Black Hole Quantum Mechanics : A Very Short Introduction

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Abstract

A firm understanding of black holes involves understanding the quantum mechanics behind them. This worksheet will introduce how stellar black holes form, the No-Hair Theorem, and Hawking radiation. This worksheet will also touch on the Information Loss Paradox that arises from Hawking radiation and current efforts to gain more knowledge about black holes.

Introduction

Black holes seem to be straight out of science fantasy, as they are bodies that take everything in that gets close to it and lets nothing escape. However, they are very much real! Photographic evidence is shown below in **Figure 1**.

Figure 1. First ever image of a black hole generated on April 10, 2019. The black hole in this image is at the center of the Messier 87 galaxy, which is around 55 million light years away from Earth.[1]

As the recent photograph demonstrates, we are still on the path of learning about black holes as a celestial phenomenon. The theory behind black holes, particularly the quantum mechanical aspects, are currently a hot topic among physicists. In this worksheet, we will introduce a few of these quantum mechanical aspects that are crucial to our current understanding of black holes. Before the quantum mechanical aspects of black holes can be discussed, however, some necessary information about general relativity must be established since a large portion of our understanding of black holes comes from general relativity as well.

Preliminary I - General Relativity[2]

General Relativity first emerged from Albert Einstein in 1916 when he began studying whether space-time (the union of space and time dimensions) could have curvature. Over a decade prior, Einstein published his theory of Special Relativity, which was built on the following postulates:

1. The laws of physics are the same in any intertial frame of reference
2. Nothing can ever surpass the speed of light

Based on these postulates, time is relative to whatever frame of reference a system is in, rather than being absolute as Newtonian physics predicts! Incorporating this result, Einstein determined that space-time can be described by the field equations

$$G^{\mu\nu} = \kappa T^{\mu\nu}$$

where $G^{\mu\nu}$ is the Einstein field tensor, $T^{\mu\nu}$ is the momentum-energy tensor, and $\kappa = \frac{8\pi G}{c^4}$. The math behind this equation is outside of the scope of this worksheet. However, the major implication of this equation that we must understand is that mass and energy affect the curvature of space-time, and the manifestation of that curvature is gravity.

A few months after Einstein first published his Theory of General Relativity, Karl Schwarzschild derived the first solution to the field equations for a non-rotating spherical mass. This solution, known as the Schwarzschild metric, is

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \left(1-\frac{2M}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

where the expression is in spherical coordinates. The importance of this solution will be established later.

With this knowledge in hand, we can now begin to probe into some fundamental concepts in black hole quantum mechanics. Going forward this worksheet will be within the scope of stellar black holes.

Theory

Black Hole Formation[3, 4]

Stellar black holes form via the gravitational collapse of dying stars. As a star approaches the end of its lifetime and exhausts all of its fuel, it's temperature begins to drop and the star begins to succumb to it's own weight. Depending on the initial mass of the star before this process begins happening, degeneracy pressure of the fermions inside of the star may sustain the star and prevent it from collapsing into itself. This is seen in white dwarfs and neutron stars. If the star, however, is massive enough, then the degeneracy pressure of fermions is not sufficient to prevent gravitational collapse. When this is the case, the dying star forms a black hole.

A key parameter when describing black hole formation is the Schwarzschild radius, defined as

$$r_s = \frac{2GM}{c^2}$$

where G is the gravitational constant, M is mass, and c is the speed of light. This radius emerges from the Schwarzschild metric introduced earlier in the worksheet and corresponds to a singularity (infinite quantity) in Einstein's field equations.

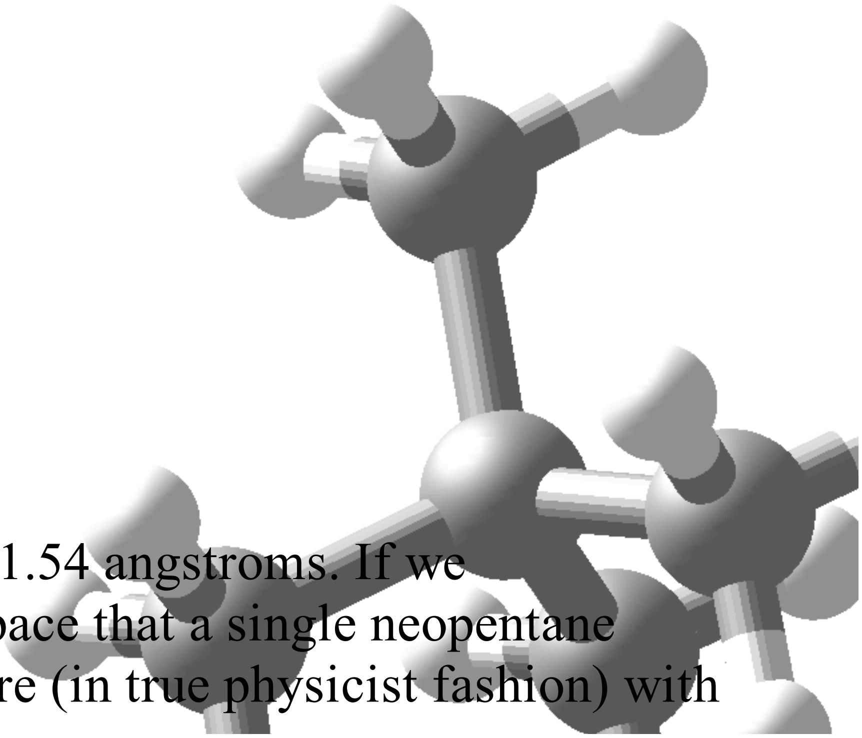
To help gain a physical understanding of this, consider neopentane.

```
> Digits := 15;
                              Digits := 15                              (1)
```

```
> with(QuantumChemistry);
[AOLabels, ActiveSpaceCI, ActiveSpaceSCF, AtomicData, BondAngles, BondDistances, Charges,    (2)
    ChargesPlot, ContractedSchrodinger, CorrelationEnergy, CoupledCluster, DensityFunctional,
    DensityPlot3D, Dipole, DipolePlot, Energy, ExcitationEnergies, ExcitationSpectra,
    ExcitationSpectraPlot, ExcitedStateEnergies, ExcitedStateSpins, FullCI, GeometryOptimization,
    HartreeFock, Interactive, Isotopes, MOCoefficients, MODiagram, MOEnergies, MOIntegrals,
    MOOccupations, MOOccupationsPlot, MOSymmetries, MP2, MolecularData,
    MolecularDictionary, MolecularGeometry, NuclearEnergy, NuclearGradient,
    OscillatorStrengths, Parametric2RDM, PlotMolecule, Populations, Purify2RDM, RDM1,
    RDM2, RTM1, ReadXYZ, Restore, Save, SaveXYZ, SearchBasisSets, SearchFunctionals,
    SkeletalStructure, Thermodynamics, TransitionDipolePlot, TransitionDipoles,
    TransitionOrbitalPlot, TransitionOrbitals, Variational2RDM, VibrationalModeAnimation,
    VibrationalModes, Video]
```

```
> neopentane := MolecularGeometry("neopentane");
neopentane := [["C", 0, 0, 0], ["C", −1.34670000, 0.37270000, 0.63640000], ["C", 0.89610000,   (3)
    1.24490000, −0.06770000], ["C", −0.23590000, −0.53850000, −1.41830000], ["C",
    0.68640000, −1.07910000, 0.84960000], ["H", −1.21060000, 0.76130000, 1.65200000],
    ["H", −1.86160000, 1.14280000, 0.05100000], ["H", −2.00950000, −0.49770000,
    0.69850000], ["H", 1.86690000, 1.01000000, −0.51840000], ["H", 1.08280000, 1.65320000,
    0.93190000], ["H", 0.43180000, 2.03480000, −0.66880000], ["H", −0.87370000,
    −1.42940000, −1.40260000], ["H", −0.72570000, 0.21090000, −2.05010000], ["H",
    0.70930000, −0.81380000, −1.89960000], ["H", 0.86840000, −0.72340000, 1.86990000],
    ["H", 1.65240000, −1.36660000, 0.41960000], ["H", 0.06930000, −1.98220000,
    0.91650000]]
```

```
> PlotMolecule(neopentane);
```



The length of a C-C bond is 1.54 angstroms. If we aproximated the region of space that a single neopentane molecule occupies as a sphere (in true physicist fashion) with a radius equivalent to that of a C-C bond, we would need to jam in 1.04 x 10^{17} kg of mass into this sphere for neopentane to become a black hole! So, any spherical object with a given mass will turn into a black hole when compressed down to its Schwarzschild radius.

No-Hair Theorem[3]

A black hole can be entirely described by 3 external observabes: mass, charge, and angular momentum. This is known as the No-Hair Theorem, which was named after John Wheeler described black holes as "having no hair." In this context, "hair" refers to information associated with anything that falls into the black hole. There is no rigorous mathematical proof for this as of yet, however.

Hawking Radiation[3, 5, 6]

So far, we have established that nothing can escape a black hole. But, if we incorporate quantum effects into our understanding of black holes, something peculiar emerges.

Suppose we have an observer at infinity distance away from the event horizon of a black hole. In a vacuum near the event horizon, we initially have

$$\boldsymbol{a}_i|0\rangle = 0 \ \forall \ i$$

where $\boldsymbol{a}_i$ is the annihilation operator. Near the event horizon, quantum fluctuations may result in particle creation, where a virtual particle-antiparticle pair are created. The pair has the state

$$\left|\psi\right\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_c|0\rangle_b \pm |1\rangle_c|1\rangle_b\right)$$

where the pair is maximally entangled. The particle-antiparticle pair exist for a certain amount of time as dictated by the uncertainty principle

$$\Delta\mathrm{E}\Delta\mathrm{T} \geq \frac{\hbar}{2}$$

before coming back together and annihilating each other. But, since partile creation is happening near an event horizon, it is possible for one of the created particles to tunnel through the event horizon, leaving the other to escape away towards infinity. As a result, the observer sees a flux of particles coming towards them from the direction of the black hole. This phenomenon, known as Hawking radiation (which Stephen Hawking first proved in 1974), is illistrated in **Figure 2**.

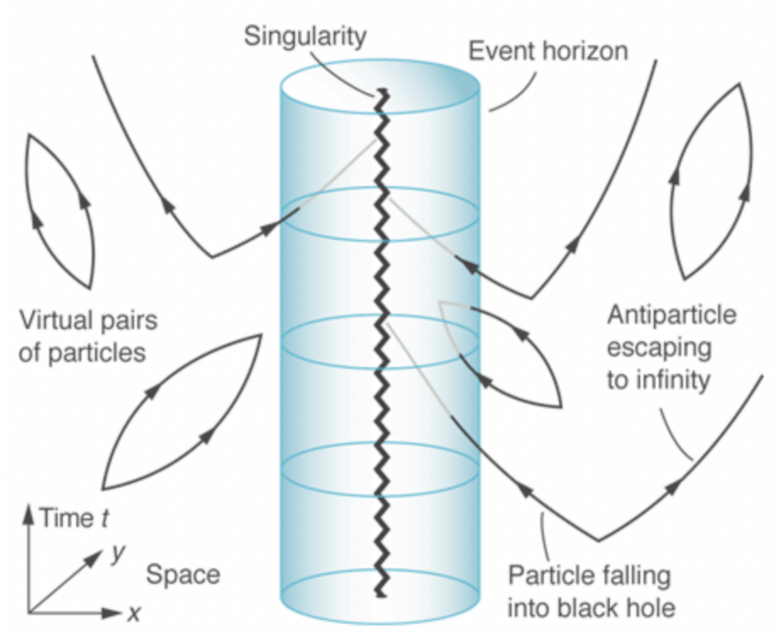


Figure 2. Particle creation near an event horizon. Rather than annihilate each other, it is possible for a particle/antiparticle generated from particle creation to pass through the event horizon while the other escapes away to infinity. The escaped particle/antiparticle is what the observer sees as Hawking radiation.[7]

Hawking radiation results in black holes having a temperature characterized by

$$T_{BH} = \frac{\hbar \kappa}{2\pi c k_B}$$

where κ is the surface gravity of the black hole and k_B is the Boltzmann constant. If we were to generate a spectrum of Hawking radiation, we would see that it matches almost perfectly to that of a blackbody!

Conflicting Results[3]

Hawking radiation will lead to black holes losing mass and over time will evaporate on the timescale of

$$\tau \propto \frac{G^2 M^3}{\hbar c^4}$$

This implies that the initial state of the infalling particle/antiparticle cannot be recovered from Hawking radiation. However, this contradicts the principle of unitarity! Quantum mechanics dictates that information must be preserved in any form of evolution or transformation. In a completely classical view of black holes, there is no information loss as we can predict that anything taken in by the black hole becomes part of it, and thus the initial properties of the thing in question become part of the black hole's characteristic quantities. With Hawking radiation, however, the escaped particle/antiparticle is entangled with the other particle/antiparticle that tunneled through the event horizon. So, the radiation that one observes is a mixed state. If a black hole were to completely evaporate, then all the information we gain from radiation will be in a mixed state that we cannot use to obtain any information about the initial states of the particles and antiparticles before the Hawking radiation mechanism occurs. In other words, it appears that any information on a created particle/antiparticle pair's initial state is lost forever! This is known as the Information Loss paradox and has been a puzzle that has plagued physicists since its emergence.

Discussion and Conclusion

Towards a Theory of Quantum Gravity[3]

Hawking radiation and the resulting information paradox is a specific example of a more fundamental problem in physics: quantum gravity. As they currently stand, general relativity and quantum mechanics do not agree with each other when quantum effects must be taken into account. The challenge that theoretical physicists face now in order to resolve the paradox is to successfully unify general relativity and quantum mechanics.

Observational Efforts[8]

Apart from theoretical physicists putting pen to paper, or chalk to chalkboard, experimental physicists are also embarking on this task through observations with the Laser Interferometer Gravitaitonal-Wave Observatory (LIGO). LIGO aims to detect gravitational waves (transverse waves produced from accelerating masses) from high-energy celestial events that otherwise could not be studied with electromagnetic radiation. An example of such a celestial event in this context would be black hole collisions. Because of the very nature of black holes, physicists cannot use methods that involve electromagnetic radiation to directly observe them. So, LIGO is crucial in obtaining black hole data as it is currently the only observatory of its kind that can utilize gravitational waves to gain more information on black holes!

The knowledge we currently have of black holes is incomplete. However, this is an exciting time for physicists and black hole enthusiasts in general as there are still some mysteries to resolve. The day that black holes can be unified under general relativity and quantum mechanics is still yet to come, but the day that it does happen will revolutionize physics as we know it!

Selected References

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