



# UNIVERSITY OF WATERLOO

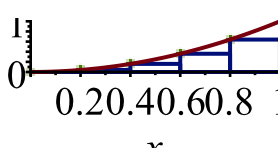
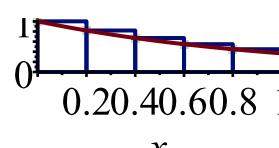
## Riemann Sums

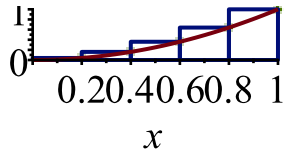
Department of Applied Mathematics

University of Waterloo

Created for: MATH 137

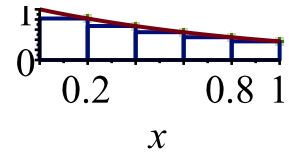
Created by: Francis J. Poulin and Emilee Carson using Maple 2015

Comments	Example 1	Example 2
Define the function, the interval in question ([a,b]) and the number of iterations, N.	$f := x \rightarrow x^2 :$ $a := 0 :$ $b := 1 :$ $N := 5 :$	$g := x \rightarrow \exp(-x) :$ $a := 0 :$ $b := 1 :$ $N := 5 :$
We can use the built in package RiemannSum to do all the work for us.	<i>with(Student[Calculus1]) :</i>	<i>with(Student[Calculus1]) :</i>
Use RiemannSum to calculate the Riemann Sum using the left method. In example 1, we look at the Riemann Sums for the simple parabola.	$RiemannSum(f(x), x = a .. b,$ $output = plot, partition = N,$ $method = left)$  <p>A left Riemann sum approximation of <math>\int_0^1 x^2 dx</math></p>	$RiemannSum(g(x), x = a .. b,$ $output = plot, partition = N,$ $method = left)$  <p>A left Riemann sum approximation of <math>\int_0^1 \exp(-x) dx</math></p>
Repeat the same process, this time using the right method.	$RiemannSum(f(x), x = a .. b,$ $output = plot, partition = N,$ $method = right)$	$RiemannSum(g(x), x = a .. b,$ $output = plot, partition = N,$ $method = right)$



A right Riemann sum approximation

$$\int_0^1 x^2 dx \approx 0.1667$$

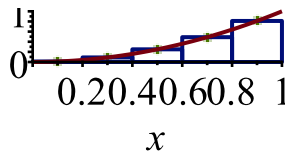


A right Riemann sum approximation

$$\int_0^1 (1-x) dx \approx 0.4167$$

And finally, using the midpoint method.

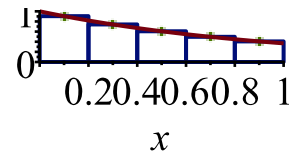
*RiemannSum(f(x), x = a..b, output = plot, partition = N, method = midpoint)*



A midpoint Riemann sum approximation of

$$\int_0^1 x^2 dx \approx 0.3333$$

*RiemannSum(g(x), x = a..b, output = plot, partition = N, method = midpoint)*



A midpoint Riemann sum approximation of

$$\int_0^1 (1-x) dx \approx 0.5000$$