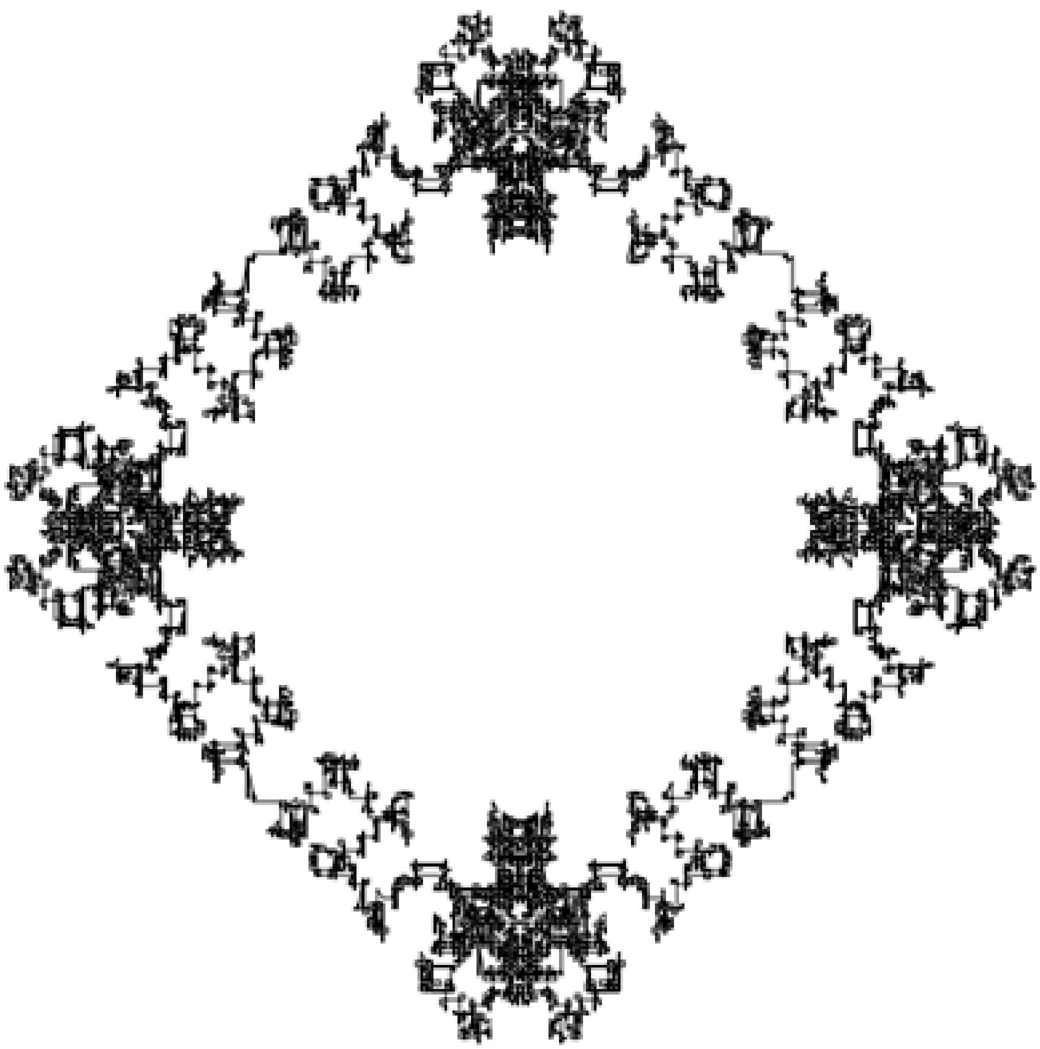
Medallion and Frieze Patterns

Introduction

Bruijn (1991) describes a sequence that can be used to generate visualizations that look like medallions and friezes.



For each step in the sequence, two numbers (each of which can be either 0 or 1) are generated. These determine if a step is up, down, left or right.

00	Left
10	Up
01	Down
11	Right

Each step k can be hence be encoded as a complex number z_k, parameterized with respect to two integers p and q.

$$z_k = z_{k-1} + \frac{(1+i)}{2}\,\delta(2k-1) + \frac{1-i}{2}\,\delta(2k)$$

$$\delta = n \rightarrow \begin{cases} 1 & \frac{n\cdot(n-1)}{2}\cdot p \bmod 2\cdot q < q \\ -1 & \textit{otherwise} \end{cases}$$

where 0 < p < 2 q.

- An odd p results in a closed shaped called a medallion.
- An even p, however, results in an open pattern called a frieze. These are horizontal patterns that repeat in one direction.

Here, we implement the algorithm in Maple, and reproduce the visualizations from Bruijn (1991)

Reference:

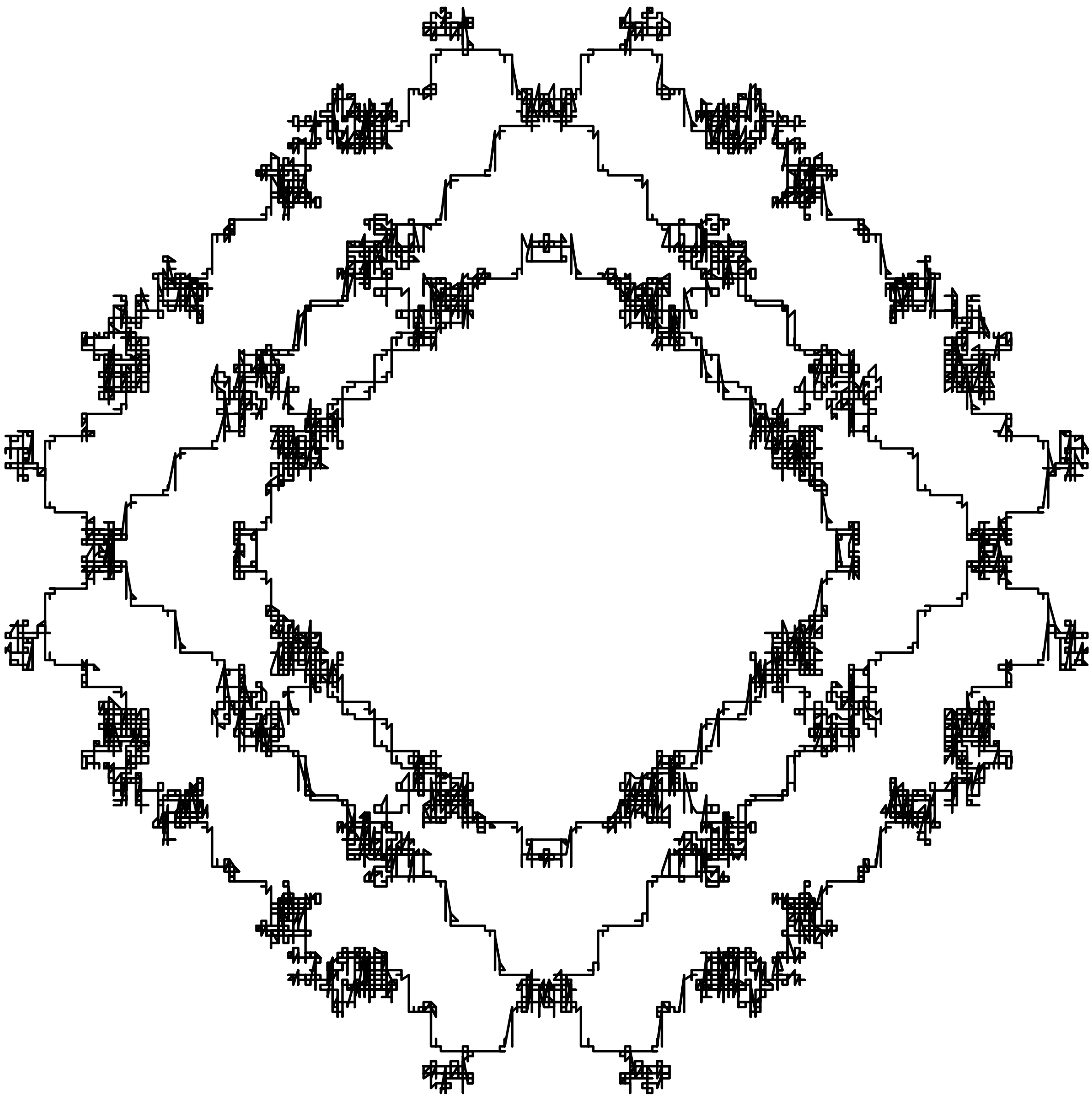
Arithmetical medallions and friezes, Nieuw Archief Wiskunde, de Bruijn, N. G., (4) vol 9 (1991) 339 -350

Code

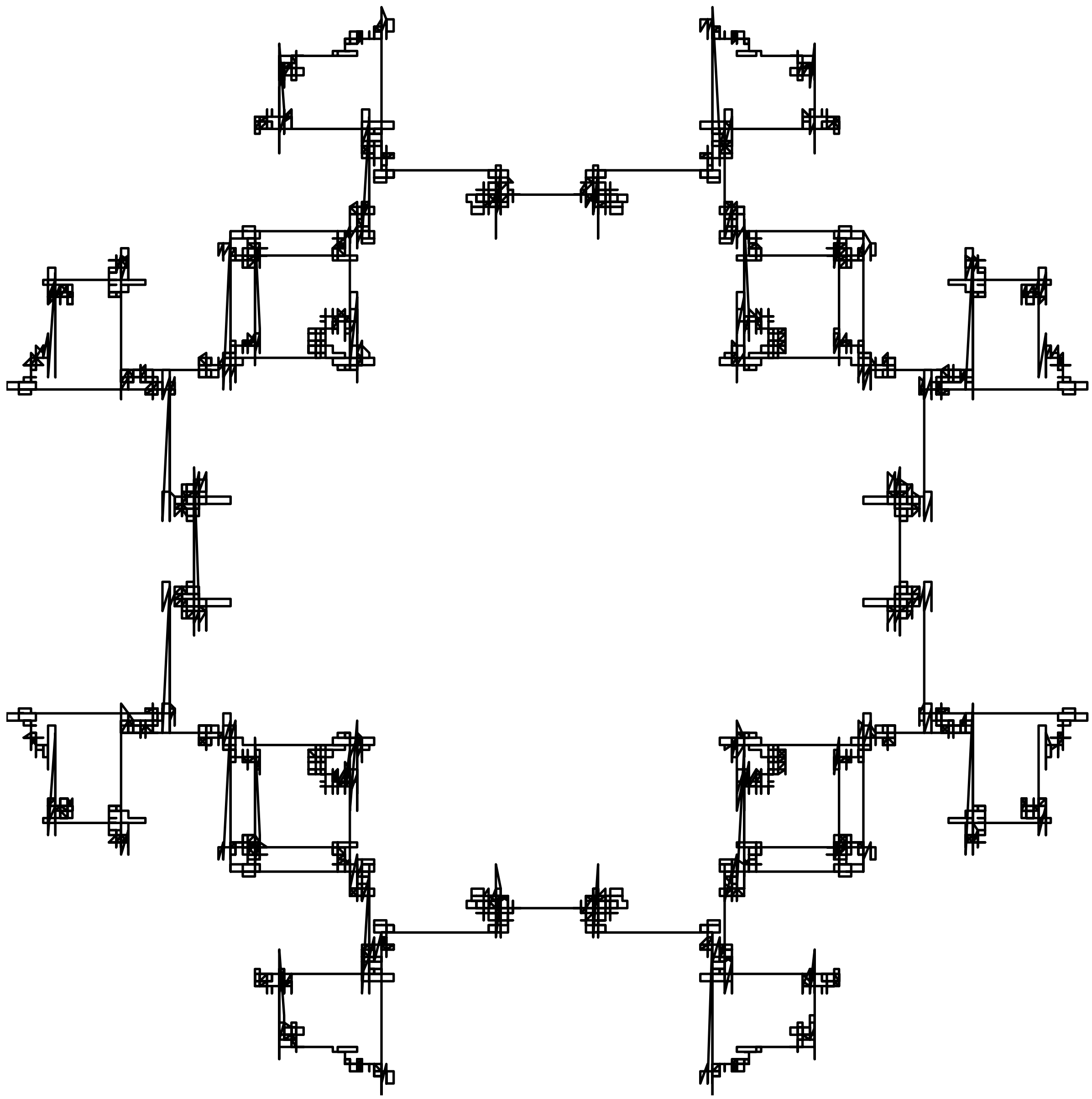
```
f := proc(p, q)
   local N, z, delta, k:
   delta := n -> piecewise(n * (n - 1) / 2 * p mod 2 * q < q
,
     1, -1):
   N := 2 * q + 1:
   z := Vector(N + 1, 'datatype' = complex[8]):
   z[1] := 0 + 0 * I:
   for k from 2 to N do
      z[k] := z[k - 1] + (1 + I) / 2 * delta(2 * k + 1) +
          (1 - I) / 2 * delta(2 * k );
   end do:
   return plot(Re(z), Im(z), 'scaling' = 'constrained',
     'axes' = 'none', 'thickness' = 0, 'color' = 'black',
     'size' = [600, 600], 'background' = 'white')
end proc:
```

Visualizations

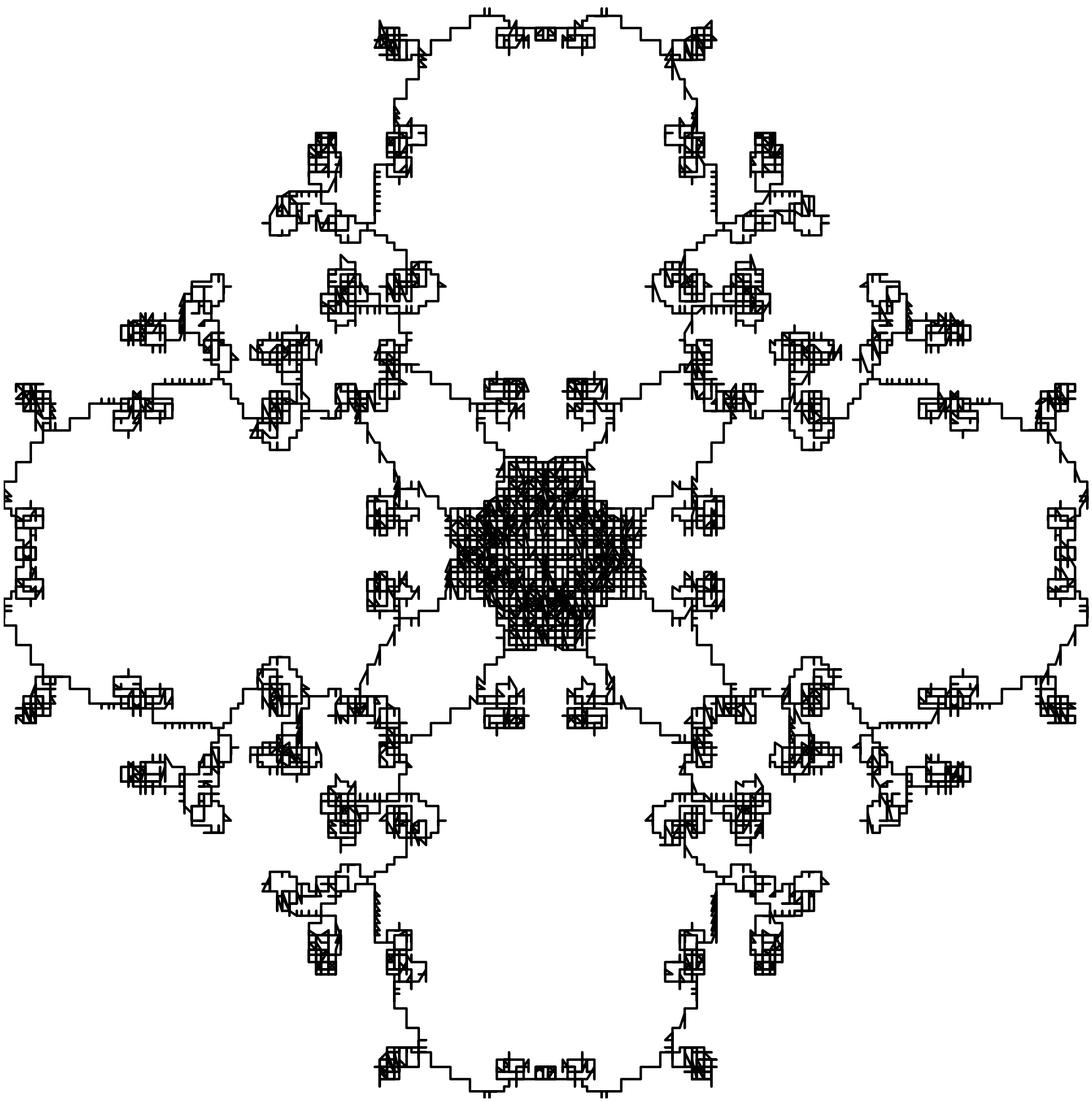
$f(501, 15001)$



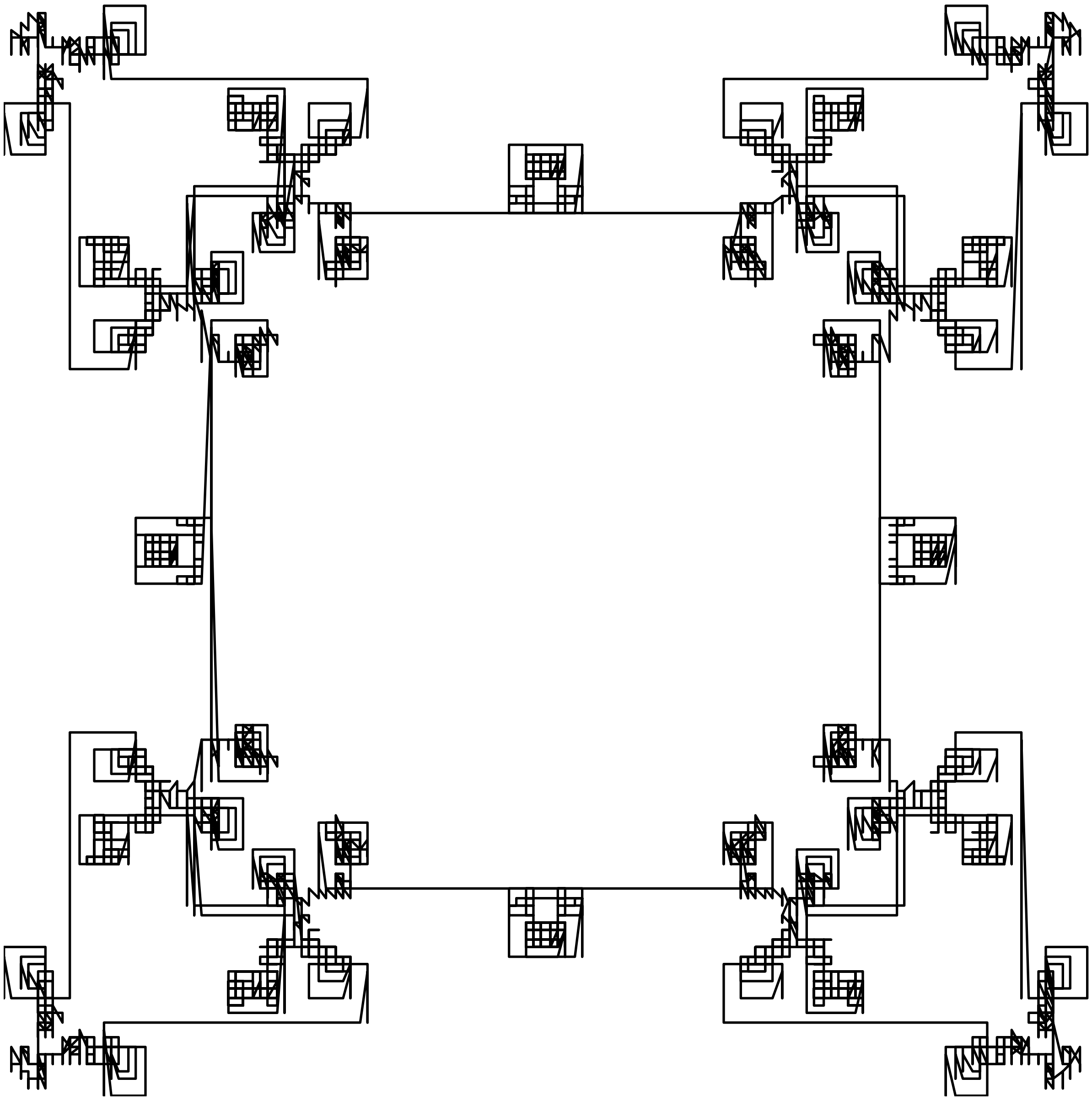
$f(23,8819)$



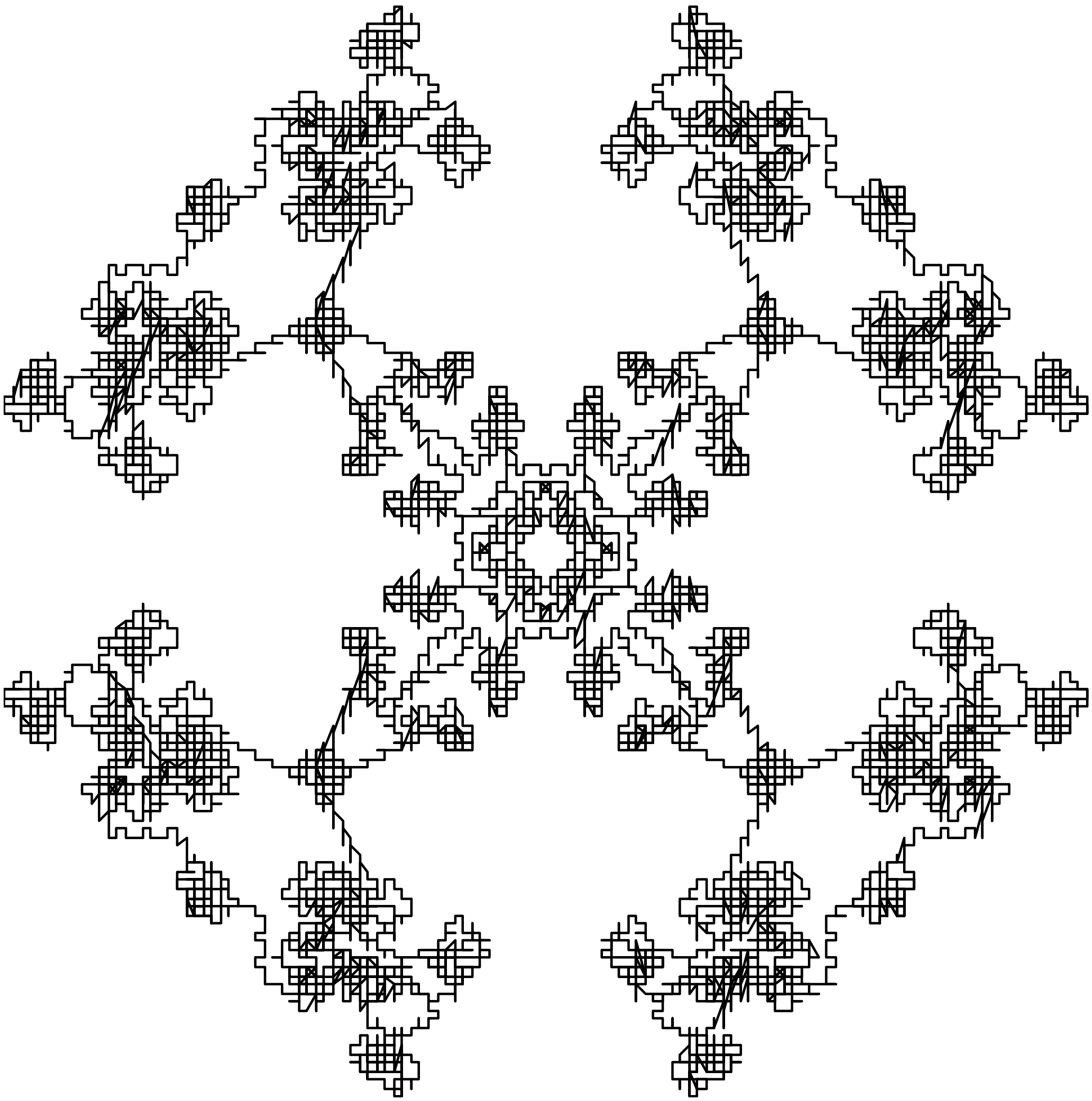
$f(\,2673, 13337\,)$



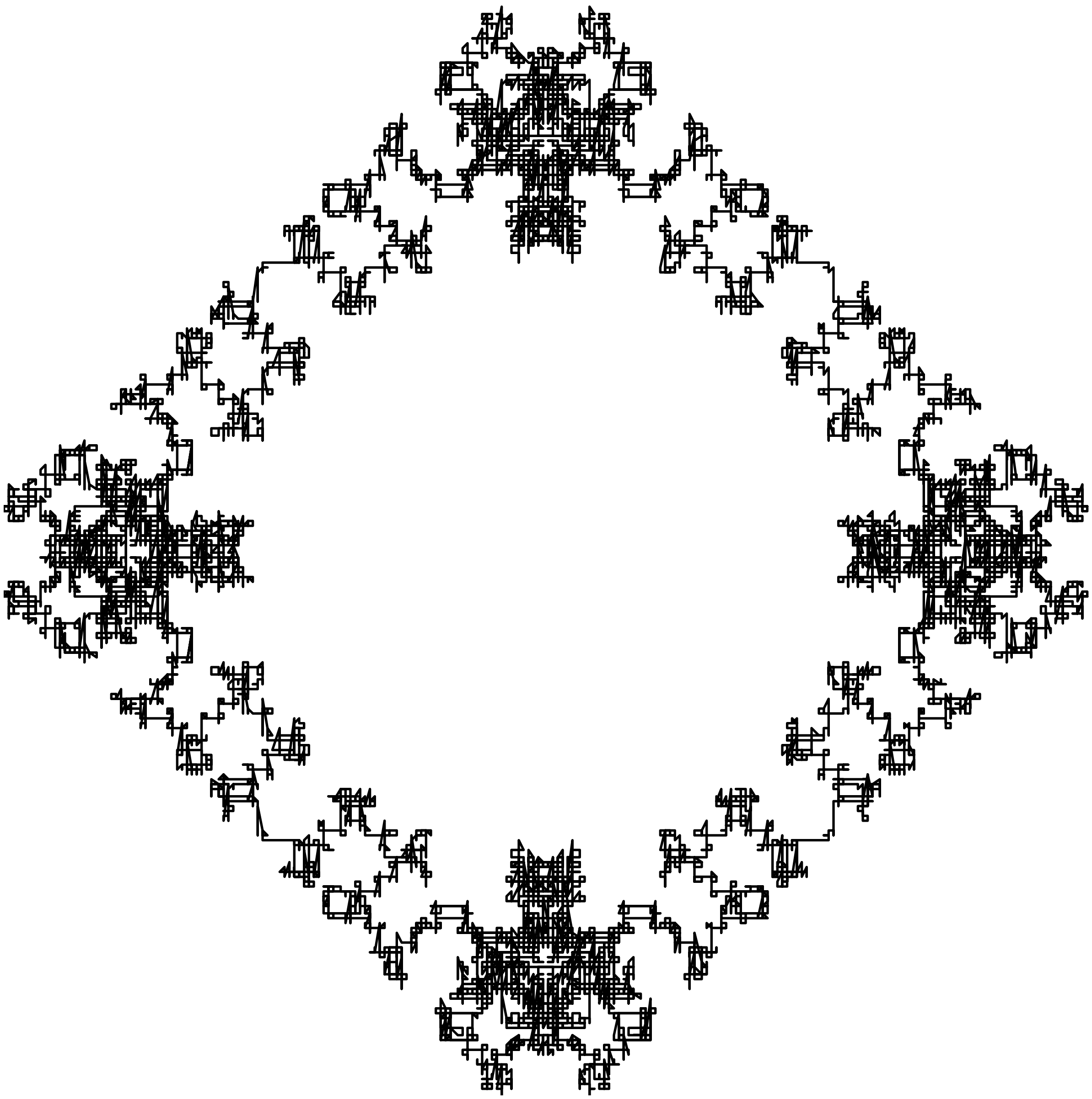
$f(7753, 7759)$



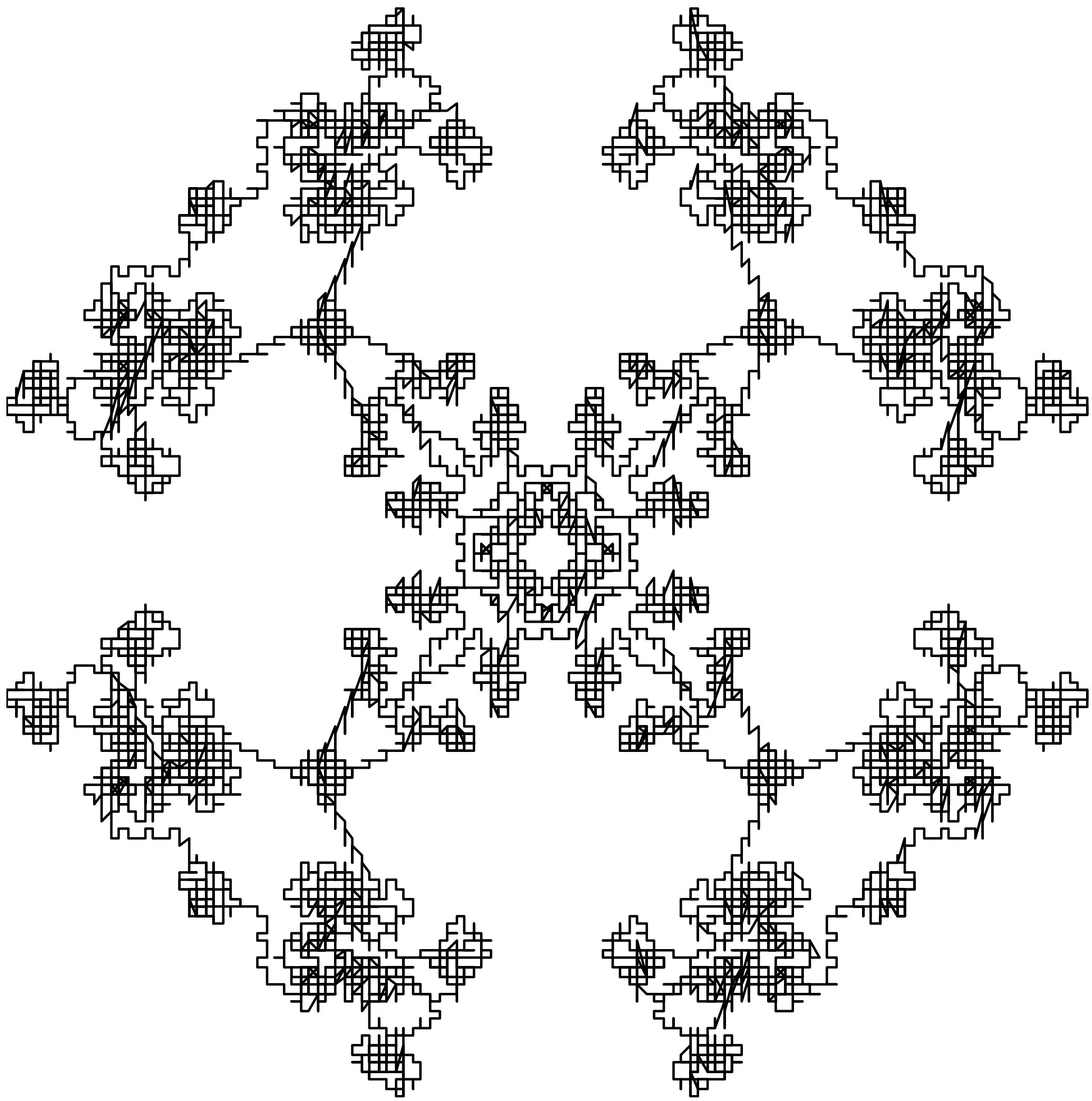
$f(4751, 7911)$



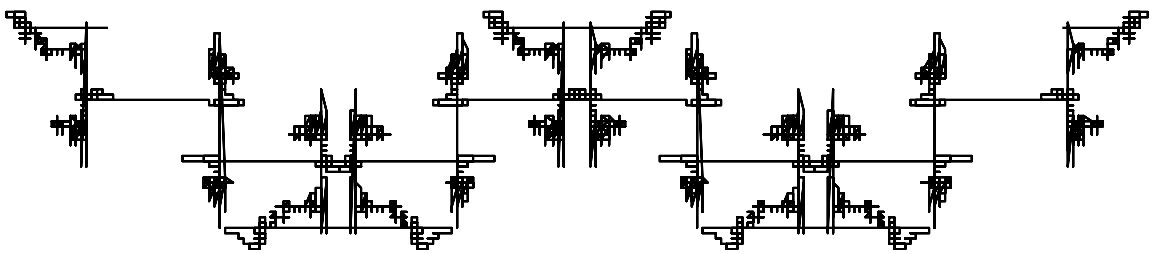
$f(501, 15067)$



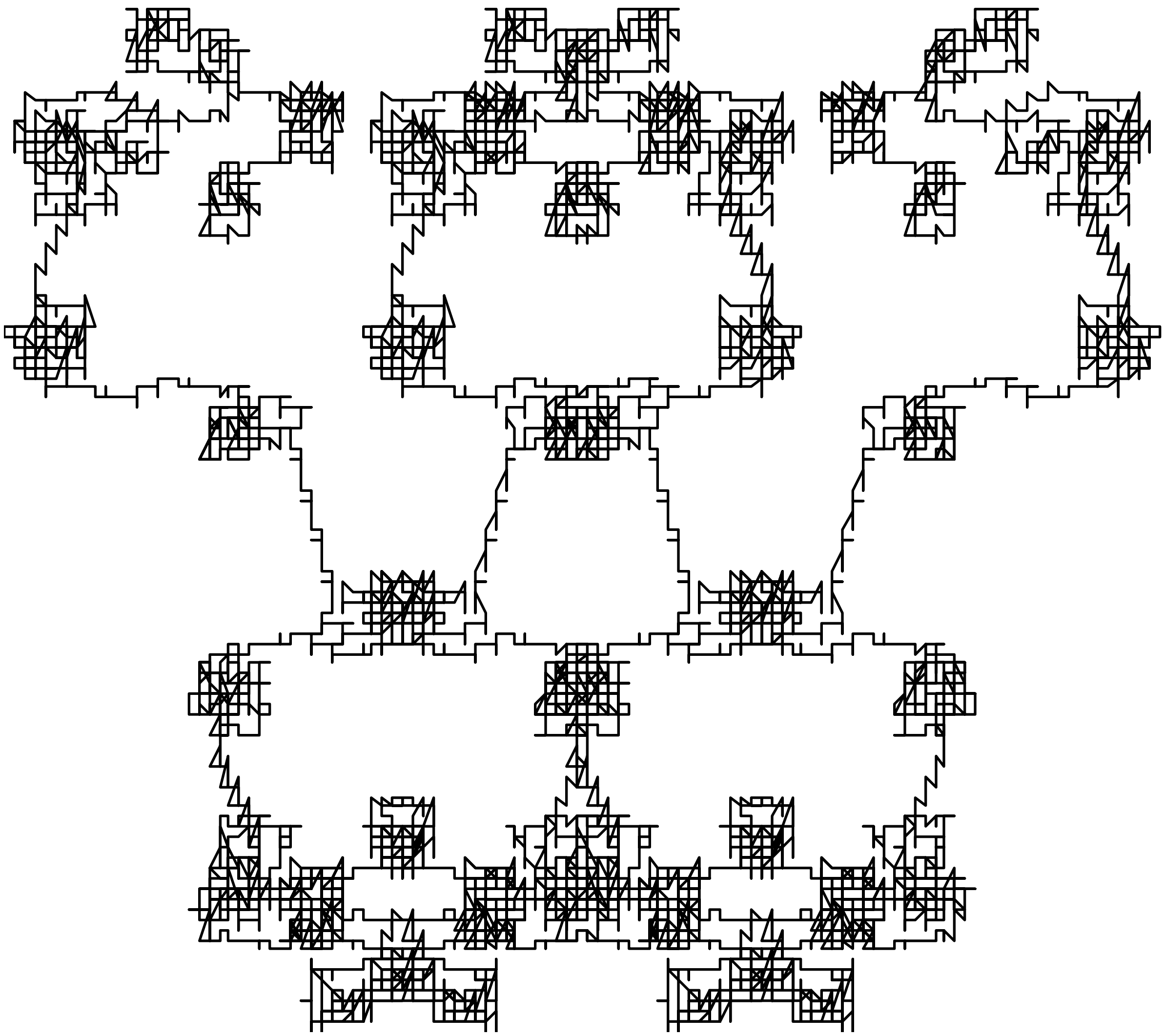
$f(4751, 7911)$



$f(6,5003)$



$f(1002, 7001)$



$f(3346, 13597)$