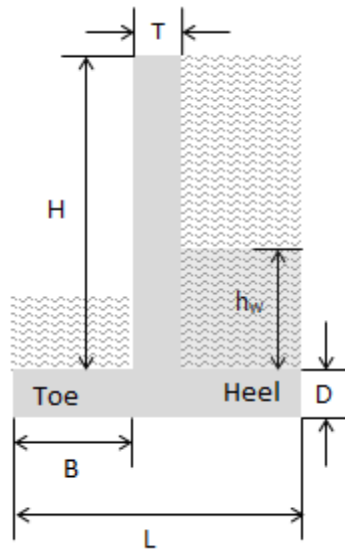


Concrete Retaining Wall

This document analyzes the design of a concrete retaining wall, employing the Strength Design Method as outlined in ACI 318.

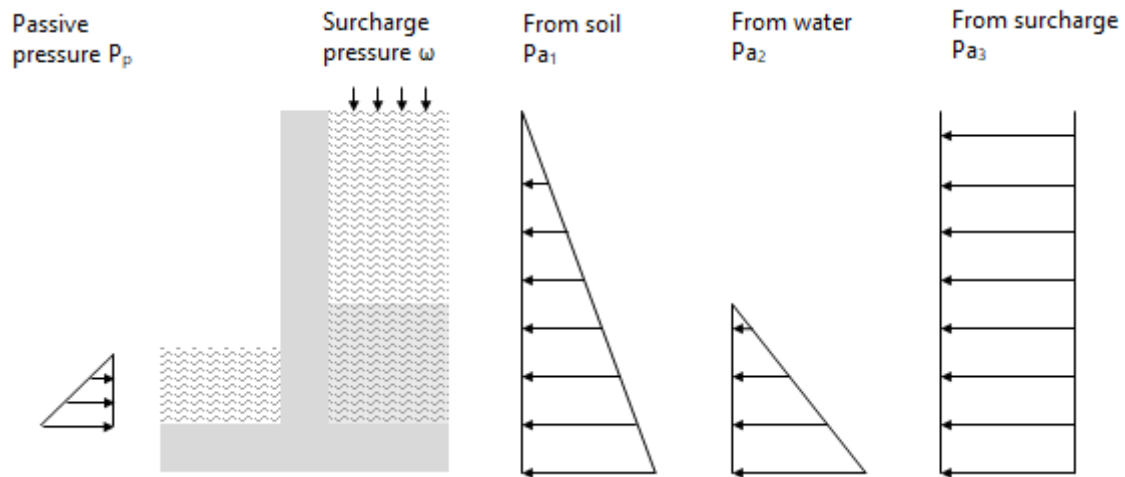


1. Parameters

Dead load (unfactored)	$DL := 0 \text{ kN}$
Live load (unfactored)	$LL := 0 \text{ kN}$
External moment dead load (unfactored)	$EMDL := 0 \text{ kN}$
External moment live load (unfactored)	$EMLL := 0 \text{ kN}$
Surcharge	$w := 12 \text{ kPa}$
Compressive strength of concrete	$f_c := 32 \text{ MPa}$
Rebar yield stress (<60 ksi, ACI 318. 11.5.2)	$f_y := 460 \text{ MPa}$
Soil bearing capacity	$q_{all} := 100 \text{ kPa}$
Unit weight of soil	$Y_{soil} := 18 \text{ kN} \cdot \text{m}^{-3}$

Unit weight of water	$Y_{\text{water}} := 10 \text{ kN}\cdot\text{m}^{-3}$
Unit weight of concrete	$Y_{\text{concrete}} := 25 \text{ kN}\cdot\text{m}^{-3}$
Coefficient of soil friction	$\phi := 30 \text{ deg}$
Height of wall	$H := 2 \text{ m}$
Height of soil	$h_s := 2 \text{ m}$
Height of soil on toe	$h_{\text{soil_toe}} := 0.6 \text{ m}$
Height of water	$h_w := 1 \text{ m}$
Height of surcharge	$h_{\text{surcharge}} := 0.8 \text{ m}$
Base width of footing	$L_f := 1500 \text{ mm}$
Thickness of footing	$D_f := 250 \text{ mm}$
Thickness of wall	$T := 250 \text{ mm}$
Concrete cover	$cc := 75 \text{ mm}$
Diameter of bars	$d_b := 12 \text{ mm}$
Load factor dead load	$LF_{\text{dead}} := 1.2$
Load factor live load	$LF_{\text{live}} := 1.6$
Load factor lateral load	$LF_{\text{lateral}} := 1.6$
Strip length of wall to analyze	$b := 1 \text{ m}$
Width of toe	$B := (L_f - T) / 2 = 625 \text{ mm}$
Strength reduction factor for flexure (ACI 318 9.3.2.1)	$\phi_{\text{flexure}} := 0.9$
Strength reduction factor for shear (ACI 313 9.3.2.3)	$\phi_{\text{shear}} := 0.75$

2. Forces on Retaining Wall



Soil pressure coefficients

$$k_a := \frac{1 - \sin(\phi)}{1 + \sin(\phi)} = 0.33$$

$$k_p := \frac{1 + \sin(\phi)}{1 - \sin(\phi)} = 3$$

Lateral force from soil

$$P_{a1} := \frac{1}{2} \cdot Y_{\text{soil}} \cdot k_a \cdot h_s^2 \cdot b = 12.00 \text{ kN}$$

Level arm from base

$$L_{\text{base_soil}} := \frac{1}{3} \cdot h_s = 0.667 \text{ m}$$

Force from water

$$P_{a2} := \frac{1}{2} \cdot Y_{\text{water}} \cdot h_w^2 \cdot b = 5 \text{ kN}$$

Level arm from base

$$L_{\text{base_water}} := \frac{1}{3} \cdot h_w = 0.33 \text{ m}$$

Force from surcharge

$$P_{a3} := w \cdot k_a \cdot h_{\text{surcharge}} \cdot b = 3.20 \text{ kN}$$

Level arm from surcharge

$$L_{\text{base_surcharge}} := \frac{1}{2} \cdot h_{\text{surcharge}} = 0.40 \text{ m}$$

Passive pressure

$$P_p := \frac{1}{2} \cdot Y_{\text{soil}} \cdot k_p \cdot h_{\text{soil_toe}}^2 \cdot b = 9.72 \text{ kN}$$

3. Check Wall for Shear

Nominal shear (equal to lateral forces on wall, and ignoring passive pressure)

$$S_n := Pa1 + Pa2 + Pa3 = 20.200 \text{ kN}$$

Ultimate shear

$$S_u := LF_{\text{lateral}} \cdot S_n = 32.320 \text{ kN}$$

Allowable shear according to ACI 318

$$\phi V_c := \phi_{\text{shear}} \cdot 0.17 \cdot 1000 \frac{\text{m}^{-1}}{\text{m}^2} \cdot \text{kg} \frac{\text{m}^{-1}}{\text{s} \cdot \text{N}} \cdot \sqrt{f_c} \cdot (T - cc) - 0.5 \cdot d_b$$

$$\phi V_c = 121.891 \text{ kN}$$

$$\begin{cases} \text{"pass"} & S_u < \phi V_c \\ \text{"fail"} & \text{otherwise} \end{cases} = \text{"pass"}$$

4. Design the Wall Stem for Flexure

Nominal moment

$$M_n := Pa1 \cdot L_{\text{base_soil}} + Pa2 \cdot L_{\text{base_water}} + Pa3 \cdot L_{\text{base_surcharge}} = 10.947 \text{ kN m}$$

Ultimate moment

$$M_u := LF_{\text{lateral}} \cdot Pa1 \cdot L_{\text{base_soil}} + LF_{\text{lateral}} \cdot Pa2 \cdot L_{\text{base_water}} + LF_{\text{live}} \cdot Pa3 \cdot L_{\text{base_surcharge}} = 17.515 \text{ kN m}$$

Reinforcement ratio

$$\omega := 0.85 \cdot \left(1 - \left(\left(1 - \frac{2}{0.85} \cdot m^{-1} \cdot \left(\frac{M_u}{\phi_{\text{flexure}} \cdot f_c \cdot (T - cc) - 0.5 \cdot d_b} \right)^2 \right) \right)^{0.5} \right)$$

$$\omega = 0.022$$

$$\rho := \omega \cdot \frac{f_c}{f_y} = 1.50 \times 10^{-3}$$

Minimum required reinforcement ratio

$$\rho_{\text{min}} := 0.002$$

$$A_s := \rho \cdot (T - cc - 0.5 \cdot d_b) \cdot b = 253.548 \text{ mm}^2$$

$$A_{s_{\text{min}}} := \rho_{\text{min}} \cdot b \cdot T = 500.000 \text{ mm}^2$$

Required main vertical bars.
Try 10-200 m spacing both sides

$$A_{s_{act_vert}} := \frac{2 \cdot \pi \cdot (10 \text{ mm})^2}{4 \cdot 200} \cdot 1000 = 785.398 \text{ mm}^2$$

Required horizontal bars.
Try 10-250 mm spacing both sides

$$A_{s_{act_horiz}} := \frac{2 \cdot \pi \cdot (10 \text{ mm})^2}{4 \cdot 250} \cdot 1000 = 628.319 \text{ mm}^2$$

$$\left\{ \begin{array}{l} \text{"OK"} \\ \text{"Increase bar dia or reduce spacing"} \end{array} \right. \quad A_{s_{min}} < A_{s_{act_vert}} \text{ and } \max(A_s, A_{s_{min}}) < A_{s_{act_horiz}} = \text{"OK"} \\ \text{otherwise} \end{array}$$

5. Stability Check

Check that the overturning moment and sliding forces are within reasonable engineering limits.

Weight due to soil on toe

$$W_{soil_toe} := Y_{soil} \cdot h_{soil_toe} \cdot \frac{L_f - T}{2} \cdot b = 6.750 \times 10^3 \text{ N}$$

Weight due to footing

$$W_{footing} := 0.9 \cdot Y_{concrete} \cdot L_f \cdot D_f \cdot b = 8.438 \times 10^3 \text{ N}$$

Weight of wall

$$W_{wall} := 0.9 \cdot Y_{concrete} \cdot T \cdot H \cdot b = 1.125 \times 10^4 \text{ N}$$

Weight of soil on heel

$$W_{soil_heel} := Y_{soil} \cdot h_s \cdot \frac{L_f - T}{2} \cdot b = 22500 \text{ N}$$

Weight of water

$$W_{water} := Y_{water} \cdot \frac{L_f - T}{2} \cdot h_w \cdot b = 6250 \text{ N}$$

Weight of surcharge

$$W_{surcharge} := w \cdot \frac{L_f - T}{2} \cdot b = 7500 \text{ N}$$

Total weight

$$W_{total} := W_{soil_toe} + W_{footing} + W_{wall} + W_{soil_heel} + W_{water} + W_{surcharge}$$

$$W_{total} = 62.688 \text{ kN}$$

Check for Overturning

Overturning moment

$$M_{overturning} := Pa1 \cdot L_{base_soil} + Pa2 \cdot L_{base_water} + Pa3 \cdot L_{base_surcharge}$$

$$M_{\text{overturning}} = 10.947 \text{ kN m}$$

Righting moment (for a conservative design, ignore $M_{\text{soil_toe}}$)

From soil on toe $M_{\text{soil_toe}} := W_{\text{soil_toe}} \cdot \frac{L_f - T}{4} = 2.109 \text{ kN m}$

From wall $M_{\text{wall}} := W_{\text{wall}} \cdot \frac{L_f}{2} = 8.438 \text{ kN m}$

From footing $M_{\text{footing}} := W_{\text{footing}} \cdot \frac{L_f}{2} = 6.328 \text{ kN m}$

From soil $M_{\text{soil_heel}} := W_{\text{soil_heel}} \cdot \left(\frac{L_f - T}{2} + T + \frac{L_f - T}{4} \right) = 26.719 \text{ kN m}$

From water $M_{\text{water}} := W_{\text{water}} \cdot \left(\frac{L_f - T}{2} + T + \frac{L_f - T}{4} \right) = 7.422 \text{ kN m}$

From surcharge $M_{\text{surcharge}} := W_{\text{surcharge}} \cdot \left(\frac{L_f - T}{2} + T + \frac{L_f - T}{4} \right) = 8.906 \text{ kN m}$

Total righting moment $M_{\text{total}} := M_{\text{wall}} + M_{\text{footing}} + M_{\text{soil_heel}} + M_{\text{water}} + M_{\text{surcharge}}$

$$M_{\text{total}} = 57.813 \text{ kN m}$$

$$\frac{M_{\text{total}}}{M_{\text{overturning}}} = 5.281$$

Factor of safety $\left\{ \begin{array}{l} \text{"OK for overturning"} \quad \frac{M_{\text{total}}}{M_{\text{overturning}}} > 2 \\ \text{"Increase sizes"} \quad \text{otherwise} \end{array} \right. = \text{"OK for overturning"}$

Check for Sliding

For stability against sliding ensure that $\frac{\text{resisting force}}{\text{sliding force}} > 1.5$

Sliding force $F_{\text{sliding}} := Pa1 + Pa2 + Pa3 = 2.020 \times 10^4 \text{ N}$

Resisting force (passive pressure)
 $W_{\text{soil_toe}}$ is ignored for a conservative design)

$$F_{\text{total}} := W_{\text{wall}} + W_{\text{footing}} + W_{\text{soil_heel}} + W_{\text{water}} + W_{\text{surcharge}}$$

$$F_{\text{total}} = 55.938 \text{ kN}$$

$$\frac{F_{\text{total}}}{F_{\text{sliding}}} = 2.769$$

Factor of safety

$$\left\{ \begin{array}{l} \text{"OK for sliding"} \quad \frac{F_{\text{total}}}{F_{\text{sliding}}} > 1.5 \\ \text{"Increase sizes"} \quad \text{otherwise} \end{array} \right. = \text{"OK for sliding"}$$

6. Check Required Length of Base

Maximum soil pressure

$$q_{\text{max}} := \frac{DL + LL + F_{\text{total}}}{b \cdot L_f} + \frac{6 \cdot (M_n + \text{EMDL} + \text{EMLL})}{b \cdot L_f^2}$$

$$q_{\text{max}} = \frac{P}{A} + \frac{6 \cdot M}{b \cdot d^2} \leq q_{\text{all}}$$

$$q_{\text{max}} = 66.483 \text{ kPa}$$

$$\left\{ \begin{array}{l} \text{"OK for soil bearing"} \quad q_{\text{max}} < q_{\text{all}} \\ \text{"Increase wall footing"} \quad \text{otherwise} \end{array} \right. = \text{"OK for soil bearing"}$$

Maximum ultimate soil pressure (considering 1 m strip)

$$q_{u_{\text{max}}} := \frac{LF_{\text{dead}} \cdot DL + LF_{\text{live}} \cdot LL + LF_{\text{dead}} \cdot (W_{\text{wall}} + W_{\text{footing}}) + LF_{\text{live}} \cdot W_{\text{surcharge}} + LF_{\text{lateral}} \cdot (W_{\text{soil_heel}} + W_{\text{water}})}{b \cdot L_f} + \frac{6 \cdot (LF_{\text{dead}} \cdot \text{EMDL} + LF_{\text{live}} \cdot \text{EMLL} + M_u)}{b \cdot L_f^2}$$

$$q_{u_{\text{max}}} = 101.122 \text{ kPa}$$

$$q_{u_{\text{min}}} := \frac{LF_{\text{dead}} \cdot DL + LF_{\text{live}} \cdot LL + LF_{\text{dead}} \cdot (W_{\text{wall}} + W_{\text{footing}}) + LF_{\text{live}} \cdot W_{\text{surcharge}} + LF_{\text{lateral}} \cdot (W_{\text{soil_heel}} + W_{\text{water}})}{b \cdot L_f} - \frac{6 \cdot (LF_{\text{dead}} \cdot \text{EMDL} + LF_{\text{live}} \cdot \text{EMLL} + M_u)}{b \cdot L_f^2}$$

$$q_{u_{\text{min}}} = 7.711 \times 10^3 \text{ Pa}$$

If q_{u_min} is in tension (+) solve for the required length. Ignore when q_{u_min} is in compression (-).

Eccentricity

$$e := \frac{LF_{dead} \cdot EMDL + LF_{live} \cdot EMLL + M_u}{LF_{dead} \cdot DL + LF_{live} \cdot LL + LF_{dead} \cdot (W_{wall} + W_{footing}) + LF_{live} \cdot W_{surcharge} + LF_{lateral} \cdot (W_{soil_heel} + W_{water})}$$

$$e = 0.215 \text{ m}$$

$$\text{Given } a = \frac{2 \cdot P}{q_{u_max} \cdot b}$$

$$a := \frac{2 \cdot (LF_{dead} \cdot DL + LF_{live} \cdot LL + LF_{dead} \cdot (W_{wall} + W_{footing}) + LF_{live} \cdot W_{surcharge} + LF_{lateral} \cdot (W_{soil_heel} + W_{water}))}{q_{u_max} \cdot b}$$

$$a = 1.614 \text{ m}$$

$$L := 2 \cdot \left(e + \frac{a}{3} \right) = 1.505 \text{ m}$$

$$L := \text{ceil}(10 \cdot L) \cdot 0.1 = 1.600 \text{ m}$$

7. Adequacy of Footing Thickness for Wide Beam Shear

When q_{u_min} is in compression

$$q_c = q_{u_min} + y$$

Solving for y by similar triangles

$$y := \left(\frac{L_f - T}{2} + T + (D_f - cc - 0.5 \cdot d_b) \right) \cdot \frac{q_{u_max} - q_{u_min}}{L_f}$$

$$y = 6.501 \times 10^4 \text{ Pa}$$

$$q_c := q_{u_min} + y = 72.725 \text{ kPa}$$

$$L_d := \frac{L_f - T}{2} - (D_f - cc - 0.5 \cdot d_b) = 456.000 \text{ mm}$$

When q_{u_min} is in tension

$$q_c = y$$

$$V_{u_compression} := \frac{1}{2} \cdot (q_c + q_{u_max}) \cdot L_d \cdot b = 39.637 \text{ kN}$$

Solving for y from similar triangles

$$y := \left(a - \left(\frac{L - T}{2} - (D_f - cc - 0.5 \cdot d_b) \right) \right) \cdot \frac{q_{u_max}}{a}$$

$$y = 6.943 \times 10^4 \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$$

$$q_c := y$$

$$L_d := \frac{L - T}{2} - (D_f - c_c - 0.5 \cdot d_b) = 0.506 \text{ m}$$

$$V_{u_tension} := \frac{1}{2} \cdot (q_c + q_{u_max}) \cdot L_d \cdot b = 43.149 \text{ kN}$$

$$V_{u_govern} := \begin{cases} V_{u_compression} & q_{u_min} > 0 \\ V_{u_tension} & \text{otherwise} \end{cases} = 3.964 \times 10^4 \text{ N}$$

Allowable shear

$$\phi V_c := \phi_{shear} \cdot 0.17 \text{ kg}^{\frac{1}{2}} \cdot \text{m}^{\frac{1}{2}} \cdot \text{s}^{-1} \cdot \sqrt{f_c} \cdot (D_f - c_c - 0.5 \cdot d_b) \cdot 1000$$

$$\phi V_c = 1.219 \times 10^5 \text{ N}$$

$$\begin{cases} \text{"OK for shear"} & V_{u_govern} < \phi V_c \\ \text{"Increase thickness of wall"} & \text{otherwise} \end{cases} = \text{"OK for shear"}$$

8. Check Wall Thickness for Flexure

When q_{u_min} is in compression

$$q_c = q_{min} + y$$

Solving for y from similar triangles

$$y := (q_{u_max} - q_{u_min}) \cdot \left(\frac{L_f - T}{2} + T \right) \cdot \frac{1}{L_f} = 5.449 \times 10^4 \text{ Pa}$$

$$q_c := q_{u_min} + y = 62.201 \text{ kPa}$$

$$M_{u_compression} := q_c \cdot \frac{L_f - T}{2} \cdot 0.5 \text{ m} \cdot \frac{L_f - T}{2} + 0.5 \cdot (q_{u_max} - q_c) \cdot \frac{L_f - T}{2} \cdot \frac{2}{3} \text{ m} \cdot \frac{L_f - T}{2} = 17.217 \text{ kN m}$$

When q_{u_min} is in tension

$$q_c = y$$

Solving for y from similar triangles

$$q_c := \frac{q_{u_{\max}} \cdot \left(a - \frac{L-T}{2} \right)}{a} = 58.841 \text{ kPa}$$

$$M_{u_{\text{tension}}} := q_c \cdot \frac{L-T}{2} \cdot \frac{1}{2} \cdot \frac{L-T}{2} + \frac{1}{2} \cdot (q_{u_{\max}} - q_c) \cdot \frac{L-T}{2} \cdot \frac{2}{3} \cdot \frac{L-T}{2}$$

$$M_{u_{\text{tension}}} = 19.826 \text{ kN m}$$

$$M_{u_{\text{govern}}} := \begin{cases} M_{u_{\text{compression}}} & q_{u_{\min}} > 0 \\ M_{u_{\text{tension}}} & \text{otherwise} \end{cases}$$

$$M_{u_{\text{govern}}} = 17.217 \text{ kN m}$$

$$\text{fsolve}(M_{u_{\text{govern}}} = \phi_{\text{flexure}} \cdot f_c \cdot b \cdot (D_f - cc - 0.5 \cdot d_b)^2 \cdot \omega \cdot (1 - 0.59 \cdot \omega), \omega)[1] = 0.021$$

Theoretical reinforcement ratio required for flexure

$$\omega := 0.85 \cdot \left(1 - \left(\left(1 - \frac{2}{0.85} \cdot m^{-1} \cdot \left(\frac{M_{u_{\text{govern}}}}{\phi_{\text{flexure}} \cdot f_c \cdot (D_f - cc - 0.5 \cdot d_b)^2} \right) \right)^{\frac{1}{2}} \right) \right)$$

$$\omega = 0.021$$

$$\rho := \omega \cdot \frac{f_c}{f_y} = 0.001$$

$$\rho_{\min} := 0.002$$

$$A_s := \rho \cdot 1\text{m} \cdot (D_f - cc - 0.5 \cdot d_b) = 2.49 \times 10^2 \text{ mm}^2$$

$$A_{s_{\min}} := \rho_{\min} \cdot 1\text{m} \cdot D_f = 5.00 \times 10^2 \text{ mm}^2$$

Required Main Bars

Try 10-200mm spacing both sides A_s act

$$A_{s_{\text{act_main_bars}}} := \frac{2 \cdot \pi \cdot (10 \text{ mm})^2 \cdot 1000}{4 \cdot 200} = 785.398 \text{ mm}^2$$

Required Secondary Bars

$$A_{s_{\text{act_secondary_bars}}} := \frac{2 \cdot \pi \cdot (10 \text{ mm})^2 \cdot 1000}{4 \cdot 250} = 628.319 \text{ mm}^2$$

Try 10-250 mm spacing both sides A_s act

$$\text{reinforcement} := \begin{cases} \text{"OK"} & \max(A_s, A_{s_{\min}}) < A_{s_{\text{act_main_bars}}} \text{ and } A_{s_{\min}} < A_{s_{\text{act_secondary_bars}}} \\ \text{"Increase bar diameter or reduce spacing"} & \text{otherwise} \end{cases}$$

reinforcement = "OK"