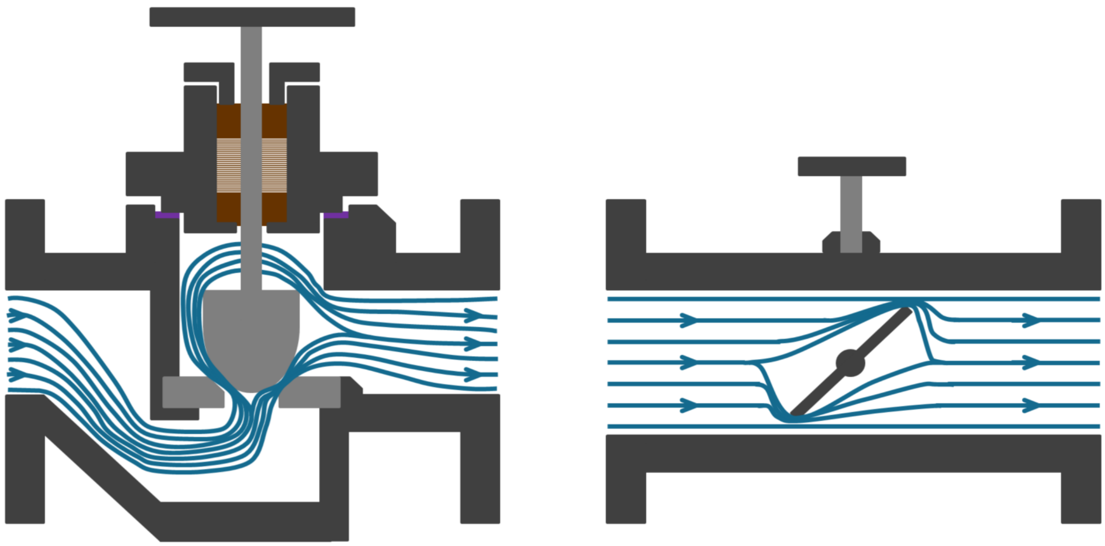
Fundamentals of Valve 2 - Compressible flow

This document is a part of the series of the fundamental calculations related to Valves. In this worksheet, the model defined in the standard ISO 6358 and Valve sizing equation which is defined in ANSI/ISA 75.01 are compared as the basic knowledge of Valve.



References:
- Peter Beater (2007), "Pneumatic Drives - System Design, Modeling and Control", Springer, ISBN 978-3-540-69471-7
- Emerson Automation Solutions (2017), "Control Valve Handbook 5th Edition", Fisher.

1. Calculation for Compressible flow

In this section, 2 types of equations for Compressible flow are compared.

1-1. ISO 6358

The following equation is defined as the standard ISO 6358, and can cover choked flow and subsonic flow of the compressible fluid.

$$Eq_{ISO} := mflow = \begin{cases} p_1 \cdot C \cdot \rho_0 \cdot \sqrt{\dfrac{T_0}{T_1}} \cdot \sqrt{1 - \left(\dfrac{\dfrac{p_2}{p_1} - b}{1 - b} \right)^2} & \dfrac{p_2}{p_1} > b \quad \text{(Subsonic)} \\ p_1 \cdot C \cdot \rho_0 \cdot \sqrt{\dfrac{T_0}{T_1}} & \text{otherwise} \quad \text{(Chocked)} \end{cases}$$

Mass flow rate : $mflow\ \frac{kg}{s}$

Upstream pressure : p_1 **Pa**

Downstream pressure : p_2 **Pa**

Density of air
at reference condition : $\rho_0\ \frac{kg}{m^3}$

Temperature of air
at reference condition : T_0 **K**

Upstream temperature : T_1 **K**

Sonic conductance : $C\ \frac{m^3}{s \cdot Pa}$

Critical pressure ratio : b

1-2. ISA 75.01

The next equation is for the turbulent flow without attached fittings, defined in ISA 75.01.
And as the flow coefficient, C_v is used.

$$Eq_{ISA1} := mflow = \begin{cases} C_v \cdot N_6 \cdot Y \cdot \sqrt{x \cdot p_1 \cdot \rho_1} & x < F_\kappa \cdot x_T \quad \text{(Subsonic)} \\ 0.667 \cdot C_v \cdot N_6 \cdot \sqrt{F_\kappa \cdot x_T \cdot p_1 \cdot \rho_1} & \text{otherwise} \quad \text{(Choked)} \end{cases}$$

Mass flow rate : mflow $\frac{lb}{h}$

Upstream pressure : p_1 psi

Downstream pressure : p_2 psi

Upstream density of air : ρ_1 $\frac{lb}{ft^3}$

Specific hear ratio factor : F_κ

Expansion factor : Y

Ratio of pressure differential to Upstream pressure : x

Flow coefficient : C_v

Pressure differential ratio factor of a control valve without attached fittings at choked flow : x_T

Numerical constant : N_6

Pressure differential ratio is obtained with $x = \frac{p_1 - p_2}{p_1}$

And, if introduce pressure ratio p_r,

$$Eq_{ISA2} := x = 1 - p_r$$

Expansion factor is calculated with the following equation.

$$Eq_{ISA3} := Y = 1 - \frac{x}{3 \cdot F_\kappa \cdot x_T}$$

Specific hear ratio factor is defined with the specific hear ratio.

$$Eq_{ISA4} := F_\kappa = \frac{\kappa}{1.40}$$

2. Comparison with plotting

In the following plot, the comparison of two equations is shown. Air is used as the fluid.

At first, ISO 6358 equation is converted by using pressure ratio p_r.

$$Eq_{ISO\_B} := \text{eval}\left(Eq_{ISO}, \left[\frac{p_2}{p_1} = p_r\right]\right)$$

$$Eq_{ISO\_B} = \text{mflow} = \begin{cases} p_1 \cdot C \cdot \rho_0 \cdot \sqrt{\frac{T_0}{T_1}} \cdot \sqrt{1 - \frac{(p_r - b)^2}{(1-b)^2}} & b < p_r \\ p_1 \cdot C \cdot \rho_0 \cdot \sqrt{\frac{T_0}{T_1}} & \text{otherwise} \end{cases}$$

And, regarding the ISA equation, combine several equations.

$$Eq_{ISA\_all} := \text{subs}\left(Eq_{ISA3}, Eq_{ISA4}, Eq_{ISA2}, Eq_{ISA1}\right)$$

$$Eq_{ISA\_all} = \text{mflow} = \begin{cases} C_v \cdot N_6 \cdot \left(1 - \frac{0.467 \cdot (1 - p_r)}{\kappa \cdot x_T}\right) \cdot \sqrt{(1 - p_r) \cdot p_1 \cdot \rho_1} & 1 - p_r < 0.714 \cdot \kappa \cdot x_T \\ 0.564 \cdot C_v \cdot N_6 \cdot \sqrt{\kappa \cdot x_T \cdot p_1 \cdot \rho_1} & \text{otherwise} \end{cases}$$

Ther following conditions are commonly used for both equations.

Upstream pressure $p_{1\_SI} := 1\ \mathbf{MPa}$

Upstream temperature $T_{1\_K} := 293.15\ \mathbf{K}$

Upstream density

$$\rho_{1\_SI} := \text{ThermophysicalData:-Property}\left(\text{DMASS}, P = p_{1\_SI}, T = T_{1\_K}, \text{Air}\right)$$

$$\rho_{1\_SI} = 11.925\ \frac{\mathbf{kg}}{\mathbf{m}^3}$$

2-1. ISO 6358

The following parameters are applied to the ISO 6358 equation.

Sonic conductance $C := 1.0 \cdot 10^{-7} \frac{m^3}{s \cdot Pa}$

Critical pressure ratio $b := 0.225$

Reference temperature $T_0 := 293.15\ K$

Reference density

$\rho_0 :=$ ThermophysicalData:-Property(DMASS, P = 100 kPa, T = T_0, Air)

$\rho_0 = 1.189 \frac{kg}{m^3}$

Thus, the plot data is obtained with the following.

$p_{ISO} :=$ plot(eval(rhs(Eq_{ISO_B}), [$p_1 = p_{1\_SI}$, $T_1 = T_{1\_K}$, $\rho_1 = \rho_{1\_SI}$]), p_r = 0.3 ..0.999, color = red, legend = "ISO 6358")

2-2. ISA 75.01

The following parameters are applied to the ISA 75.01 equation.

Specific heat ratio $\kappa := 1.4$

Flow coefficient $C_v := 2.44$

Pressure differential ratio factor (without attached fittings) $x_T := 0.775$

Numerical constant $N_6 := 63.3$

Upstream pressure $p_{1\_FPS} := p_{1\_SI} \cdot \left(\frac{1}{\mathbf{psi}} \right)$

Upstream density $\rho_{1\_FPS} := \rho_{1\_SI} \cdot \left(\frac{1}{\frac{\mathbf{lb}}{\mathbf{ft}^3}} \right)$

Thus, the plot data is obtained with the following

$$p_{ISA} := \text{plot}\left(\text{eval}\left(\text{rhs}\left(Eq_{ISA\_all}\right), \left[\rho_1 = \rho_{1\_FPS}, p_1 = p_{1\_FPS}\right]\right) \cdot \left(\frac{\frac{\mathbf{lb}}{\mathbf{h}}}{\frac{\mathbf{kg}}{\mathbf{s}}} \right) \cdot \left(\frac{\mathbf{kg}}{\mathbf{s}} \right), \right.$$
$$\left. p_r = 0.3 \,..0.999, \text{color} = \text{blue}, \text{legend} = \text{"ISA 75.01"} \right)$$

2-3. Comparison

There are slightly difference between the characteristics of 2 equations even if adjusting some parameter values to match.

$\text{plots:-display}\left(\left[p_{ISO}, p_{ISA}\right]\right) =$