

Financial Markets

Topics on Commodities Futures Trading

Session # 27

Alejandro Reynoso

▼ 1. Introduction

▼ 1.1 History

In the 1840s, Chicago had become a commercial center with railroad and telegraph lines connecting it with the East. Around this same time, the McCormick reaper was invented which eventually led to higher wheat production. Midwest farmers came to Chicago to sell their wheat to dealers who, in turn, shipped it all over the country.

He brought his wheat to Chicago hoping to sell it at a good price. The city had few storage facilities and no established procedures either for weighing the grain or for grading it. In short, the farmer was often at the mercy of the dealer.

1848 saw the opening of a central place where farmers and dealers could meet to deal in "spot" grain - that is, to exchange cash for immediate delivery of wheat.

The futures contract, as we know it today, evolved as farmers (sellers) and dealers (buyers) began to commit to future exchanges of grain for cash. For instance, the farmer would agree with the dealer on a price to deliver to him 5,000 bushels of wheat at the end of June. The bargain suited both parties. The farmer knew how much he would be paid for his wheat, and the dealer knew his costs in advance. The two parties may have exchanged a written contract to this effect and even a small amount of money representing a "guarantee."

Such contracts became common and were even used as collateral for bank loans. They also began to change hands before the delivery date. If the dealer decided he didn't want the wheat, he would sell the contract to someone who did. Or, the farmer who didn't want to deliver his wheat might pass his obligation on to another farmer. The price would go up and down depending on what was happening in the wheat market. If bad weather had come, the people who had contracted to sell wheat would hold more valuable contracts because the supply would be lower; if the harvest were bigger than expected, the seller's contract would become less valuable. It wasn't long before people who had no intention of ever buying or selling wheat began trading the contracts. They were speculators, hoping to buy low and sell high or sell high and buy low.

▼ 1.2 Futures Contracts

A cash commodity must meet three basic conditions to be successfully traded in the futures market:

It has to be standardized and, for agricultural and industrial commodities, must be in a basic, raw, unprocessed state. There are futures contracts on wheat, but not on flour. Wheat is wheat (although different types of wheat have different futures contracts). The miller who needs a wheat futures to help him avoid losing money on his flour transactions with customers wouldn't need a flour futures. A given amount of wheat yields a given amount of flour and the cost of converting wheat to flour is fairly fixed, hence predictable.

Perishable commodities must have an adequate shelf life, because delivery on a futures contract is deferred.

The cash commodity's price must fluctuate enough to create uncertainty, which means both risk and potential profit.

Most exchange trading floors are divided into pits (or rings) where traders stand facing one another. These are more or less shallow octagonal areas with raised steps around the edge. Each pit is designated for trading one or more futures contracts. For instance, at the Chicago Board of Trade (CBOT) there are large pits for trading T-bonds, soybean, and corn futures among many others. The Commodities Exchange Center in New York houses more than one futures exchange. There you will find trading pits for such diverse commodities as coffee, sugar, frozen orange juice, cocoa, gold, cotton, and heating oil.

Every futures exchange is set up in about the same way. Like the stock exchanges, the people trading on the floor must be members of the exchange itself. The members support the exchange by dues and assessments. Non-members - average investors, for instance - trade through brokerage firms whose officers or partners hold memberships.

The exchange provides the place to trade and support facilities, such as phones and price-reporting and dissemination systems. It does not set prices or buy or sell for itself. However, its employees scrutinize operations and strictly enforce exchange rules and federal commodity trading regulations.

Unlike a stock, which represents equity in a company and can be held for a long time, if not indefinitely, futures contracts have finite lives. They are primarily used for hedging commodity price-fluctuation risks or for taking advantage of price movements, rather than for the buying or selling of the actual cash commodity. The word "contract" is used because a futures contract requires delivery of the commodity in a stated month in the future unless the contract is liquidated before it expires.

The buyer of the futures contract (the party with a long position) agrees on a fixed purchase price to buy the underlying commodity (wheat, gold or T-bills, for example) from the seller at the expiration of the contract. The seller of the futures contract (the party with a short position) agrees to sell the underlying commodity to the buyer at expiration at the fixed sales price. As time passes, the contract's price changes relative to the fixed price at which the trade was initiated. This creates profits or losses for the trader.

In most cases, delivery never takes place. Instead, both the buyer and the seller, acting independently of each other, usually liquidate their long and short positions before the contract expires; the buyer sells futures and the seller buys futures.

Arbitrageurs in the futures markets are constantly watching the relationship between cash and futures in order to exploit such mispricing. If, for example, an arbitrageur realized that gold futures in a certain month were overpriced in relation to the cash gold market and/or interest rates, he would immediately sell those contracts knowing that he could lock in a risk-free profit. Traders on the floor of the exchange would notice the heavy selling activity and react by quickly pushing down the futures price, thus bringing it back into line with the cash market. For this reason, such opportunities are rare and fleeting. Most arbitrage strategies are carried out by traders from large dealer firms. They monitor prices in the cash and futures markets from "upstairs" where they have electronic screens and direct phone lines to place orders on the exchange floor.

▼ 2. Term Structure: Backwardation and Contango

▼ 2.1 Context

While the word contango may sound mysterious, it is used to describe a fairly normal pricing situation in futures. **A market is said to be in contango when the forward price of a futures contract is above the expected future spot price. Normal backwardation, which is essentially the opposite of contango, occurs when the forward price of a futures contract is below the expected future spot price. Because contango and backwardation are known states in the market, traders can employ strategies that attempt to exploit them. Contango and backwardation are frequently seen in commodity markets where certain factors prompt the price discrepancy between expected future spot prices and the price of futures contracts.** Here, we will introduce the two market states of contango and normal backwardation, explain why they happen and how they affect futures traders.

A brief review of exactly what a futures contract is can be helpful when trying to understand contango and backwardation. A futures contract is a legally binding agreement to buy or sell a specified financial instrument or physical commodity at a predetermined price in the future. The buyer of a futures contract (who is long the contract) expects price to increase, while the seller (who is short the contract) anticipates that price will decrease. A futures contract is an obligation to do something in the future.

Some futures contracts are settled in cash, while others call for physical delivery. Many commodity futures are physically delivered; however, market participants can hedge or speculate in the contract without ever taking physical delivery by entering an offsetting position before the contract expiration date.

Commodities are bought and sold on two separate but associated markets: The cash market, which involves the buying and selling of physical commodities, and the futures market, which involves the buying and selling of a future obligation. In a cash market, transactions are settled on the spot (thus the name "spot" market), meaning that the exchange between the buyer and seller takes place in the present. An instrument's spot price is its market price, or the price at which it could be bought or sold today (often the futures nearest expiration is considered the spot

futures).

In a futures market, on the other hand, any exchange between the buyer and seller takes place at some predetermined time in the future. The various futures contracts (for example, the February 2013, March 2013 and April 2013 light sweet crude oil futures contracts) state the price that will be paid and the date of delivery. Few contracts result in physical delivery because most are closed with offsetting positions before expiration

A futures curve is a graphical representation of the current prices for the various delivery dates of a particular instrument. A trader can graph a futures curve for a specific instrument by plotting contract expiration dates along an x-axis with the corresponding futures prices along a y-axis. The current price is the spot price (the price at which the contract could be bought and sold today), and the various futures contracts' prices are plotted going forward in time. “

A futures curve that shows prices that increase as time moves forward is called a normal curve, sometimes referred to as a normal market. This type of curve reflects that the cost to carry increases with longer expiration. In general, traders are willing to pay a premium to avoid the costs associated with transporting, storing and insuring a commodity; therefore, the furthest-out contracts typically are priced higher. A normal futures curve appears upward sloping.

An inverted curve, or inverted market, on the other hand, exists when the prices for faraway deliveries are below the current spot price. Prices for the commodity may be higher today because of a temporary shortage in the cash market brought on by a variety of factors such as weather, natural disaster, war or another geopolitical event.

For example, if a Gulf Coast hurricane disrupts oil refinery production, near-term contracts may be priced higher than those with further-out expirations. Similarly, silver might be in tight supply because investors with physical silver are holding on to it; therefore, the price of the current contract may be higher than later ones. The prices for future deliveries will fall because the supply disruption is expected to end. When these prices are plotted on a graph, the resulting curve appears downward sloping

Patterns over time

While a futures curve presents a snapshot in time — a fixed representation of futures prices against contract maturities — contango and backwardation are observed over time and exist depending on whether futures prices are rising or falling.

Contango and normal backwardation are influenced by differences in the futures price and the spot price for a given commodity — a difference known as the “basis.”

The price of a futures contract — whether it is above or below the spot price — will converge to the spot price as its expiration date approaches. This is called convergence. The futures price and spot price converge because of arbitrage, supply and demand. Here's an example of how it works:

- Assume silver futures are trading above the spot price.
- Traders short silver futures and buy the underlying to take advantage of the arbitrage opportunity.
- The shorting of the futures contracts increases the supply of contracts, which results

in a drop in futures prices (more supply = lower prices).

- The buying of the underlying increases the demand, which results in an increase in spot prices.
- As this continues, futures and spot prices eventually converge.

If a contract is above the spot price, price eventually will move down to be in line with the spot price. Because price must converge with the spot price upon expiration, contango implies that futures prices must fall over time.

A market is said to be in backwardation when the futures price is less than the expected future spot price. Again, because price must converge with the spot price as expiration draws near, backwardation implies that the futures price must rise over time.

The most recent and severe example occurred in the crude oil market and affected the long-only indexes. One of the positives of investing in commodities is that the normal state of backwardation puts the wind at the back of a long-only investment. Investors would make profits as they rolled from the near month to a further out contract. However, contango conditions negatively affected this strategy and placed a penalty each time the index rolled into a more expensive contract.

In general, speculators often are long if a contract is trading in backwardation, and short if it is trading in contango. Keep in mind, however, these market states can and do change. One of the more well-known examples of the potential negative effects of contango and backwardation is the case of Metallgesellschaft AG (MG), a former German metals and oils conglomerate.

In late 1993 and early 1994, MG's American affiliate, MG Refining and Marketing Inc. (MGRM), reported losses on positions in energy futures and swaps. The company had been using a hedging system that relied on normal backwardation markets for profits. What the company did not expect was a shift to contango, which ultimately led to losses of about \$1.3 billion. After MGRM's catastrophic losses, a \$1.9 billion rescue effort led by more than 100 German and international banks prevented MG from going into bankruptcy.

Contango and backwardation, however, are not necessarily good reasons to stay out of a market. Investors and traders should be aware of these dynamic market states and understand that just because a market is in backwardation right now, doesn't mean that it won't soon be trading in contango.

Contango and backwardation, while exotic sounding, are normal conditions in futures markets. Investors and traders can maintain awareness of these market states by evaluating the spot price of an underlying and the prices of near and far futures contracts

▼ 2.2 Arbitrages in a Contango Market

Here, we are going to introduce a strategy that could help us exploit how to profit from the cost of carry in the futures market. We all know that, in futures trading, contango and backwardation are two interesting shapes of the futures curve that have

long been noted by investors. In the terminology of the futures market, contango is loosely referred to the market condition wherein the price of a futures contract is trading above the spot price, while the backwardation is the opposite condition.

The reason for contango or backwardation is a result of the cost or benefit of holding the actual asset underlying the futures contract. We would not expand all the behind the contango and backwardation. Interested readers could find more explanations on this subject. Here, we are focusing on the strategy side.

Let's start from scratch. **We would like to simplify matters and define the futures curve to be in contango if the futures price of the nearest contract is lower than the futures prices of the second nearest contract, and similarly for backwardation.** For the purpose of clear illustration, we start our discussion with a simple but unrealistic idea, and will build from there. Suppose at one specific point in time we observe a contango in the futures market. In particular, the futures price of the nearest contract (call it contract one) at the time is lower than the futures price of the second nearest contract (call it contract two). If we were able to enter into a long position on contract one and a short position on contract two, we would be able to exit our long position as contract one delivers, hold the asset and lock in a higher price as we deliver the asset when contract two expires, thereby earning a riskless profit.

This naive strategy will not work in reality because of various factors involved when we hold the asset, most prominently the storage costs for storing the asset until the maturity of contract two, which will be more than sufficient to cancel out all the profits.

Now we realized that if there is a tradable asset that can closely track the spot price of the futures contracts, which we define to be the futures price of the nearest contract, then a contango or backwardation situation can be translated into arbitrage. The reasoning is as follows. Suppose that we have found the tracking asset, in times of contango we can long that asset and short the second nearest contract. When the nearest contract matures—note that we are not holding that contract but holding an asset that tracks the futures price—we can unwind the long position just like exiting a futures long position. In this way, we have successfully solved the problem related to simply trading in futures: we no longer need to hold the asset because unwinding our long position in that asset does not lead to delivery. In times of backwardation, we can do the reverse, namely, short the tracking asset and long the second nearest contract, and then unwinding our short position in that asset as the nearest futures contract expires.

Now our problem is how to find an asset that tracks the spot prices of futures well so that we can successfully implement our arbitrage strategy. In this article, we use WTI oil commodities futures as our example. We successfully located an index that does not conduct front-end rolling, so that the weighted average of the underlying contracts' maturity can remain constant over time, Standard & Poor's 500 Energy Sector Equally Weighted Index (SPXEWEN), which contains stocks of large oil companies, offshore engineering companies, oil pipeline companies and other energy related companies in the United States. These companies' share prices are largely affected by the oil price; as a result, SPXEWEN Index has tracked the spot prices of oil much better than the other indices. We can replicate the index performance by simply constructing a portfolio of all index members. Since

SPXEWEN Index is an equally weighted equity index, it is unnecessary to closely monitor the price of each stock to maintain a specified weight. We can replicate the index performance by simply constructing a portfolio of all index members. Since SPXEWEN Index is an equally weighted equity index, it is unnecessary to closely monitor the price of each stock to maintain a specified weight.

▼ 2.3 Roll Yields for Markets in Backwardation and Contango

▼ 2.2.1 *The Term Structure*

The amount of return generated in a backwardated futures market that is achieved by rolling a short-term contract into a longer-term contract and profiting from the convergence toward a higher spot price. Profiting from roll yield is a common goal for many strategies used by traders in the futures market.

Backwardation occurs when a futures contract will trade at a higher price as it approaches expiration compared to when the contract is farther away from expiration. Rolling into less expensive futures contracts allows the trader to consistently profit from the rise in a futures' price as it nears expiration.

The biggest risk to this strategy is that the market will shift to contango (opposite as backwardation). This type of changing market has led to major losses by various hedge funds in the past and is the reason why it should only be attempted by experienced traders.

Step 1. Restart and Load the necessary packages.	<pre>restart : with(Statistics) : with(plots) :</pre>
Step 2. Write a procedure for the dynamics of the spot price.	<pre>SpotPrice := proc(X0, n, μ, σ) local Y, A, x, i; Y := RandomVariable(Normal(μ, σ)) : A := Sample(Y, n) : x[1] := X0 : for i from 2 by 1 to n do x[i] := x[i - 1] + A[i] : end do: seq([i, x[i]], i = 1 .. n); end proc:</pre>
Step 3. Develop procedures for yield	<pre>TermStructure := proc(S, T, c) local F, i, L : F := S · (1 + c) :</pre>

curves in Backwardation and Contango.

- We will consider n to be the length of the contract
- We will interpolate the nodes of the curve.
- We will define the parameter c as the degree of contango in the lower end of the curve.

```
L[1] := S :
for i from 2 by 1 to T do
L[i] := L[i - 1] + 0.5 · (F - L[i - 1]) :
end do:
seq([i, L[i]], i = 1 .. T) :
end proc:
```

Step 4. Plot the Term Structure for different values of T and c .

```
S := 100 :
T := 12 :
c := 0.2 :

if c > 0 then
note :=
    "Market in Contango with coefficient =" :
else
note :=
    "Market in Backwardation with
    coefficient="
end if:

B := [_TermStructure(S, T, c)] :
listplot(B, gridlines = true,
title
    = [typeset("(Mirror)Term Structure for #
    of periods =", T, "\n",
    note, " = ", c),
font = [TIMES, Italic, 11]], color = red) ;
```

*(Mirror)Term Structure for # of
periods = 12
Market in Contango with
coefficient = 0.2*



▼ 2.2.2 Procedures for Generating Futures Prices from Spot Prices

Step 1. Make a procedure for the Stochastic Process of the Spot Price	<pre> restart : with(Statistics) : with(plots) : SpotPrice :=proc(X0, n, μ, σ) local Y, A, x, i; Y := RandomVariable(Normal(μ, σ)) : A := Sample(Y, n) : x[1] := X0 : for i from 2 by 1 to n do x[i] := x[i - 1] + A[i] : end do: seq(x[i], i = 1 ..n); end proc: </pre>
Step 2. Make a procedure for the Term Structure of the Futures contracts	<pre> _TermStructure :=proc(S, T, c, θ) local F, r, L, i : F := S · (1 + c) : L[1] := S : for i from 2 by 1 to T do L[i] := L[i - 1] + θ · (F - L[i - 1]) : end do: seq(L[i], i = 1 ..T) : end proc: </pre>
Step 3. Generate an adjustment factor to describe the dynamics of a given contract along the curve.	<pre> _AdjustmentFactor :=proc(T, c, θ) local TS, i, _AF : TS := _TermStructure(1, T, c, θ) : for i from 1 by 1 to T do _AF[i] := TS[T - i + 1] : end do: seq(_AF[i], i = 1 ..T) : end proc: </pre>
Step 4. Generate a Procedure for the equivalent Contango or backwardation for	<pre> _CurveEquivalence :=proc(T, t, c, θ) local TS : TS := _TermStructure(1, T, c, θ) : </pre>

<i>contracts of different length.</i>	<pre> TS[$T - t - 1$] - 1; end proc: </pre>
Step 5. Generate a procedure that describes the convergence of the futures price along its corresponding curve.	<pre> _Convergence := proc(X0, n, mu, sigma, c, T, T1, T2, T3, T4, theta) local N, i, M, SP, P, part11, part2, part12, part13, part14, M1, M2, M3, M4, c1, c2, c3, c4 : SP := SpotPrice(X0, n, mu, sigma) : for i from 1 by 1 to T1 do P[i] := SP[i] : c1 := _CurveEquivalence(T, T1, c, theta) : M1[i] := (_AdjustmentFactor(T1, c1, θ) [i]) · P[i] : end do: for i from T1 + 1 by 1 to n do P[i] := SP[i] : M1[i] := SP[i] : end do: part11 := [seq([i, M1[i]], i = 1 ..n)] : part2 := [seq([i, P[i]], i = 1 ..n)] : for i from 1 by 1 to T2 do P[i] := SP[i] : c2 := _CurveEquivalence(T, T2, c, theta) : M2[i] := (_AdjustmentFactor(T2, c2, θ) [i]) · P[i] : end do: for i from T2 + 1 by 1 to n do P[i] := SP[i] : M2[i] := SP[i] : end do: part12 := [seq([i, M2[i]], i = 1 ..n)] : for i from 1 by 1 to T3 do P[i] := SP[i] : c3 := _CurveEquivalence(T, T3, c, theta) : M3[i] := (_AdjustmentFactor(T3, c3, θ) [i]) · P[i] : end do: for i from T3 + 1 by 1 to n do </pre>

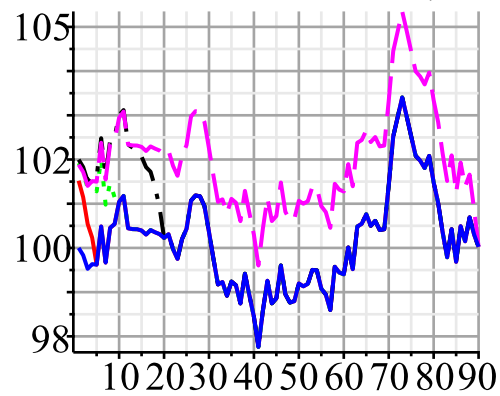
	<pre> P[i] := SP[i] : M3[i] := SP[i] : end do: part13 := [seq([i, M3[i]], i = 1 ..n)] : for i from 1 by 1 to T4 do P[i] := SP[i] : c4 := _CurveEquivalence(T, T4, c, theta) : M4[i] := (_AdjustmentFactor(T4, c4, θ) [i]) ·P[i] : end do: for i from T4 + 1 by 1 to n do P[i] := SP[i] : M4[i] := SP[i] : end do: part14 := [seq([i, M4[i]], i = 1 ..n)] : part11, part12, part13, part14, part2; end proc: </pre>
Step 5. Check the results for a given set of parameters.	<pre> X0 := 100 : n := 90 : mu := 0 : sigma := 0.5 : T4 := 90 : T3 := 20 : T2 := 10 : T1 := 5 : T := 100 : c := 0.02 : θ := 0.3 : part := _Convergence(X0, n, mu, sigma, c, T, T1, T2, T3, T4, θ) : part11 := part[1] : part12 := part[2] : part13 := part[3] : part14 := part[4] : part2 := part[5] : H1 := listplot(part11, color = red, gridlines = true) : H2 := listplot(part12, color = green, linestyle = dot) : H3 := listplot(part13, color = black, linestyle = dashdot) : H4 := listplot(part14, color = magenta, linestyle = dash) : </pre>

```

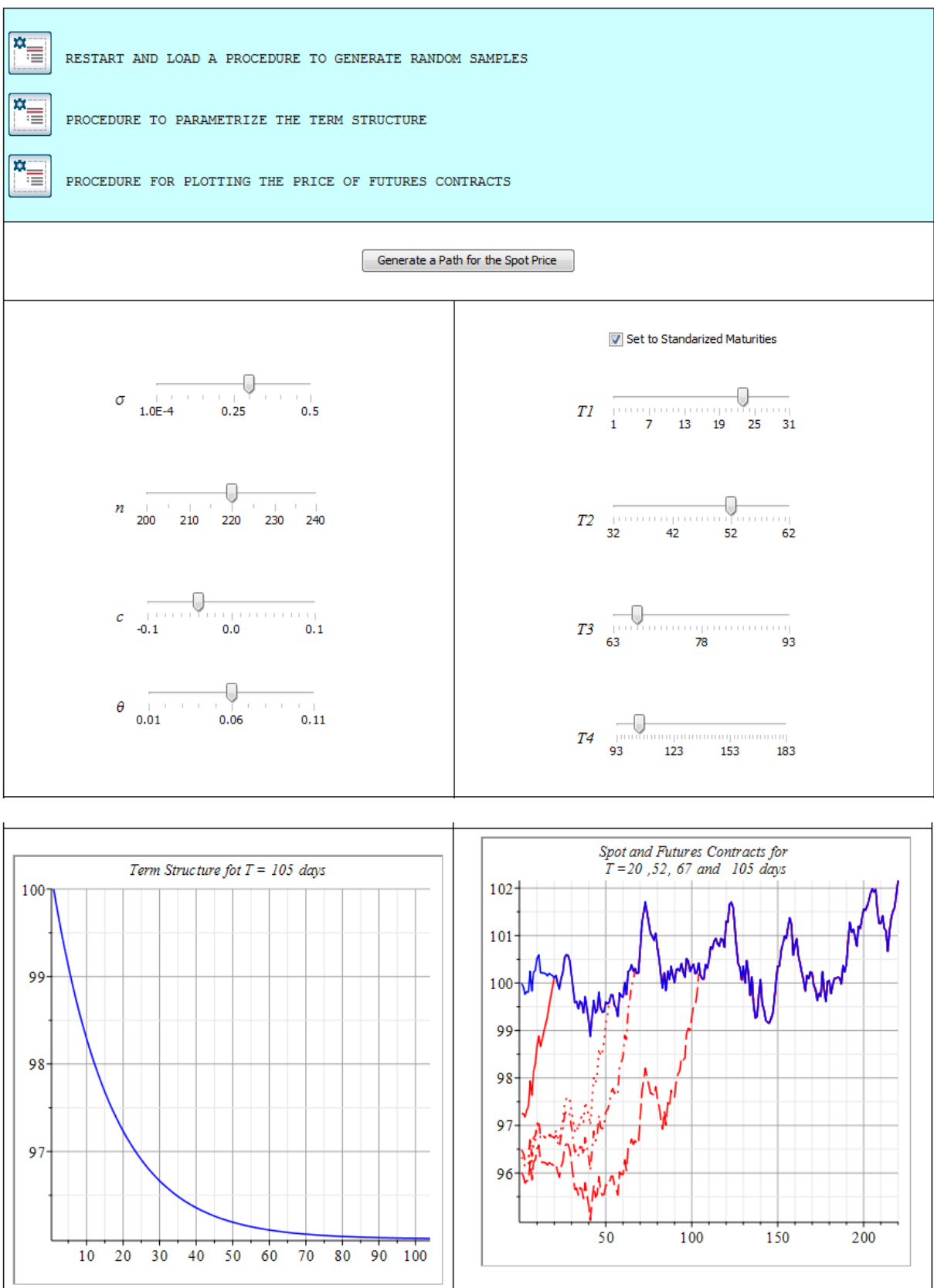
H5 := listplot(part2, color = blue) :
display(H1, H2, H3, H4, H5, title
= [ typeset("Spot and Futures Contracts for
", "\n", "T =",
T1, " ", T2, " ", T3, " and ", T4, " days"), font
= [ TIMES, Italic, 11 ]]);

```

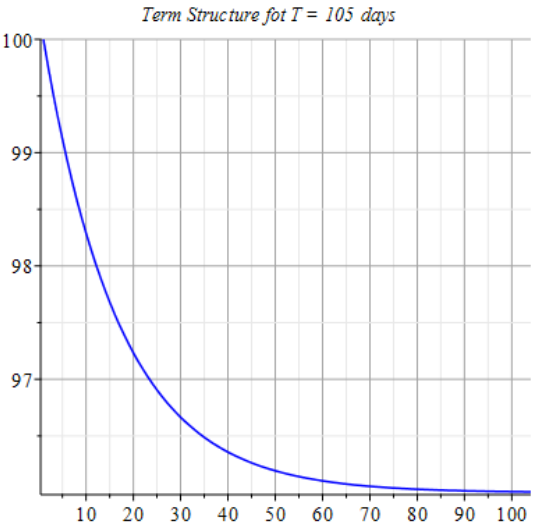
*Spot and Futures Contracts for
T = 5 , 10, 20 and 90 days*



▼ 2.2.2 A Visualization Tool for Backwardation and Contango

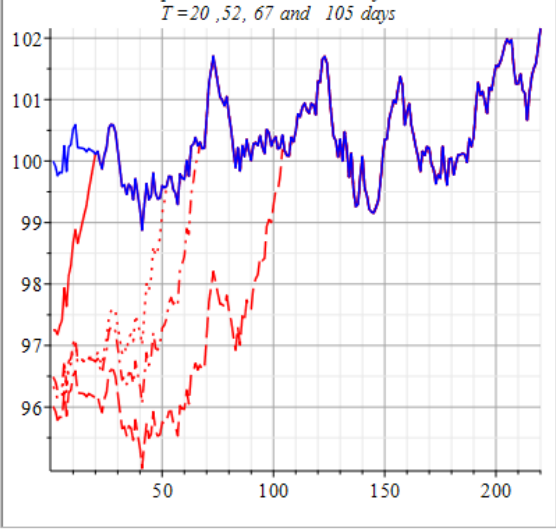


Term Structure for $T = 105$ days



A line graph showing the term structure of interest rates for T = 105 days. The x-axis represents time in days from 10 to 100, and the y-axis represents the interest rate from 97 to 100. The curve starts at 100 and decreases exponentially, approaching 97 as time increases.

Spot and Futures Contracts for $T = 20, 52, 67$ and 105 days



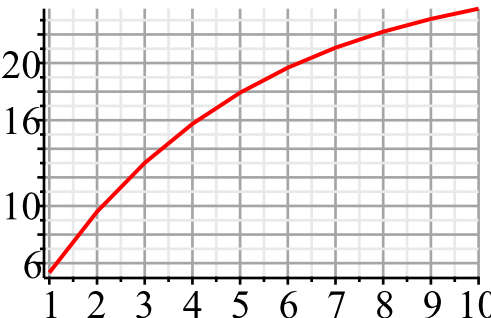
A line graph showing the spot and futures contracts for T = 20, 52, 67, and 105 days. The x-axis represents time in days from 0 to 200, and the y-axis represents the price from 96 to 102. The plot shows a blue line for the spot price and four red lines (solid and dashed) for the futures contracts. The spot price fluctuates between 99 and 102, while the futures contracts start at lower prices and converge towards the spot price as maturity increases.

▼ 3. Roll Overs: Term Structure and Tracking Error

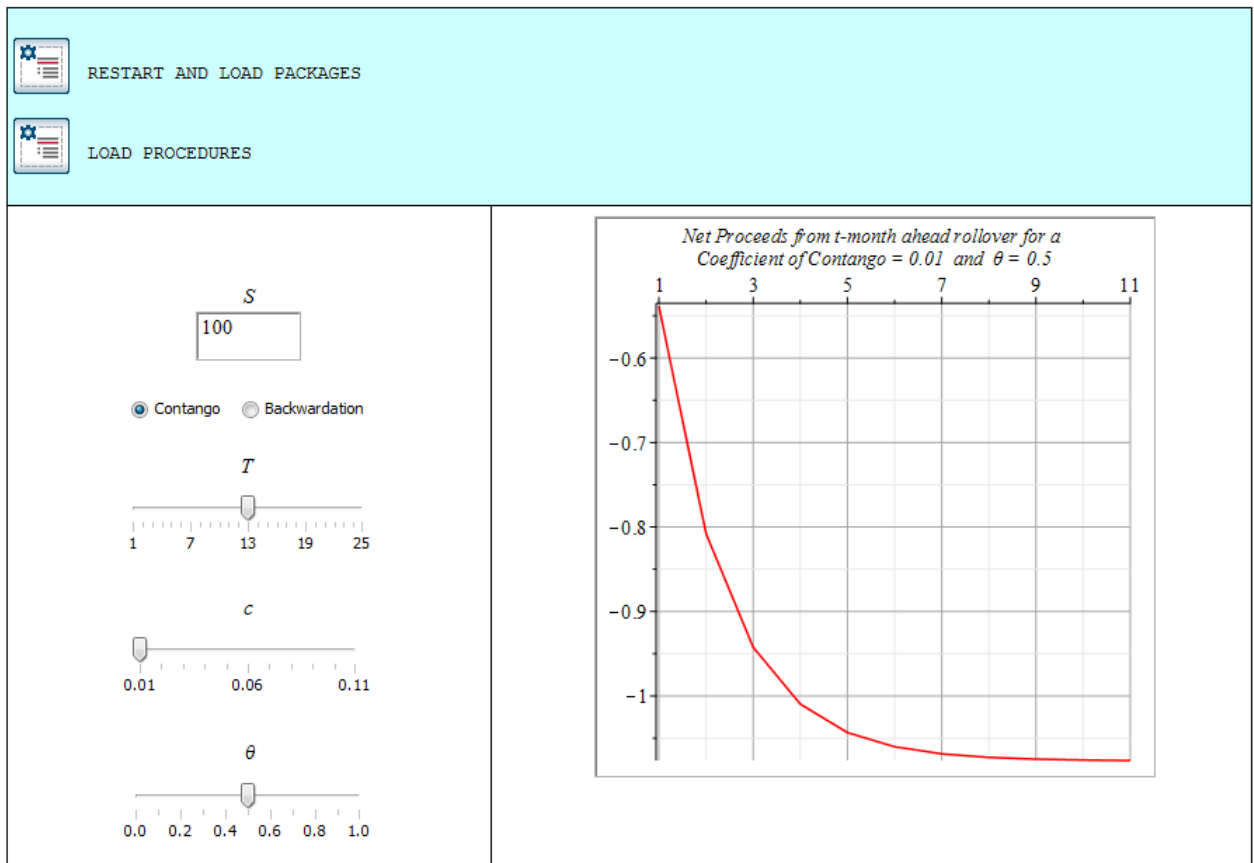
▼ 3.1 A Simple Code for the Term Structure and the Price of Next Month Futures

Step 1. Reset Maple and Load Packages	<pre>restart : with(plots) : with(Statistics) :</pre>
Step 2. Re-define the Term Structure procedure to go for a linear model.	<pre>_TermStructure := proc(S, T, c, θ) local F, i, L : F := S · (1 + c) : L[1] := S : for i from 2 by 1 to T do L[i] := L[i - 1] + θ · $\frac{1}{T}$ · (F - L[1]) + (1 - θ) · (F - L[i - 1]) : end do: seq([i, L[i]], i = 1 .. T) : end proc:</pre>
Step 3. Generate a Procedure to plot the term structure.	<pre>_GraphTermStructure := proc(S, T, c, θ) local note, B; if c > 0 then note := "Coefficient of Contango" : else note := "Coefficient of Backwardation" end if: B := [_TermStructure(S, T, c, θ)] : listplot(B, gridlines = true, title = [typeset("(Mirror)Term Structure for periods =", T, "\n", note, " = ", c, " and θ =", θ), font = [TIMES, Italic, 11]], color = red) ; end proc:</pre>
	<pre>S := 100 : T := 12 : c := 0.2 : θ := 0.8 : P := _GraphTermStructure(S, T, c, θ) : display(P);</pre>

	(Mirror)Term Structure for periods =12 Coefficient of Contango = 0.2 and $\theta = 0.8$ 
Step 5. Define a Function for the initial price of each one of the contracts on a curve of - Contract curve of T periods - With a Contango/Backwardation coefficient of c - With t periods to mature	<pre> _PriceOfContract := proc(S, T, c, t, θ) local F, L, i, TS; F := S · (1 + c) : L[1] := S : for i from 2 by 1 to T do L[i] := L[i - 1] + $\theta \cdot \frac{1}{T} \cdot (F - L[1]) + (1$ - $\theta) \cdot (F - L[i - 1])$: end do: TS := [seq(L[i], i = 1 .. T)] : TS[t] : end proc: </pre>
Step 6. Define the Roll Over Procedure - Sell the t to mature contract - Buy a longer contract	<pre> _RollOver := proc(S, T, c, t_sell, t_buy, θ) _PriceOfContract(S, T, c, t_sell, θ) - _PriceOfContract(S, T, c, t_buy, θ) : end proc: </pre>
	<pre> S := 100 : T := 12 : c := 0.2 : t_sell := 1 : t_buy := 2 : θ := 0.999 : _RollOver(S, T, c, t_sell, t_buy, θ); -1.6850000 (3.1.1) </pre>
Step 7. Write a procedure	

<i>for plotting the Roll Over Costs for different curve specifications.</i>	<pre> _GraphRollOver := proc(S, T, c, θ) local note, B; if $c > 0$ then note := "Coefficient of Contango" : else note := "Coefficient of Backwardation" end if: B := [seq(_RollOver($S, T, c, 1, t, \theta$), $t = 2 .. T - 1$)] : listplot(B, gridlines = true, title = [typeset("Cost of t-mont ahead rollover for a ", "\n", note, " = ", c, "and $\theta = ", \theta$), font = [TIMES, Italic, 11]], color = red) ; end proc: </pre>																						
	<pre> $S := 100$: $T := 12$: $c := -0.2$: $\theta := 0.8$: _GraphRollOver(S, T, c, θ); </pre> <i>Cost of t-mont ahead rollover for a Coefficient of Backwardation = -0.2 and $\theta = 0.8$</i>  <table border="1"> <caption>Approximate data points from the graph</caption> <thead> <tr> <th>t</th> <th>Cost</th> </tr> </thead> <tbody> <tr><td>1</td><td>6</td></tr> <tr><td>2</td><td>10</td></tr> <tr><td>3</td><td>13</td></tr> <tr><td>4</td><td>16</td></tr> <tr><td>5</td><td>18</td></tr> <tr><td>6</td><td>19</td></tr> <tr><td>7</td><td>20</td></tr> <tr><td>8</td><td>21</td></tr> <tr><td>9</td><td>21.5</td></tr> <tr><td>10</td><td>22</td></tr> </tbody> </table>	t	Cost	1	6	2	10	3	13	4	16	5	18	6	19	7	20	8	21	9	21.5	10	22
t	Cost																						
1	6																						
2	10																						
3	13																						
4	16																						
5	18																						
6	19																						
7	20																						
8	21																						
9	21.5																						
10	22																						

▼ 3.2 A Calculator of Roll Over Costs



▼ 3.3 Simulator of Roll Over Bias



RESTART AND LOAD PACKAGES



LOAD PROCEDURES FOR CALCULATING THE ROLL OVER COSTS



PROCEDURES FOR OUR CUSTOMIZED ROLL OVER MENU DEFINED IN STEP

Step 1. Choose a Specification for the Random Shocks

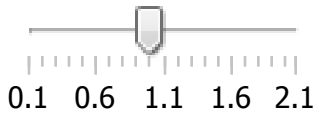
b

☐ Normal

☒ StudentT

☐ Cauchy

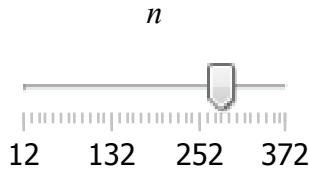
☐ Uniform



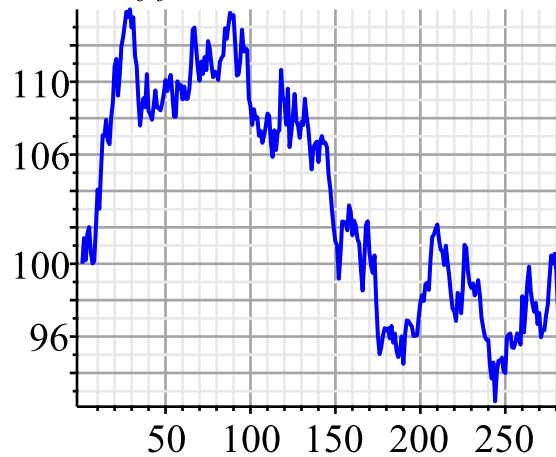
Note: For the StudentT distribution, the degrees of freedom are $10 \times b$

$$\frac{0.3891083839}{(1 + 0.1000000000 u^2)^{5.500000000}}$$

Step 2. Produce a Sample of the Random Process



StudentT Distribution with degrees of freedom = 10×1.0

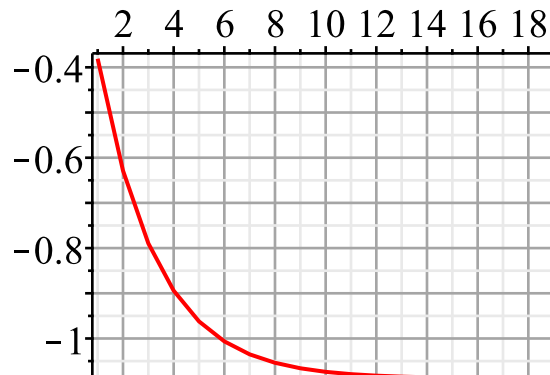


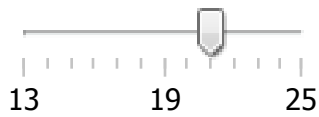
Step 3. Specify the Term Structure and the corresponding Roll Over Cost Function

S
100

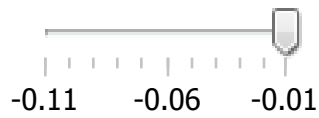
- ☐ Contango
- ☒ Backwardation
- T

Net proceeds from t -month ahead rollover for a Coefficient of Contango = 0.01 and $\theta = 0.65$

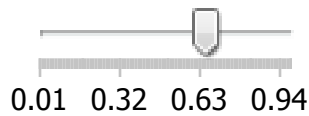




c



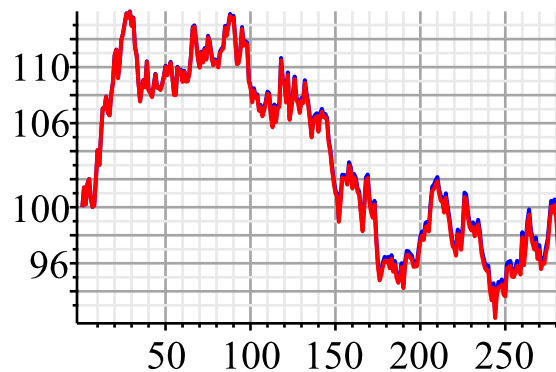
θ



Step 4. Choose the Option for Rolling Over Contracts

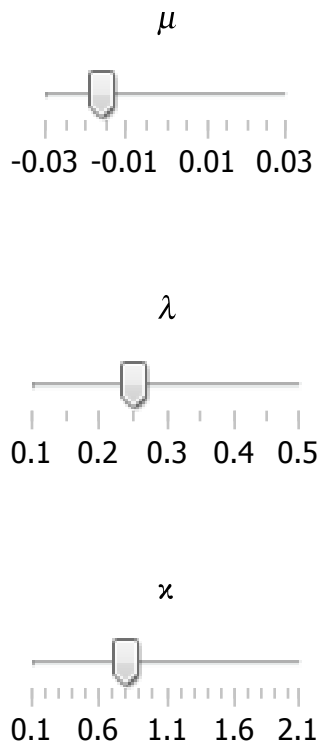
- ☐ 1 Mo ahead ☐ 2 vs 3 Mo ahead
☐ 6 vs 9 Mo ahead ☐ 9 vs 12 Mo Ahead

Index (BLUE) vs. Index Futures (RED) with Roll Over
Case of Normal Backwardation
 $c = 0.01$ and $\theta = 0.65$

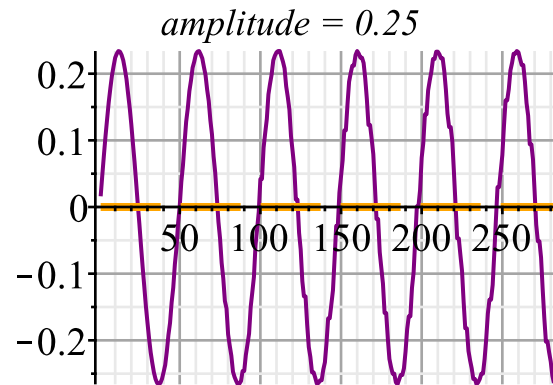


Step 5. Develop a procedure

*assuming that c is cyclical
and see what happens to the
effect of rollover.*



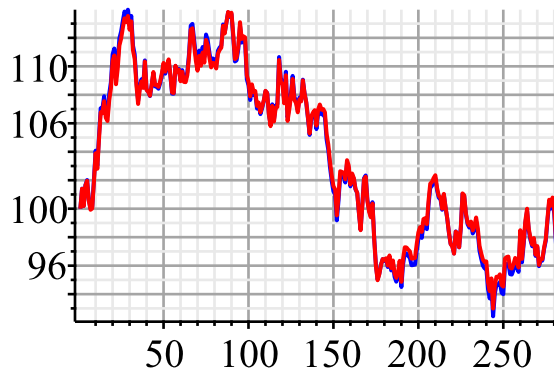
*Model for a cyclical Contango with
parameters
displacement = -0.016 frequency =
0.8*



*Step 6. Develop a procedure
assuming that c is cyclical
and see what happens to the
effect of rollover.*

- ☐ 1 Mo ahead ☐ 2 vs 3 Mo ahead
☐ 6 vs 9 Mo ahead ☐ 9 vs 12 Mo Ahead

*Index (BLUE) vs. Index Futures
(RED) with Roll Over
Market with Backwardation Bias
 $\mu = -0.016$*



▼ 3.4 Distributions of Underlying vs. Roll Overs



RESTART AND LOAD PACKAGES



LOAD PROCEDURES FOR CALCULATING THE ROLL OVER COSTS



PROCEDURES FOR OUR CUSTOMIZED ROLL OVER MENU DEFINED IN STEP 4

Step 1. Choose a Specification for the Random Shocks

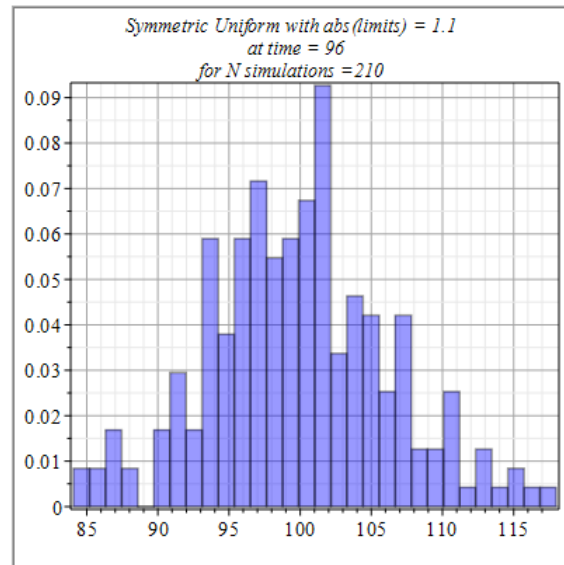
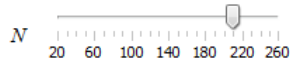


Note: For the StudentT distribution, the degrees of freedom are $10*b$

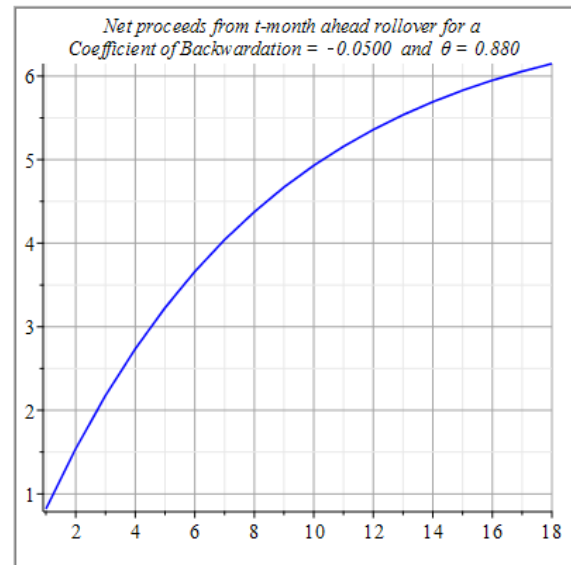
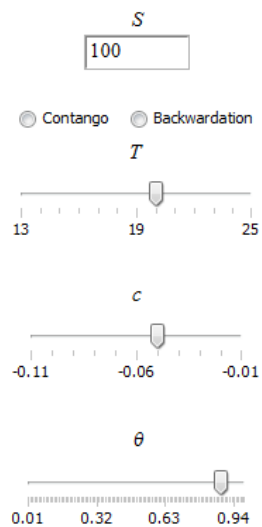
☒ Normal ☐ StudentT ☐ Cauchy ☐ Uniform

0	$u < -1.1$
0.4545454545	$u < 1.1$
0	otherwise

Step 2. Produce M Samples of the Random Process with Length n for each one.



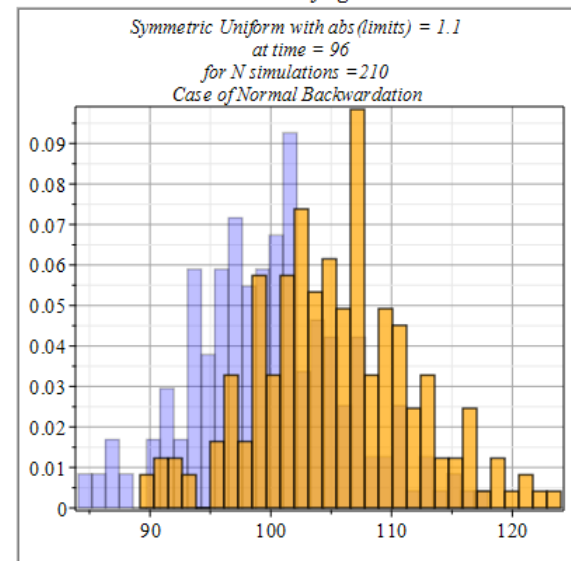
Step 3. Specify the Term Structure and the corresponding Roll Over Cost Function



Step 4. Choose the Option for Rolling Over Contracts

☐ 1 Mo ahead
 ☐ 2 vs 3 Mo ahead
 ☐ 6 vs 9 Mo ahead
 ☒ 9 vs 12 Mo Ahead

Orange= Roll Over Futures
Blue= Underlying Index



► 4. Options on Rolled Index-Future-Based Assets

► 5. VaR and Expected Loss of Rolled Over Futures

▼ 6. Arbitrages of Futures vs. Physicals and/or Spot Tracking

Assets

Here, we are going to introduce a strategy that could help us exploit how to profit from the cost of carry in the futures market. We all know that, in futures trading, contango and backwardation are two interesting shapes of the futures curve that have long been noted by investors. In the terminology of the futures market, contango is loosely referred to the market condition wherein the price of a futures contract is trading above the spot price, while the backwardation is the opposite condition.

The reason for contango or backwardation is a result of the cost or benefit of holding the actual asset underlying the futures contract. We would not expand all the behind the contango and backwardation. Interested readers could find more explanations on this subject. Here, we are focusing on the strategy side.

Let's start from scratch. We would like to simplify matters and define the futures curve to be in contango if the futures price of the nearest contract is lower than the futures prices of the second nearest contract, and similarly for backwardation. For the purpose of clear illustration, we start our discussion with a simple but unrealistic idea, and will build from there. Suppose at one specific point in time we observe a contango in the futures market. In particular, the futures price of the nearest contract (call it contract one) at the time is lower than the futures price of the second nearest contract (call it contract two). If we were able to enter into a long position on contract one and a short position on contract two, we would be able to exit our long position as contract one delivers, hold the asset and lock in a higher price as we deliver the asset when contract two expires, thereby earning a riskless profit.

This naive strategy will not work in reality because of various factors involved when we hold the asset, most prominently the storage costs for storing the asset until the maturity of contract two, which will be more than sufficient to cancel out all the profits.

Now we realized that if there is a tradable asset that can closely track the spot price of the futures contracts, which we define to be the futures price of the nearest contract, then a contango or backwardation situation can be translated into arbitrage. The reasoning is as follows. Suppose that we have found the tracking asset, in times of contango we can long that asset and short the second nearest contract. When the nearest contract matures—note that we are not holding that contract but holding an asset that tracks the futures price—we can unwind the long position just like exiting a futures long position. In this way, we have successfully solved the problem related to simply trading in futures: we no longer need to hold the asset because unwinding our long position in that asset does not lead to delivery. In times of backwardation, we can do the reverse, namely, short the tracking asset and long the second nearest contract, and then unwinding our short position in that asset as the nearest futures contract expires.

Now our problem is how to find an asset that tracks the spot prices of futures well so that we can successfully implement our arbitrage strategy. In this article, we use WTI oil commodities futures as our example. We successfully located an index that does not conduct front-end rolling, so that the weighted average of the underlying contracts' maturity can remain constant over time, Standard & Poor's 500 Energy Sector Equally Weighted Index (SPXEWEN), which contains stocks of large oil companies, offshore engineering companies, oil pipeline companies and other energy related companies in the United States. These companies' share prices are largely affected by the oil price; as

a result, SPXEWEN Index has tracked the spot prices of oil much better than the other indices. We can replicate the index performance by simply constructing a portfolio of all index members. Since SPXEWEN Index is an equally weighted equity index, it is unnecessary to closely monitor the price of each stock to maintain a specified weight. We can replicate the index performance by simply constructing a portfolio of all index members. Since SPXEWEN Index is an equally weighted equity index, it is unnecessary to closely monitor the price of each stock to maintain a specified weight.

► 7. Resources