

## Essential Maple 7

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### Chapter 1: Basics

#### Section 1.1: Getting Started

##### *Session 1.1.3: Sample Maple sessions*

##### *Third session - heat conduction by Fourier series*

```
> restart;
> plots[setoptions](colour=black);
> I1 := int( sin(k*Pi*x)^2, x=0..1 );
```

$$I1 := -\frac{1}{2} \frac{\cos(k\pi) \sin(k\pi) - k\pi}{k\pi}$$

```
> I1 := eval( I1, {sin(k*Pi)=0, cos(k*Pi)=(-1)^k} );
```

$$I1 := \frac{1}{2}$$

> **I2 := int( x^2\*(1-x^2)\*sin(k\*Pi\*x), x=0..1);**

$$I2 := -2 \frac{-12 \cos(k \pi) + k^3 \pi^3 \sin(k \pi) + 5 k^2 \pi^2 \cos(k \pi) - 12 k \pi \sin(k \pi) + k^2 \pi^2 + 12}{k^5 \pi^5}$$

> **I2 := eval( I2, {sin(k\*Pi)=0, cos(k\*Pi)=(-1)^k} );**

$$I2 := -2 \frac{-12 (-1)^k + 12 + 5 k^2 \pi^2 (-1)^k + k^2 \pi^2}{k^5 \pi^5}$$

> **I2 := collect( I2, k, factor );**

$$I2 := -2 \frac{5 (-1)^k + 1}{\pi^3 k^3} + \frac{24 ((-1)^k - 1)}{\pi^5 k^5}$$

> **c := unapply( 2\*I2, k );**

$$c := k \rightarrow -4 \frac{5 (-1)^k + 1}{\pi^3 k^3} + \frac{48 ((-1)^k - 1)}{\pi^5 k^5}$$

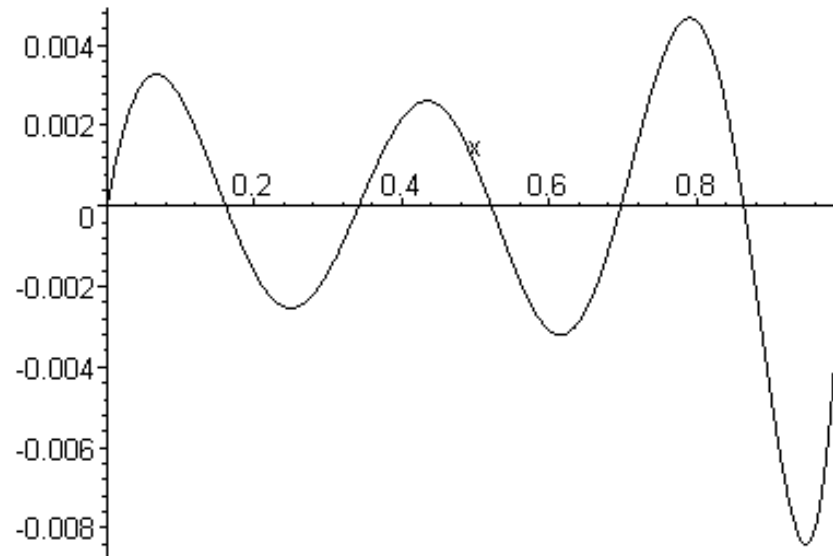
> **fn := n -> add( c(k)\*sin(k\*Pi\*x), k=1..n );**

$$fn := n \rightarrow \text{add}(c(k) \sin(k \pi x), k = 1 .. n)$$

> **f5 := fn( 5 );**

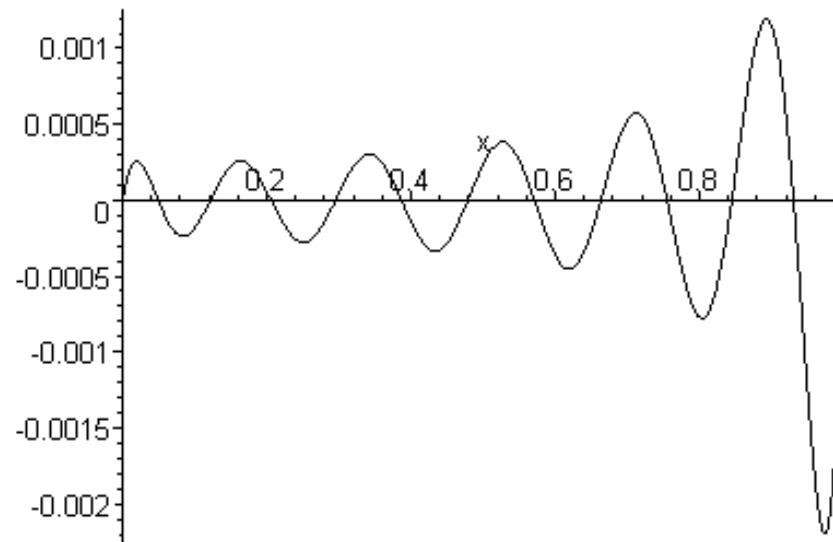
$$f5 := \left( 16 \frac{1}{\pi^3} - \frac{96}{\pi^5} \right) \sin(\pi x) - \frac{3 \sin(2 \pi x)}{\pi^3} + \left( \frac{16}{27} \frac{1}{\pi^3} - \frac{32}{81} \frac{1}{\pi^5} \right) \sin(3 \pi x) - \frac{3 \sin(4 \pi x)}{8 \pi^3} + \left( \frac{16}{125} \frac{1}{\pi^3} - \frac{96}{3125} \frac{1}{\pi^5} \right) \sin(5 \pi x)$$

> **plot( f5 - x^2\*(1-x^2), x=0..1 );**



```
> f10 := fn( 10 );
```

```
> plot( f10 - x^2*(1-x^2), x=0..1 );
```



> **un := n -> add( c(k)\*exp(-k^2\*Pi^2\*t)\*sin(k\*Pi\*x), k=1..n);**

$$un := n \rightarrow \text{add}\left(c(k) e^{(-k^2 \pi^2 t)} \sin(k \pi x), k = 1 \dots n\right)$$

> **u5 := un(5);**

$$u5 := \left(16 \frac{1}{\pi^3} - \frac{96}{\pi^5}\right) e^{(-\pi^2 t)} \sin(\pi x) - \frac{3 e^{(-4 \pi^2 t)} \sin(2 \pi x)}{\pi^3} + \left(\frac{16}{27} \frac{1}{\pi^3} - \frac{32}{81} \frac{1}{\pi^5}\right) e^{(-9 \pi^2 t)} \sin(3 \pi x) - \frac{3 e^{(-16 \pi^2 t)} \sin(4 \pi x)}{8 \pi^3} \\ + \left(\frac{16}{125} \frac{1}{\pi^3} - \frac{96}{3125} \frac{1}{\pi^5}\right) e^{(-25 \pi^2 t)} \sin(5 \pi x)$$

> **diff( u5, t ) - diff( u5, x, x );**

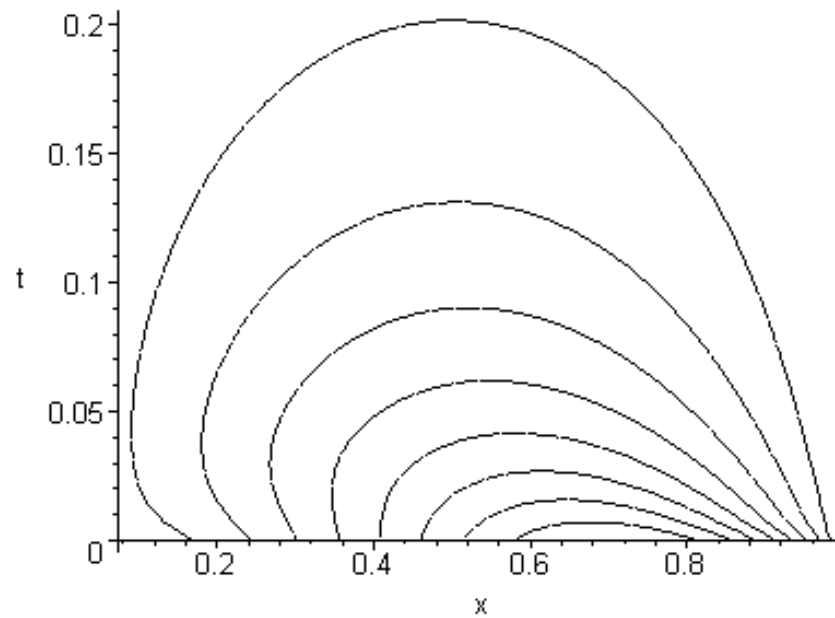
0

> **u10 := un(10);**

> **diff( u10,t ) - diff( u10, x, x );**

0

> **plots[contourplot](u10, x=0..1, t=0..0.21, grid=[30,30], colour=black);**



- > **restart;**
- > **plots[setoptions](colour=BLACK);**
- > **read "C:/books/ess/programs/fouriersine.mpl";**
- > **macro(FSS=FourierSineSeries);**

*FSS*

- > **f := FSS(x^2\*(1-x^2),x);**

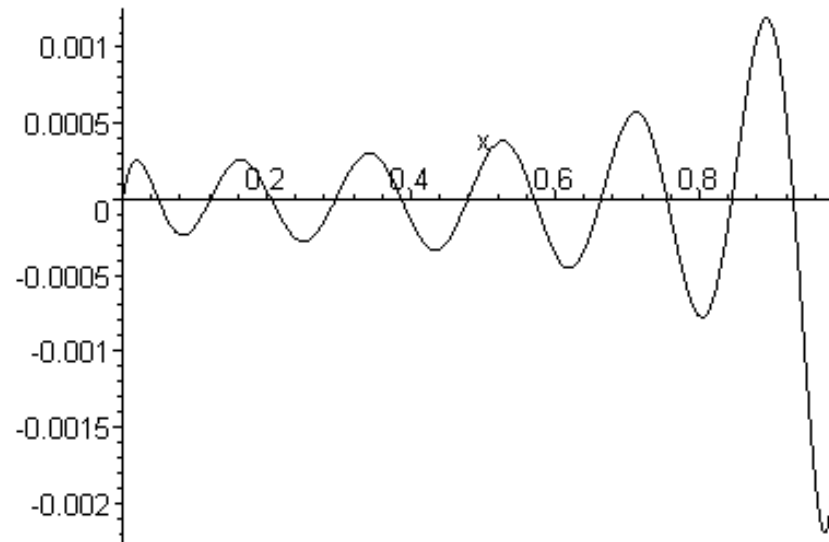
$f := n \rightarrow \text{add}(c(k) \sin(k \pi x), k = 1 .. n)$

- > **f5 := f(5);**

$$f5 := \left( 16 \frac{1}{\pi^3} - \frac{96}{\pi^5} \right) \sin(\pi x) - \frac{3 \sin(2 \pi x)}{\pi^3} + \left( \frac{16}{27} \frac{1}{\pi^3} - \frac{32}{81} \frac{1}{\pi^5} \right) \sin(3 \pi x) - \frac{3 \sin(4 \pi x)}{8 \pi^3} + \left( \frac{16}{125} \frac{1}{\pi^3} - \frac{96}{3125} \frac{1}{\pi^5} \right) \sin(5 \pi x)$$

- > **f10 := f(10);**

> **plot(f10-x^2\*(1-x^2),x=0..1);**

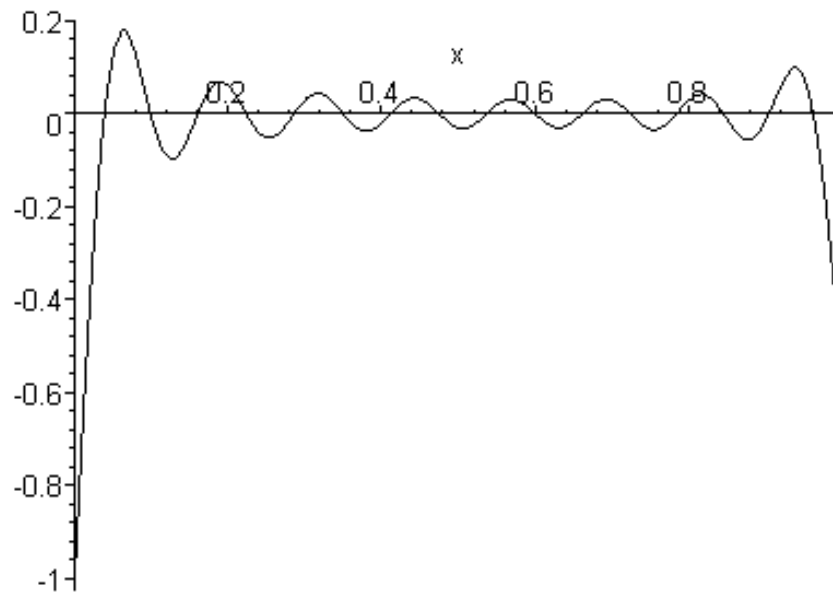


> **f := FSS(cos(x),x);**

$$f := n \rightarrow \text{add}(c(k) \sin(k \pi x), k = 1 .. n)$$

> **f15 := f(15);**

> **plot(f15-cos(x),x=0..1);**



> **restart;**

> **plots[setoptions](colour=black);**

> **FourierSineSeries2 := proc( f::operator )**

**local k, z;**

**description "FSS2(f::operator):"**

**" Improved Fourier series of f";**

**(n,x) -> add( limit( 2\*int( sin(k\*Pi\*z)\*f(z), z=0..1 ),**

**k=j**

**) \* sin(j\*Pi\*x),**

**j=1..n**

**)**

**end proc;**

*FourierSineSeries2 := proc(f:operator)*

**local k, z;**

**description "FSS2(f:operator): Improved Fourier series of f";**

**(n, x) → add(limit(2\*int(sin(k\*π\*z)\*f(z), z = 0 .. 1), k = j)\*sin(j\*π\*x), j = 1 .. n)**

**end proc**

> **macro(FSS2=FourierSineSeries2);**

*FSS2*

> **FSS2( x^2\*(1-x^2), x );**

Error, invalid input: FourierSineSeries2 expects its 1st argument, f, to be of type operator, but received x^2\*(1-x^2)

> **f := FSS2( x -> x^2\*(1-x^2) );**

$$f := (n, x) \rightarrow \text{add} \left( \left( \lim_{k \rightarrow j} 2 \int \sin(k \pi z) (x \rightarrow x^2 (1 - x^2))(z) dz = 0 .. 1 \right) \sin(j \pi x), j = 1 .. n \right)$$

> **f5 := f( 5, x );**

$$f5 := 16 \frac{(-6 + \pi^2) \sin(\pi x)}{\pi^5} - \frac{3 \sin(2 \pi x)}{\pi^3} + \frac{16}{81} \frac{(-2 + 3 \pi^2) \sin(3 \pi x)}{\pi^5} - \frac{3 \sin(4 \pi x)}{8 \pi^3} + \frac{16}{3125} \frac{(-6 + 25 \pi^2) \sin(5 \pi x)}{\pi^5}$$

> **fc := FSS2( x -> cos(Pi\*x) );**

$$fc := (n, x) \rightarrow \text{add} \left( \left( \lim_{k \rightarrow j} 2 \int \sin(k \pi z) (x \rightarrow \cos(\pi x))(z) dz = 0 .. 1 \right) \sin(j \pi x), j = 1 .. n \right)$$

> **fc( 10, x );**

$$\frac{8 \sin(2 \pi x)}{3 \pi} + \frac{16 \sin(4 \pi x)}{15 \pi} + \frac{24 \sin(6 \pi x)}{35 \pi} + \frac{32 \sin(8 \pi x)}{63 \pi} + \frac{40 \sin(10 \pi x)}{99 \pi}$$