

Classroom Tips and Techniques: Integration by Parts in Maple

Robert J. Lopez
Emeritus Professor of Mathematics and Maple Fellow
Maplesoft

▼ Introduction

Maple implements "integration by parts" with two different commands. One was designed in a pedagogical setting, and the other, for a "production" setting.

In this article, we compare the functionalities of the two commands, [Parts](#) (in *IntegrationTools*) as a "production" tool, and [Rule\[parts\]](#) (in *Student:-CalculusI*) as a didactic command.

In the following illustrations, we will write the formula as

$$\int u \, dv = uv - \int v \, du$$

so that the factor u is to be differentiated, and the companion factor, dv , is to be integrated.

▼ Initialize

The following two integrals

$$\begin{aligned} \Phi_1 &:= \tan(x) \int x \sin(x) \\ &\quad dx : \\ \Phi_2 &:= \int J_0^6(x) \cos(x) \\ &\quad dx : \end{aligned}$$

and their sum

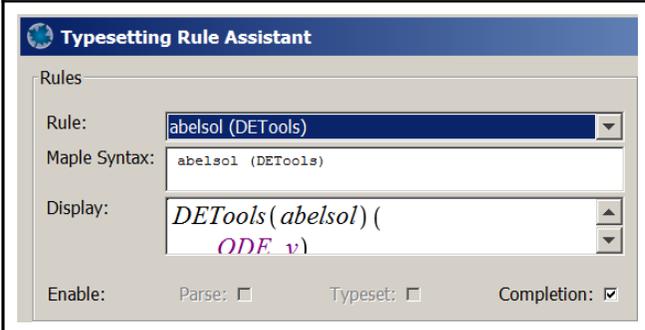
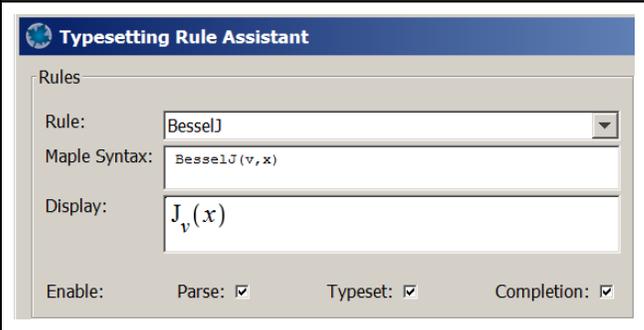
$$\begin{aligned} \Phi_3 &:= \Phi_1 + \Phi_2 \\ \tan(x) \left(\int x \sin(x) \, dx \right) &+ \int J_0(x)^6 \cos(x) \, dx \end{aligned}$$

will be used as the basis for the discussion of parts integration.

In the second integral the textbook notation $J_0(x)$ stands for a Bessel function of order zero. Maple knows this function as "BesselJ" but the following modifications to Maple allow the textbook notation to be understood.

```
interface(typesetting=extended) :
Typesetting:-
  EnableParseRule("BesselJ") :
```

Alternatively, the interactive Rules Assistant accessed from the View menu can be used, provided extended typesetting has been implemented.

	
<p>Figure 1 Rules Assistant in default state</p>	<p>Figure 2 Rules Assistant after parse rule enabled</p>

▼ The Parts Command from the *IntegrationTools* Package

Table 1 makes four applications of the **Parts** command from the *IntegrationTools* package.

$\text{IntegrationTools:-Parts}(\Phi_1, x)$ $\tan(x) \left(-x \cos(x) - \left(\int -\cos(x) \, dx \right) \right)$
$\text{IntegrationTools:-Parts}(\Phi_1 = \cos(x), x)$ $\tan(x) \left(-x \cos(x) - \left(\int -\cos(x) \, dx \right) \right) = \cos(x)$
$\text{IntegrationTools:-Parts}(\Phi_2, x)$ $x \left(\int \frac{J_0(x)^6 \cos(x)}{x} \, dx \right) - \left(\int \int \frac{J_0(x)^6 \cos(x)}{x} \, dx \, dx \right)$
$\text{IntegrationTools:-Parts}(\Phi_3, x)$ <p style="color: magenta; text-decoration: underline;">Error, (in IntegrationTools:-Parts) multiple integrals</p>

detected

Table 1 The **Parts** command from the *IntegrationTools* package applied to the integrals in Table 1

Table 1 immediately reveals that only the term taken as u need be provided. The command then extracts dv and provides v . The table also reveals that the **Parts** command is not deterred by a multiplicative factor in front of the integral. It also reveals that the command applies both to an expression as well as an equation. However, it further shows that if an inappropriate choice of u is made, a formal antiderivative representing v is devised, and the integration-by-parts formula is applied. Finally, the table reveals that if an expression contains more than one integral, an error results. The user must find a way to impose the command on just one integral in the expression.

Table 2 lists several ways that the **Parts** command could be applied just to Φ_1 in expression Φ_3 . Each requires that the integral to which **Parts** is to be applied can be singled out by some distinguishing feature. In the first method, the **evalindets** command is used to place the **Parts** command onto the operand of Φ_3 that is of Maple type ``*``, that is, a product. The operands of Φ_3 are identified by applying the **op** command, as in

$$\text{op}(\Phi_3) = \tan(x) \left(\int x \sin(x) \, dx \right), \int J_0(x)^6 \cos(x) \, dx$$

In the second method, Φ_3 is decomposed by the alternate usage of the **op** command, the **Parts** command is applied to the relevant integral (the first operand), and the transformed Φ_3 reconstructed by adding back the second operand. Of course, this approach can be tedious, but that might be preferable to grappling with the mysteries of Maple types.

$$\text{evalindets}(\Phi_3, \text{`*`}, \text{IntegrationTools:-Parts}, x)$$
$$\tan(x) \left(-x \cos(x) - \left(\int -\cos(x) \, dx \right) \right) + \int J_0(x)^6 \cos(x) \, dx$$

$$\text{IntegrationTools:-Parts}(\text{op}(1, \Phi_3), x) + \text{op}(2, \Phi_3)$$
$$\tan(x) \left(-x \cos(x) - \left(\int -\cos(x) \, dx \right) \right) + \int J_0(x)^6 \cos(x) \, dx$$

Table 2 Applying **Parts** to a particular integral in a sum of integrals

▼ Rule[parts] from the *Student[Calculus1]* Package

The *Student[Calculus1]* package contains code for single-stepping through problems in calculus. Limits, derivatives, and integrals can be evaluated step-wise, with the user signaling to Maple the relevant rules to apply. Unfortunately, the syntax for this single-stepping by rules is more complicated than the typical instructor would want to expose to a class of calculus students.

Fortunately, this functionality has been embedded in pop-up tools that apply the rules via buttons in

point-and-click interfaces called *Tutors*. Figure 3 (below) shows the [Integration Methods](#) tutor applied to Φ_1 . The first button to be pressed is the "Parts" button, which then launches the dialog in which both u and v are to be entered. Note that here, Maple uses the paradigm

$$\int f dg = fg - \int g df$$

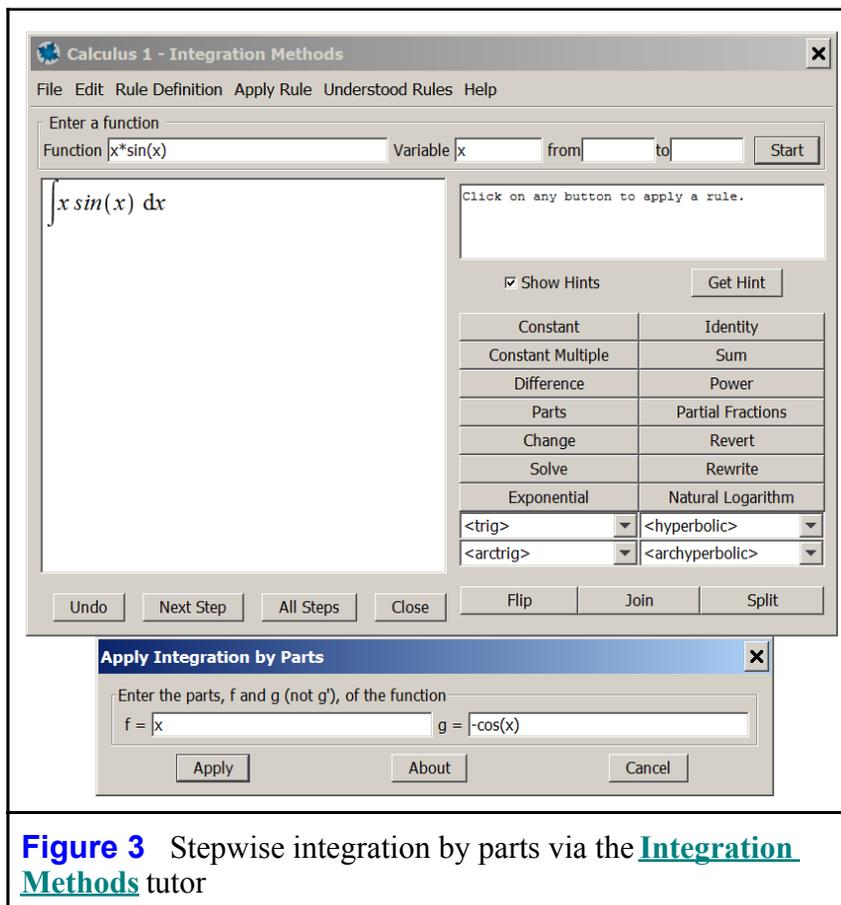


Figure 3 Stepwise integration by parts via the [Integration Methods](#) tutor

In Table 3, the [Rule\[parts\]](#) command is applied to the integrals examined in Table 1.

<p><i>Student:-CalculusI:-Rule[parts, x, -cos(x)](Φ_1)</i></p> $\tan(x) \left(\int x \sin(x) dx \right) = \tan(x) \left(-x \cos(x) - \left(\int -\cos(x) dx \right) \right)$
<p><i>Student:-CalculusI:-Rule[parts, x, -cos(x)]($\Phi_1 = \cos(x)$)</i></p> <p><u>Error, (in Student:-CalculusI:-Rule[parts, x, `+`(`-`(cos(x)))) input expression does not have any incomplete calculus operations</u></p>
<p><i>Student:-CalculusI:-Rule[parts, x, -cos(x)]($\Phi_1 - \cos(x)$)</i></p> $\tan(x) \left(\int x \sin(x) dx \right) - \cos(x) = \tan(x) \left(-x \cos(x) - \left(\int -\cos(x) dx \right) \right) - \cos(x)$

$$\text{Student:-Calculus1:-Rule[parts, x, -\cos(x)](\Phi_3)$$

$$\tan(x) \left(\int x \sin(x) \, dx \right) + \int J_0(x)^6 \cos(x) \, dx = \tan(x) \left(-x \cos(x) - \left(\int -\cos(x) \, dx \right) \right) + \int J_0(x)^6 \cos(x) \, dx$$

Table 3 [Rule\[parts\]](#) from the *Student:-Calculus1* package applied to the integrals in Table 1

Unlike the [Parts](#) command, [Rule\[parts\]](#) requires the user to provide both u and v (not dv). But like the [Parts](#) command in the *IntegrationTools* package, [Rule\[parts\]](#) is not hampered by a multiplier in front of the integral. However, unlike the [Parts](#) command, it is indeed impeded by an equation. It must act on an expression only. Finally, [Rule\[parts\]](#) acts intelligently on an expression containing more than one integral.

▼ Conclusion

The [Parts](#) command in the *IntegrationTools* package maps across equations, but not across expressions containing more than one integral. Also, it acts formally on an integral for which the supplied u is inappropriate. But it need only be given u , not u and v .

In the didactic environment, [Rule\[parts\]](#) will remain the best alternative, mainly because it requires the student to provide v as well as u , and behaves nicely in an equation and in the presence of more than one integral. Despite its shortcoming in an expression containing more than one integral, in a "production-mode" the [Parts](#) command would be the tool of choice.

Legal Notice: © Maplesoft, a division of Waterloo Maple Inc. 2017. Maplesoft and Maple are trademarks of Waterloo Maple Inc. This application may contain errors and Maplesoft is not liable for any damages resulting from the use of this material. This application is intended for non-commercial, non-profit use only. Contact Maplesoft for permission if you wish to use this application in for-profit activities.