^4yz - 62x^4z^2 + 99x^3y^3 - 50x^3y^2z + 31x^3yz^2 - 47x^3z^3 - 90x^2y^4 - x^2y^3z - 86x^2y^2z^2 - 50x^2yz^3 \\
 & + 78x^2z^4 + 80xy^5 + 66xy^4z - 53xy^3z^2 - 72xy^2z^3 - 53xyz^4 - 42xz^5 + 9y^6 - 66y^5z + 39y^4z^2 - 98y^3z^3 - 88y^2z^4 + 62yz^5 - 78z^6 \\
 & + 63x^5 + 92x^4y + 77x^4z - 12x^3y^2 - 26x^3yz - 91x^3z^2 + 94x^2y^3 + 23x^2y^2z + 88x^2yz^2 + 17x^2z^3 - 29xy^4 - 19xy^3z - 87xy^2z^2 \\
 & - 61xyz^3 + 88xz^4 - 32y^5 + 94y^4z - 36y^3z^2 - 43y^2z^3 - 55yz^4 + 62z^5 + 66x^4 - 62x^3y - 47x^3z - 84x^2y^2 - 53x^2yz + 72x^2z^2 \\
 & - 47xy^3 + 79xy^2z - 23xyz^2 - 76xz^3 + 68y^4 + 40y^3z - 73y^2z^2 + 25yz^3 + 11z^4 - 61x^3 + 85x^2y - 99x^2z + 43xy^2 - 37xyz - 65xz^2 \\
 & + 22y^3 + 25y^2z + 9yz^2 + 88z^3 - 85x^2 + 31xy + 25xz + 4y^2 + 40yz + z^2 + 28x + 61y + 30z + 81
 \end{aligned}$$

> **compact1 := collect(p1, [x,y], distributed, VK:-veil);**

$$\begin{aligned}
 compact1 := & -55x^6y + 97x^5y^2 + 57x^4y^3 + 54x^3y^4 - 85x^7 + 41x^2y^5 - 86xy^6 - 61y^7 - K_2x - K_3y + K_1 - K_5x^4y^2 - K_6x^4y + K_4x^5y \\
 & + K_{19}x^2y^3 + K_{20}x^2y^2 + K_{21}x^2y + K_{23}xy^4 + K_{25}xy^2 - K_{10}x^6 + 7K_{12}x^5 - K_{13}y^3 - 3K_{28}y^6 - K_7x^3y^3 - K_8x^3y^2 - K_9x^3y \\
 & + K_{11}y^2 + K_{14}x^4 + K_{15}y^4 + K_{16}x^3 + K_{17}y^5 - 2K_{18}x^2y^4 + 10K_{22}xy^5 - K_{24}xy^3 - K_{26}xy + K_{27}x^2
 \end{aligned}$$

> **K[1] = VK:-unveil(K[1]);**

$$K_1 = 81 + 40z^7 + 11z^4 + 88z^3 + 62z^5 - 78z^6 + z^2 + 30z$$

> **K[2] = VK:-unveil(K[2]);**

$$K_2 = 76z^3 - 88z^4 + 34z^6 - 25z + 42z^5 + 65z^2 - 28$$

> **zero := VK:-unveil(compact1, infinity) - p1:**

> **normal(zero);**

0

> **p2 := sort(randpoly([r,ln(r),Y], dense, degree=6));**

$$\begin{aligned} p2 := & -5r^6 + 4r^5 Y - 28r^5 \ln(r) - 48r^4 Y^2 + 57r^4 Y \ln(r) + 10r^4 \ln(r)^2 - 14r^3 Y^3 - 68r^3 Y^2 \ln(r) + 58r^3 Y \ln(r)^2 - 7r^3 \ln(r)^3 - 39r^2 Y^4 \\ & - 4r^2 Y^3 \ln(r) + 5r^2 Y^2 \ln(r)^2 - 91r^2 Y \ln(r)^3 - 73r^2 \ln(r)^4 - 68r Y^5 - 59r Y^4 \ln(r) + 6r Y^3 \ln(r)^2 + 68r Y^2 \ln(r)^3 - 99r Y \ln(r)^4 - 14r \ln(r)^5 \\ & + 16Y^6 - 77Y^5 \ln(r) + 92Y^4 \ln(r)^2 + 59Y^3 \ln(r)^3 - 69Y^2 \ln(r)^4 - 90Y \ln(r)^5 - 58 \ln(r)^6 - 11r^5 - 11r^4 Y - 82r^4 \ln(r) - 9r^3 Y^2 + 14r^3 Y \ln(r) \\ & - 94r^3 \ln(r)^2 + 8r^2 Y^3 - 50r^2 Y^2 \ln(r) - 86r^2 Y \ln(r)^2 + r^2 \ln(r)^3 - 42r Y^4 + 6r Y^3 \ln(r) + 72r Y^2 \ln(r)^2 + 45r Y \ln(r)^3 - 67r \ln(r)^4 - 40Y^5 \\ & - 63Y^4 \ln(r) - 93Y^3 \ln(r)^2 - 56Y^2 \ln(r)^3 - 84Y \ln(r)^4 - 53 \ln(r)^5 + 38r^4 - 51r^3 Y - 35r^3 \ln(r) - 49r^2 Y^2 + 50r^2 Y \ln(r) + 43r^2 \ln(r)^2 \\ & - 47r Y^3 - 87r Y^2 \ln(r) - 28r Y \ln(r)^2 + 76r \ln(r)^3 + 21Y^4 - 90Y^3 \ln(r) + 91Y^2 \ln(r)^2 - 83Y \ln(r)^3 + 46 \ln(r)^4 - 73r^3 + 11r^2 Y + 67r^2 \ln(r) \\ & - 32r Y^2 + 72r Y \ln(r) - 61r \ln(r)^2 - 94Y^3 + 61Y^2 \ln(r) - 54Y \ln(r)^2 - 91 \ln(r)^3 + 93r^2 + 37r Y - 46r \ln(r) - 98Y^2 - 3Y \ln(r) + 10 \ln(r)^2 \\ & - 93r + 75Y - 82 \ln(r) + 39 \end{aligned}$$

> **compact2 := collect(p2, [r,ln(r)], distributed, VC:-veil);**

$$\begin{aligned} compact2 := & -28r^5 \ln(r) + 10r^4 \ln(r)^2 - 7r^3 \ln(r)^3 - 73r^2 \ln(r)^4 - 5r^6 - 14r \ln(r)^5 - 58 \ln(r)^6 - C_2 r - C_4 r^4 - C_5 r^3 - C_6 r^2 - C_{16} \ln(r) \\ & - C_{19} \ln(r)^4 - C_{20} \ln(r)^5 + C_3 r^5 + C_1 - C_7 r^2 \ln(r)^3 + 2C_9 r^3 \ln(r)^2 - C_{10} r \ln(r)^4 - C_{11} r^3 \ln(r) - C_{14} r^2 \ln(r) + C_{17} \ln(r)^2 + C_8 r^4 \ln(r) \\ & + C_{12} r^2 \ln(r)^2 + C_{13} r \ln(r)^3 + C_{15} r \ln(r)^2 + C_{18} \ln(r)^3 - C_{21} r \ln(r) \end{aligned}$$

> **zero := VC:-unveil(compact2) - p2:**

> **expand(zero);**

> **p3 := sort(randpoly([x,y,z,r,ln(r)], dense, degree=3));**

$$\begin{aligned}
 p3 := & 95 r^3 - 67 r^2 x - 5 r^2 y + 35 r^2 z - 18 r^2 \ln(r) - 36 r x^2 - 95 r x y - 62 r x z + 68 r x \ln(r) + 20 r y^2 + 8 r y z + 23 r y \ln(r) - 63 r z^2 \\
 & - 67 r z \ln(r) - 20 r \ln(r)^2 + 95 x^3 - 68 x^2 y + 98 x^2 z - 95 x^2 \ln(r) + 92 x y^2 + 8 x y z - 18 x y \ln(r) + 66 x z^2 + 40 x z \ln(r) + 43 x \ln(r)^2 - 67 y^3 \\
 & + 8 y^2 z + 93 y^2 \ln(r) + 81 y z^2 - 44 y z \ln(r) - 81 y \ln(r)^2 - 24 z^3 - 36 z^2 \ln(r) + 65 \ln(r)^3 + 46 r^2 - 65 r x + 34 r y + 19 r z + 52 r \ln(r) + 8 x^2 \\
 & + 44 x y + 68 x z + 6 x \ln(r) + 45 y^2 - 80 y z - 95 y \ln(r) + 85 z^2 + 60 z \ln(r) + 46 \ln(r)^2 + 31 r + 39 x + 63 y + 95 z - 24 \ln(r) + 63
 \end{aligned}$$

> **compact3 := collect(p3, [x,y], distributed, VK:-veil);**

$$compact3 := K_{29} - K_{30} x - K_{31} y + K_{32} y^2 - 67 y^3 + 95 x^3 - 68 x^2 y + 92 x y^2 - K_{33} x y - K_{34} x^2$$

> **compact4 := collect(compact3, [y], VC:-veil);**

$$compact4 := -67 y^3 + C_{22} y^2 - C_{23} y + C_{24}$$

> **VC:-unveil(C[22]);**

$$K_{32} + 92 x$$

> **VK:-unveil(K[32]);**

$$8 z + 93 \ln(r) + 20 r + 45$$

> **L := [seq(K[i]=VK:-unveil(K[i]), i=29..34)];**

$$\begin{aligned}
 L := & [K_{29} = 63 + 31 r + 52 r \ln(r) - 24 z^3 + 95 z + 19 r z - 18 r^2 \ln(r) + 65 \ln(r)^3 + 95 r^3 + 85 z^2 + 60 z \ln(r) - 24 \ln(r) + 46 \ln(r)^2 - 67 r z \ln(r) \\
 & - 20 r \ln(r)^2 - 36 z^2 \ln(r) + 46 r^2 + 35 r^2 z - 63 r z^2, K_{30} = -68 r \ln(r) + 65 r - 39 - 66 z^2 + 62 r z - 68 z - 43 \ln(r)^2 + 67 r^2 - 40 z \ln(r) - 6 \ln(r), \\
 & K_{31} = -63 - 8 r z + 44 z \ln(r) - 81 z^2 - 34 r + 95 \ln(r) + 80 z - 23 r \ln(r) + 81 \ln(r)^2 + 5 r^2, K_{32} = 8 z + 93 \ln(r) + 20 r + 45,
 \end{aligned}$$

$$K_{33} = -44 + 18 \ln(r) + 95 r - 8 z, K_{34} = 36 r - 8 + 95 \ln(r) - 98 z]$$

> **codegen[fortran](L,optimized);**

```
t2 = log(r)
```

```
t3 = r*t2
```

```
t5 = z**2
```

```
t9 = r*z
```

```
t11 = r**2
```

```
t14 = t2**2
```

```
t20 = z*t2
```

```
K(29) = 63+31*r+52*t3-24*t5*z+95*z+19*t9-18*t11*t2+65*t14*t2+95*t1
```

```
#1*r+85*t5+60*t20-24*t2+46*t14-67*t9*t2-20*r*t14-36*t5*t2+46*t11+35
```

```
#*t11*z-63*r*t5
```

```
K(30) = -68*t3+65*r-39-66*t5+62*t9-68*z-43*t14+67*t11-40*t20-6*t2
```

```
t48 = 95*t2
```

```
K(31) = -63-8*t9+44*t20-81*t5-34*r+t48+80*z-23*t3+81*t14+5*t11
```

```
t53 = 8*z
```

```
K(32) = t53+93*t2+20*r+45
```

```
K(33) = -44+18*t2+95*r-t53
```

```
K(34) = 36*r-8+t48-98*z
```

> **codegen[C](L,optimized);**

```
t2 = log(r);
```

```
t3 = r*t2;
```

$$t5 = z*z;$$

$$t9 = r*z;$$

$$t11 = r*r;$$

$$t14 = t2*t2;$$

$$t20 = z*t2;$$

$$\begin{aligned} K[28] = & 63.0+31.0*r+52.0*t3-24.0*t5*z+95.0*z+19.0*t9-18.0*t11*t2+65.0*t14 \\ & *t2+95.0*t11*r+85.0*t5+60.0*t20-24.0*t2+46.0*t14-67.0*t9*t2-20.0*r*t14-36.0*t5* \\ & t2+46.0*t11+35.0*t11*z-63.0*r*t5; \end{aligned}$$

$$\begin{aligned} K[29] = & -68.0*t3+65.0*r-39.0-66.0*t5+62.0*t9-68.0*z-43.0*t14+67.0*t11 \\ & -40.0*t20-6.0*t2; \end{aligned}$$

$$t48 = 95.0*t2;$$

$$\begin{aligned} K[30] = & -63.0-8.0*t9+44.0*t20-81.0*t5-34.0*r+t48+80.0*z-23.0*t3+81.0*t14+ \\ & 5.0*t11; \end{aligned}$$

$$t53 = 8.0*z;$$

$$K[31] = t53+93.0*t2+20.0*r+45.0;$$

$$K[32] = -44.0+18.0*t2+95.0*r-t53;$$

$$K[33] = 36.0*r-8.0+t48-98.0*z;$$

>

>

>