Fundamentals - Loss of Internal flow

This document shows the fundamental calculations related to the internal flow. The loss of head or pressure is caused by the friction loss and elevation. And, regarding the friction loss, the Darcy friction factor is determined based on the flow condition, Laminar or Turbulent.
If the fluid of flow is liquid, the elevation cannot be ignored.

As the examples of calculation, this document will demonstrate the flow calculation in some cases after introducing equations for friction factor.

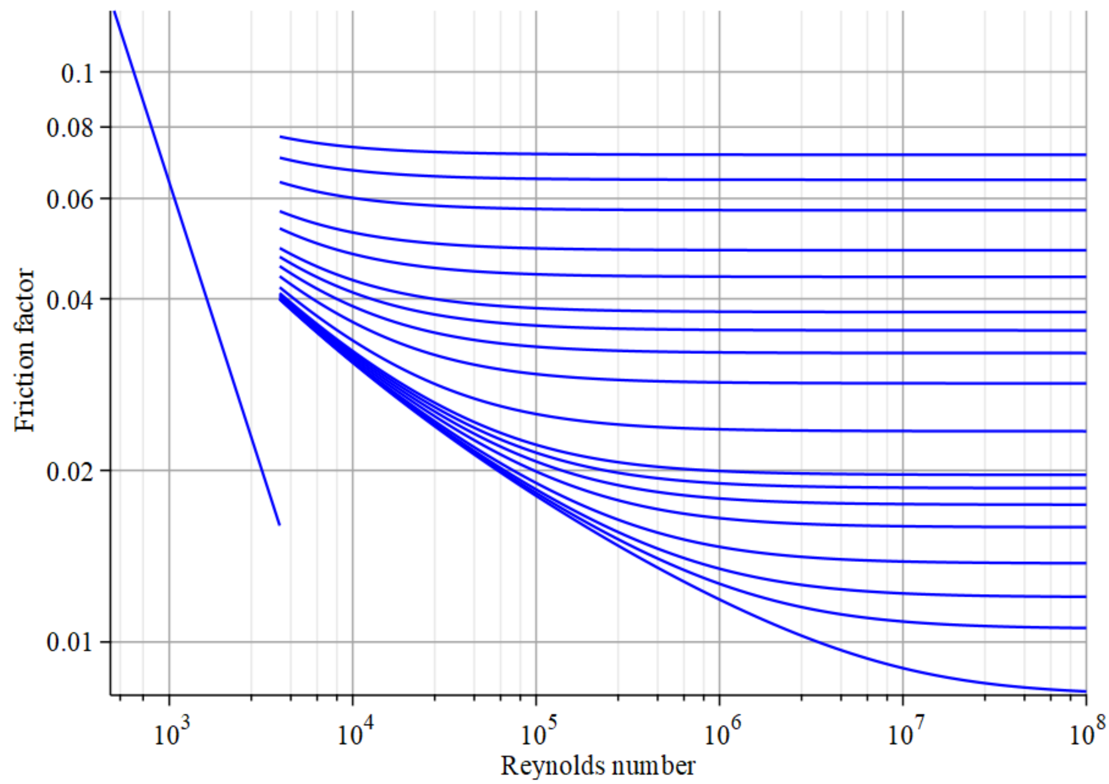


Figure 1 : Moody diagram

1. Basic equations

In this section, basic equations are introduceds.

1-1. Friction loss (Darcy-Weisbach equation)

The friction loss of internal flow is defined with the following general equation.

$$h_f = \lambda \cdot \frac{L}{D_h} \cdot \frac{v^2}{2 \cdot g}$$

h_f **m**: Head of friction loss
λ: Friction factor (Dimensionless)
L **m**: Length of flow
D_h **m**: Hydraulic diameter
g $\frac{m}{s^2}$: Gravity
v $\frac{m}{s}$: Velocity of flow

(*) With SI units

And, the above equation can be transformed by using the relationships below.

Head / Pressure	$h = \frac{p}{\rho \cdot g}$	
Velocity / Volume flow rate	$vflow = v \cdot A$	
Vloume / Mass flow rate	$mflow = \rho \cdot vflow$	

ρ $\frac{kg}{m^3}$: Density
vflow $\frac{m^3}{s}$: Volume flow rate
mflow $\frac{kg}{s}$: Mass flow rate
A m^2: Flow area

Table 1 : Friction loss equations

	Velocity	Volume flow rate	Mass flow rate
Head	$h_f = \lambda \cdot \frac{L}{D_h} \cdot \frac{v^2}{2 \cdot g}$	$h_f = \lambda \cdot \frac{L}{D_h} \cdot \frac{vflow^2}{2 \cdot g \cdot A^2}$	$h_f = \lambda \cdot \frac{L}{D_h} \cdot \frac{mflow^2}{2 \cdot g \cdot \rho^2 \cdot A^2}$
Pressure	$p_f = \lambda \cdot \frac{L}{D_h} \cdot \frac{\rho \cdot v^2}{2}$	$p_f = \lambda \cdot \frac{L}{D_h} \cdot \frac{\rho \cdot vflow^2}{2 \cdot A^2}$	$p_f = \lambda \cdot \frac{L}{D_h} \cdot \frac{mflow^2}{2 \cdot \rho \cdot A^2}$

1-2. Friction factor

For the Darcy-Weisbach equation, the Darcy friction factor is needed to calculate. And, it is difference based on the flow condition, Laminar or Turbulent, and which is defined with Reynolds number Rey.

Laminar

$$\lambda_{lam} = \frac{64}{Rey}$$

If the fuild is oil, $\lambda_{lam} = \frac{75}{Rey}$

Turbulent / Smooth pipe

Blasius' equation $\lambda_{tur} = \frac{0.3164}{Rey^{0.25}}$ $(2000 < Rey < 10^5)$

Nikuradse equation $\lambda_{tur} = 0.0032 + 0.2221 \cdot Rey^{-0.237}$ $(Rey > 10^5)$

Hermann equation $\lambda_{tur} = 0.0054 + 0.396 \cdot Rey^{-0.3}$ $(Rey < 1.5 \cdot 10^5)$

White equation $\frac{1}{\sqrt{\lambda_{tur}}} = 1.8 \cdot \log10\left(\frac{Rey}{6.8}\right)$ $(6000 < Rey < 4 \cdot 10^7)$

Turbulent / Rough pipe (ϵ is the roughness in **m**)

Colebrook-White equation

$$\frac{1}{\sqrt{\lambda_{tur}}} = -2 \cdot \log10\left(\frac{\frac{\epsilon}{D_h}}{3.7} + \frac{2.51}{Rey \cdot \sqrt{\lambda_{tur}}}\right)$$

Swamee-Jain equation

$$\lambda_{tur} = \frac{0.25}{\left(\log10\left(\frac{\frac{\epsilon}{D_h}}{3.7} + \frac{5.74}{Rey^{0.9}}\right)\right)^2}$$

$(5000 < Rey < 10^8)$

$\left(0.000001 < \frac{\epsilon}{D_h} < 0.05\right)$

Brkic

$$\lambda_{tur} = \left(-2 \cdot \log10\left(\frac{2.18 \cdot \beta}{Rey} + \frac{\frac{\epsilon}{D_h}}{3.71} \right) \right)^{-2}$$

$$\beta = \ln\left(\frac{Rey}{1.816 \cdot \ln\left(\frac{1.1 \cdot Rey}{\ln(1 + 1.1 \cdot Rey)} \right)} \right)$$

1-3. Reynolds number

Reynolds number, which is needed to define Laminar or Turbulent, can be obtained with the following equation.

$$Rey = \frac{\rho \cdot v \cdot D_h}{\mu}$$

μ **Pa·s**: Dynamic viscosity

By using the relationship of Velocity / Mass flow rate, it can be transformed.

$$Rey = \frac{mflow \cdot D_h}{\mu \cdot A}$$

1-4. Elevation

If there is the height difference between Inlet and Outlet, sometimes the head of elevation needs to be considered. The equation is the following.

$$h_e = z$$

z **m**: Height difference

By using the relationship of head/pressure, it can be transformed.

$$p_e = z \cdot \rho \cdot g$$

Appendix 1. Comparison of Friction factor of smooth pipe

Generate data of plots with 4 types of friction factor for smooth pipe on Turtubent flow

$$p_{a1\_1} := \text{plots:-loglogplot}\left(\frac{0.3164}{Rey^{0.25}}, Rey = 2000..10^7, \text{legend} = \text{"Blasius"}, \text{color} = \text{brown}\right)$$

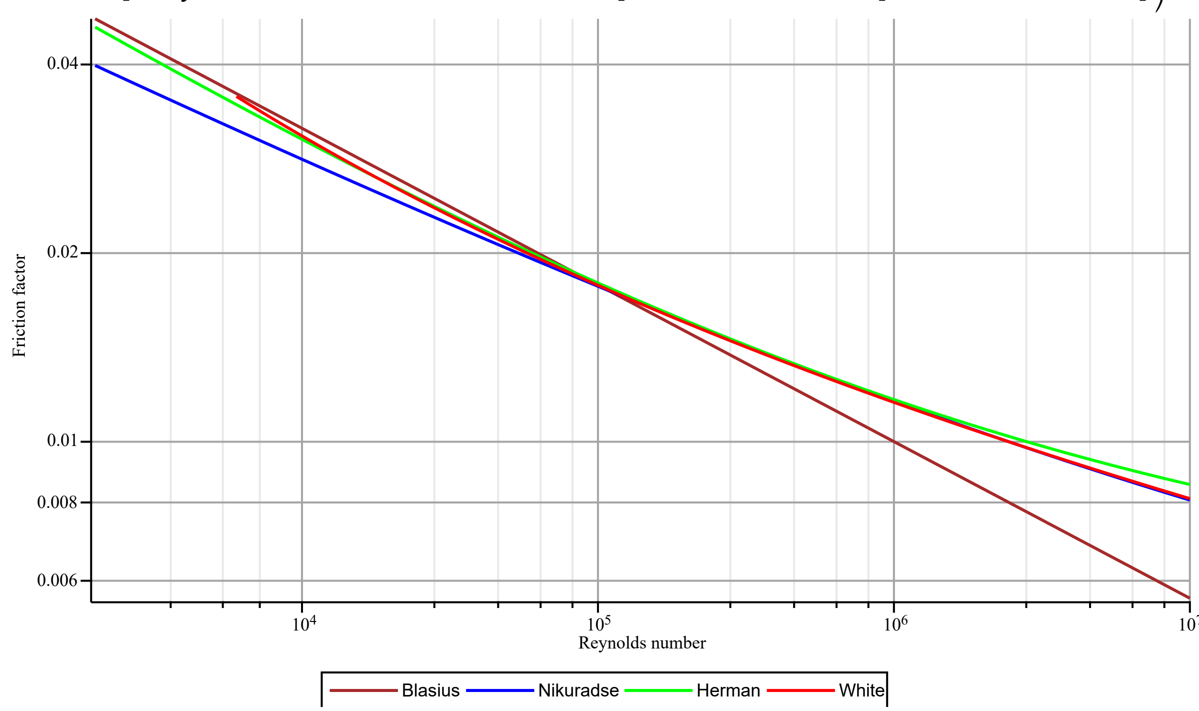
$$p_{a1\_2} := \text{plots:-loglogplot}\left(0.0032 + 0.2221 \cdot Rey^{-0.237}, Rey = 2000..10^7, \text{legend} = \text{"Nikuradse"}, \text{color} = \text{blue}\right)$$

$$p_{a1\_3} := \text{plots:-loglogplot}\left(0.0054 + 0.396 \cdot Rey^{-0.3}, Rey = 2000..10^7, \text{legend} = \text{"Herman"}, \text{color} = \text{green}\right)$$

$$p_{a1\_4} := \text{plots:-loglogplot}\left(\text{solve}\left(\frac{1}{\sqrt{\lambda_{tur}}} = 1.8 \cdot \log10\left(\frac{Rey}{6.8}\right), \lambda_{tur}\right), Rey = 6000..10^7, \text{legend} = \text{"White"}, \text{color} = \text{red}\right)$$

Comparion of friction factors

$$\text{plots:-display}\left(\left[p_{a1\_1}, p_{a1\_2}, p_{a1\_3}, p_{a1\_4}\right], \text{gridlines}, \textit{labels} = \left[\text{"Reynolds number"}, \text{"Friction factor"}\right], \text{labeldirections} = \left[\text{horizontal}, \text{vertical}\right]\right) =$$



Appendix 2. Comparison of Friction factor of rough pipe

Generate data of plots with 4 types of friction factor for smooth pipe on Turbulent flow

Roughness of pipe $\epsilon_{a1} := 0.0001\ \mathbf{m}$

Hydraulics diameter $D_{h\_a1} := 0.1\ \mathbf{m}$

Digits for numerical calculation $Digits := 16$

$$p_{a2\_1} := plots{:}\text{-}loglogplot\left(solve\left(\frac{1}{\sqrt{\lambda_{tur}}} = -2 \cdot \log10\left(\frac{\frac{\epsilon_{a1}}{D_{h\_a1}}}{3.7} + \frac{2.51}{Rey \cdot \sqrt{\lambda_{tur}}}\right), \lambda_{tur}\right), Rey = 4000..10^{8}, legend = \text{"Colebrock-White"}, color = red\right)$$

$$p_{a2\_2} := plots{:}\text{-}loglogplot\left(\frac{0.25}{\left(\log10\left(\frac{\frac{\epsilon_{a1}}{D_{h\_a1}}}{3.7} + \frac{5.74}{Rey^{0.9}}\right)\right)^{2}}, Rey = 4000..10^{8}, legend = \text{"Swamee-Jain"}, color = blue\right)$$

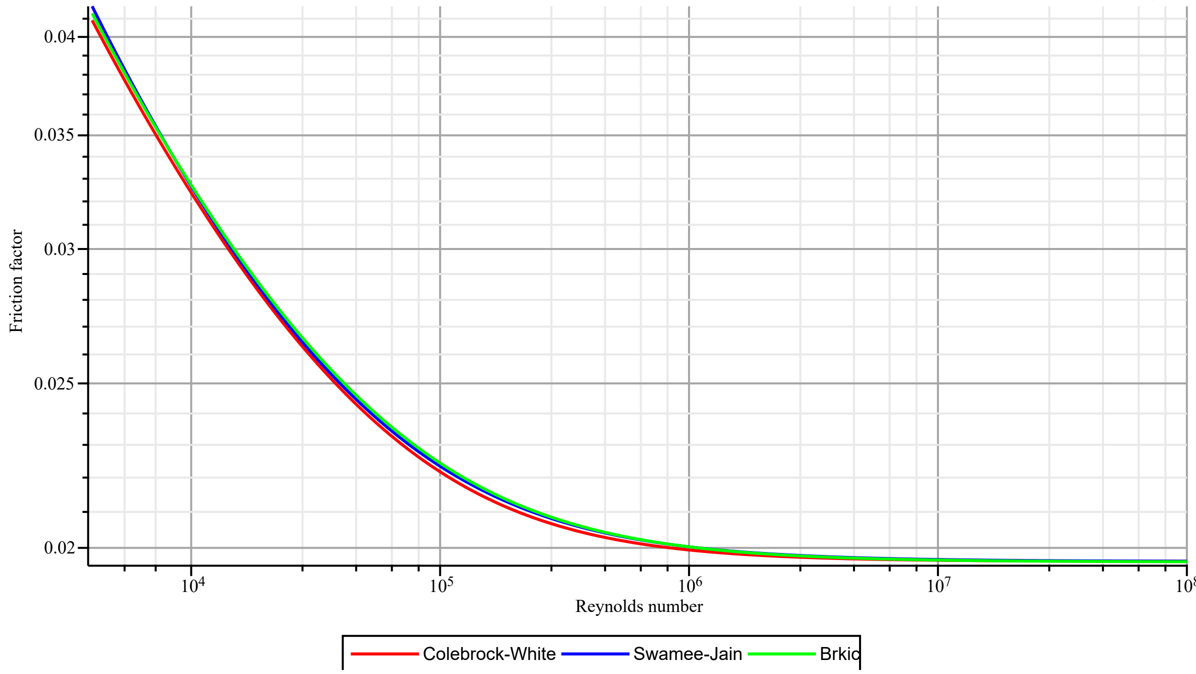
$$Eq_{br} := \left(-2 \cdot \log10\left(\frac{2.18 \cdot \beta}{Rey} + \frac{\frac{\epsilon_{a1}}{D_{h\_a1}}}{3.71}\right)\right)^{-2}$$

$$Eq_{br2} := eval\left(Eq_{br}, \left[\beta = \ln\left(\frac{Rey}{1.816 \cdot \ln\left(\frac{1.1 \cdot Rey}{\ln(1 + 1.1 \cdot Rey)}\right)}\right)\right]\right)$$

$$p_{a2\_3} := plots{:}\text{-}loglogplot\left(Eq_{br2}, Rey = 4000..10^{8}, legend = \text{"Brkic"}, color = green\right)$$

Comparion of friction factors

$$\text{plots:-display}\left(\left[p_{a2\_1}, p_{a2\_2}, p_{a2\_3}\right], \text{gridlines}, \textit{labels} = \left[\text{"Reynolds number"}, \text{"Friction factor"}\right], \text{labeldirections} = \left[\text{horizontal}, \text{vertical}\right]\right) =$$



Appendix 3. Moody diagram

In this appendix, the Moody diagram is plotted.

Laminar

$$p_{a3\_l} := \text{plots:-loglogplot}\left(\frac{64}{Rey}, Rey = 500\,..4000, color = blue\right)$$

Turbulent (Colebrock-White equation)

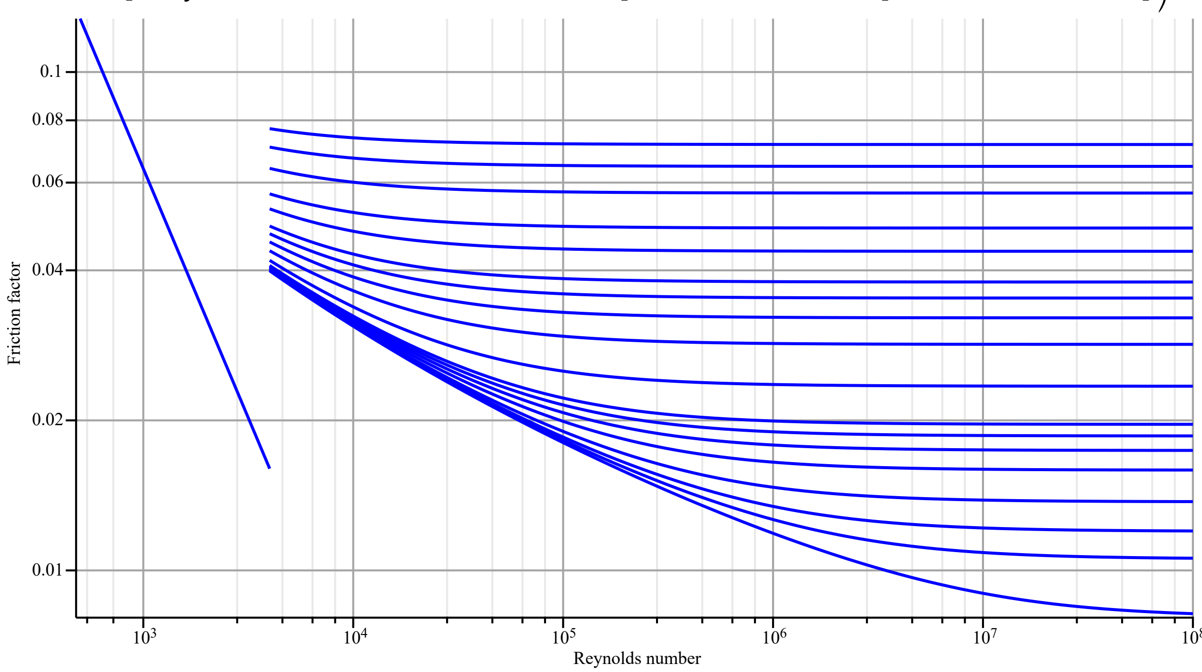
Data set of the relative roghness ($\frac{\epsilon}{D_h}$)

ep_d_list := [0.00001, 0.00005, 0.0001, 0.0002, 0.0004, 0.0006, 0.0008, 0.001, 0.002, 0.004, 0.006, 0.008, 0.01, 0.015, 0.02, 0.03, 0.04, 0.05]

$$p_{a3\_t} := \left[\text{seq}\left(\text{plots:-loglogplot}\left(\text{solve}\left(\text{eval}\left(\frac{1}{\sqrt{\lambda_{tur}}} = -2\cdot\text{log10}\left(\frac{ep\_d}{3.7} + \frac{2.51}{Rey\cdot\sqrt{\lambda_{tur}}}\right), [ep\_d = ep\_d\_list[i]]\right), \lambda_{tur}\right), Rey = 4000\,..10^8, color = blue\right), i = 1\,..nops(ep\_d\_list)\right)\right]$$

Plot the moody diagram

$$\text{plots:-display}\left(\left[p_{a3\_l}, p_{a3\_t}[\ ]\right], gridlines, labels = [\text{"Reynolds number"}, \text{"Friction factor"}], labeldirections = [horizontal, vertical]\right) =$$



3. Case studies

In this section, the mass flow rate is calculated with the equations explained above under several cases. And, in terms of checking the effect of elevation, the result of both with and without elevation care compared.

Equations to solve

Pressure loss (Darcy-Weisbach equation)

$$Eq_p := p_f = \lambda \cdot \frac{L}{D_h} \cdot \frac{mflow^2}{2 \cdot \rho \cdot A^2}$$

Darcy friction factor (Colebrock-White equation is used for Turbulent)

$$Eq_f := \lambda = \begin{cases} \frac{64}{Rey} & Rey < 4000 \\ solve\left(\frac{1}{\sqrt{\lambda_{tur}}} = -2 \cdot \log10\left(\frac{\frac{\epsilon}{D_h}}{3.7} + \frac{2.51}{Rey \cdot \sqrt{\lambda_{tur}}} \right), \lambda_{tur} \right) & otherwise \end{cases}$$

Reynolds number

$$Eq_{rey} := Rey = \frac{mflow \cdot D_h}{\mu \cdot A}$$

Elevation

$$Eq_e := p_e = z \cdot \rho \cdot g$$

Common conditions

Roughness of pipe $\epsilon := 0.0001\ m$

Hydraulics diameter $D_h := 0.1\ m$

Flow area $A := \left(\frac{D_h}{2} \right)^2 \cdot Pi = 0.008\ m^2$

Length of pipe $L := 1\ m$

Height difference between Inlut and Outlet $z := 0.5\ m$

Gravity $g := 9.81 \frac{m}{s^2}$

Digits for numerical calculation Digits := 16

3-1. Air (Low pressure, e.g. for Ventilation application)

In this case, Air is defined as the fluid of flow. And, because of the low density, the effect of elevetion is not big.

Inlet pressure $p_{in\_cs1} := 103325\ Pa$

Outlet pressure $p_{out\_cs1} := 101325\ Pa$

Design pressure drop $dp_{cs1} := p_{in\_cs1} - p_{out\_cs1} = 2000\ Pa$

Design temperature $T_{cs1\_K} := 293.15\ K$

Fluid $Fluid_{cs1} :=$ "Air"

Density of fluid at Inlet

$$\rho_{cs1} := \text{ThermophysicalData:-Property}\left(DMASS, P = p_{in\_cs1}, T = T_{cs1\_K}, Fluid_{cs1}\right)$$

$$\rho_{cs1} = 1.228 \frac{kg}{m^3}$$

Dynamic viscosity of fluid at Inlet

$$\mu_{cs1} := \text{ThermophysicalData:-Property}\left(V, P = p_{in\_cs1}, T = T_{cs1\_K}, Fluid_{cs1}\right)$$

$$\mu_{cs1} = 1.821 \times 10^{-5}\ Pa \cdot s$$

Objective equation (without elevation) by combining equations

$$Eq_{obj\_cs1} := eval\left(Eq_p, eval\left(Eq_f, Eq_{rey}\right)\right)$$

Assgin the value of parameters

$$Eq_{cs1} := eval\left(Eq_{obj\_cs1}, \left[\mu = \mu_{cs1}, \rho = \rho_{cs1}, p_f = dp_{cs1}\right]\right)$$

Obtain the mass flow rate

$$mflow_{cs1} := fsolve\left(Eq_{cs1}, mflow\right) = 1.231 \frac{kg}{s}$$

Objective equation (with elevatin) by adding the elevation equation

$$Eq_{obj\_cs2} := lhs\left(Eq_{obj\_cs1}\right) = rhs\left(Eq_{obj\_cs1}\right) - rhs\left(Eq_e\right)$$

Assign the value of parameters

$$Eq_{cs2} := eval\left(Eq_{obj\_cs2}, \left[\mu = \mu_{cs1}, \rho = \rho_{cs1}, p_f = dp_{cs1}\right]\right)$$

Obtain the mass flow rate

$$mflow_{cs2} := fsolve\left(Eq_{cs2}, mflow\right) = 1.233 \frac{kg}{s}$$

Change of mass flow rate by elevation

$$\frac{mflow_{cs2} - mflow_{cs1}}{mflow_{cs1}} = .15\%$$

Thus, there are only 0.15 % effect on the mass flow rate.

3-2. Water

In this case, Air is defined as the fluid of flow. And, because of the low density, the effect of elevetion is not big.

Inlet pressure $p_{in\_cs3} := 151325\ \mathbf{Pa}$

Outlet pressure $p_{out\_cs3} := 101325\ \mathbf{Pa}$

Design pressure drop $dp_{cs3} := p_{in\_cs3} - p_{out\_cs3} = 50000\ \mathbf{Pa}$

Design temperature $T_{cs3\_K} := 293.15\ \mathbf{K}$

Fluid $Fluid_{cs3} :=$ "Water"

Density of fluid at Inlet

$$\rho_{cs3} := \text{ThermophysicalData:-Property}\left(\text{DMASS}, P = p_{in\_cs3}, T = T_{cs3\_K}, Fluid_{cs3}\right)$$

$$\rho_{cs3} = 998.230\ \frac{\mathbf{kg}}{\mathbf{m}^3}$$

Dynamic viscosity of fluid at Inlet

$$\mu_{cs3} := \text{ThermophysicalData:-Property}\left(\text{V}, P = p_{in\_cs3}, T = T_{cs3\_K}, Fluid_{cs3}\right)$$

$$\mu_{cs3} = 0.001\ \mathbf{Pa \cdot s}$$

Objective equation (without elevation) by combining equations

$$Eq_{obj\_cs3} := eval\left(Eq_p, eval\left(Eq_f, Eq_{rey}\right)\right)$$

Assgin the value of parameters

$$Eq_{cs3} := eval\left(Eq_{obj\_cs3}, \left[\mu = \mu_{cs3}, \rho = \rho_{cs3}, p_f = dp_{cs3}\right]\right)$$

Obtain the mass flow rate

$$mflow_{cs3} := fsolve\left(Eq_{cs3}, mflow\right) = 176.461\ \frac{kg}{s}$$

Objective equation (with elevatin) by adding the elevation equation

$$Eq_{obj\_cs4} := lhs\left(Eq_{obj\_cs3}\right) = rhs\left(Eq_{obj\_cs3}\right) - rhs\left(Eq_e\right)$$

Assign the value of parameters

$$Eq_{cs4} := eval\left(Eq_{obj\_cs4}, \left[\mu = \mu_{cs3}, \rho = \rho_{cs3}, p_f = dp_{cs3}\right]\right)$$

Obtain the mass flow rate

$$mflow_{cs4} := fsolve\left(Eq_{cs4}, mflow\right) = 184.929\ \frac{kg}{s}$$

Change of mass flow rate by elevation

$$\frac{\left(mflow_{cs4} - mflow_{cs3}\right)}{mflow_{cs3}} = 4.80\%$$

Thus, there are 4.8 % effect on the mass flow rate.