Dark Matter: Coma Cluster **

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Problem: This worksheet makes use of data about the huge Coma cluster (Abell 1656) of more than 1000 galaxies (Rowe 2010) at a distance of approximately 92 Mpc (Mattson, 2015) to calculate the percentage of dark matter in the cluster. The problem is based on Goldberg (2013). The data are from Biviano et al. (1995) and include a set of 26 galaxies near the cluster centre.

Hints:

Use the relativistic redshift formula when calculating the redshifts of extremely distant objects.
Use the virial theorem to estimate the cluster mass.
Create lists of the x and y coordinates of the galaxies (columns D and E, respectively).
Compute the maximum radius in arcseconds,
Create a matrix and list of the radial velocities, expressed as redshifts (column C).
Calculate the average redshift of the galaxies in the cluster.
Calculate the standard deviation of the redshifts.
Calculate the 1sd velocity dispersion of the galaxies.
Calculate the characteristic radius in Mpc.
Use the virial theorem to solve for the mass.
Estimate the baryonic mass of 1000 galaxies, each with 10^{11} solar-type stars on average.
Calculate the percentage of total mass contained in the non-baryonic (dark) matter.

Data:

with(*Statistics*) :

with(*ListTools*) :

In the following spreadsheet, column A contains galaxy identification numbers, B: radial velocities (km/s), C: radial velocities expressed as redshifts z, D: number of arcseconds away from the cluster centre in the x direction, E: number of arcseconds away from the cluster centre in the y direction.

Coma Cluster (selected galaxies near centre)					
	A	B	C	D	E
1	4469	7452	0.025174	1310	-1414
2	4479	5749	0.019364	1322	-1982
3	4522	7606	0.025701	1380	-1728
4	4535	7653	0.025862	1393	-1857
5	4579	4915	0.016531	1451	-1396
6	4578	5186	0.017451	1451	-2113
7	4597	4915	0.016531	1478	-1723
8	4630	7335	0.024774	1525	-1721
9	4692	8318	0.028142	1594	-1944
10	4714	7226	0.024401	1624	-1410
11	4792	7173	0.024220	1721	-1743
12	4794	7304	0.024668	1724	-1712
13	4829	6055	0.020405	1762	-1291
14	4852	7694	0.026002	1791	-1532
15	4907	5504	0.018531	1855	-1522
16	4918	4811	0.016179	1867	-2030
17	4928	7442	0.025140	1877	-1694
18	4937	5709	0.019228	1887	-1506
19	4943	8203	0.027747	1896	-1712
20	4956	6819	0.023010	1913	-1778
21	5038	6205	0.020916	2060	-2028
22	5051	7323	0.024733	2076	-1806
23	5102	8122	0.027469	2141	-1591
24	5136	7012	0.023670	2182	-2146
25	5284	7545	0.025492	2434	-1509
26	5296	7310	0.024688	2454	-1872

$Mpc := 3.086e22\,m$; *# 1 megaparsec in metres*

$$3.086\ 10^{22}\ \mathrm{m} \tag{1}$$

$Msun := 2e30\,kg$; *# solar mass*

$$2.\ 10^{30}\ \mathrm{kg} \tag{2}$$

$G := 6.67e{-11}\,N\,\frac{m^2}{kg^2}$;*# gravitational constant*

$$\frac{6.67\ 10^{-11}\ \mathrm{N\ m^2}}{\mathrm{kg}^2} \tag{3}$$

$dMpc := 92$; *#distance to the Coma Cluster in Mpc*

$$92 \tag{4}$$

Useful Equations:

$$U = -\frac{3}{5}\,\frac{GM^2}{R}$$ *# gravitational potential energy*

$$K = \frac{3}{2}\,M\sigma_v^2$$ *# kinetic energy*

U = -2K: # *Virial theorem*

Solution:

Create lists of the x and y coordinates of the galaxies (columns D and E, respectively).

xarcsec := *Matrix*(⟨⟨1310⟩, ⟨1322⟩, ⟨1380⟩, ⟨1393⟩, ⟨1451⟩, ⟨1451⟩, ⟨1478⟩, ⟨1525⟩, ⟨1594⟩, ⟨1624⟩, ⟨1721⟩, ⟨1724⟩, ⟨1762⟩, ⟨1791⟩, ⟨1855⟩, ⟨1867⟩, ⟨1877⟩, ⟨1887⟩, ⟨1896⟩, ⟨1913⟩, ⟨2060⟩, ⟨2076⟩, ⟨2141⟩, ⟨2182⟩, ⟨2434⟩, ⟨2454⟩⟩);

26 x 1 Matrix
Data Type: anything
Storage: rectangular
Order: Fortran_order **(5)**

xx := *convert*(*xarcsec*, *list*)

[1310, 1322, 1380, 1393, 1451, 1451, 1478, 1525, 1594, 1624, 1721, 1724, 1762, 1791, 1855, 1867, 1877, 1887, 1896, 1913, 2060, 2076, 2141, 2182, 2434, 2454] **(6)**

yarcsec := *Matrix*(⟨⟨-1414⟩, ⟨-1982⟩, ⟨-1728⟩, ⟨-1857⟩, ⟨-1396⟩, ⟨-2113⟩, ⟨-1723⟩, ⟨-1721⟩, ⟨-1944⟩, ⟨-1410⟩, ⟨-1743⟩, ⟨-1712⟩, ⟨-1291⟩, ⟨-1532⟩, ⟨-1522⟩, ⟨-2030⟩, ⟨-1694⟩, ⟨

$-1506\rangle, \langle -1712\rangle, \langle -1778\rangle, \langle -2028\rangle, \langle -1806\rangle, \langle -1591\rangle, \langle -2146\rangle, \langle -1509\rangle, \langle -1872\rangle\rangle);$

$$\left[\begin{array}{c} \textit{26 x 1 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{array}\right] \quad \textbf{(7)}$$

$yy := convert(yarcsec, list)$

$[-1414, -1982, -1728, -1857, -1396, -2113, -1723, -1721, -1944, -1410, -1743, -1712, -1291, -1532, -1522, -2030, -1694, -1506, -1712, -1778, -2028, -1806, -1591, -2146, -1509, -1872]$ **(8)**

$\max([1310, 1322, 1380, 1393, 1451, 1451, 1478, 1525, 1594, 1624, 1721, 1724, 1762, 1791, 1855, 1867, 1877, 1887, 1896, 1913, 2060, 2076, 2141, 2182, 2434, 2454])$

$$2454 \quad \textbf{(9)}$$

$\min([-1414, -1982, -1728, -1857, -1396, -2113, -1723, -1721, -1944, -1410, -1743, -1712, -1291, -1532, -1522, -2030, -1694, -1506, -1712, -1778, -2028, -1806, -1591, -2146, -1509, -1872])$

$$-2146 \quad \textbf{(10)}$$

Computing the maximum radius in arcseconds:

$rarcsec := evalf\left(\sqrt{(\max(xx))^2 + (\min(yy))^2}\right)$

$$3259.974234 \quad \textbf{(11)}$$

Create a matrix and list of the radial velocities, expressed as redshifts (column C).

$z := Matrix(\langle\langle 0.25174e\text{-}1\rangle, \langle 0.19364e\text{-}1\rangle, \langle 0.25701e\text{-}1\rangle, \langle 0.25862e\text{-}1\rangle, \langle 0.16531e\text{-}1\rangle, \langle 0.17451e\text{-}1\rangle, \langle 0.16531e\text{-}1\rangle, \langle 0.24774e\text{-}1\rangle, \langle 0.28142e\text{-}1\rangle, \langle 0.24401e\text{-}1\rangle, \langle 0.24220e\text{-}1\rangle, \langle 0.24668e\text{-}1\rangle, \langle 0.20405e\text{-}1\rangle, \langle 0.26002e\text{-}1\rangle, \langle 0.18531e\text{-}1\rangle, \langle 0.16179e\text{-}1\rangle, \langle 0.25140e\text{-}1\rangle, \langle 0.19228e\text{-}1\rangle, \langle 0.27747e\text{-}1\rangle, \langle 0.23010e\text{-}1\rangle, \langle 0.20916e\text{-}1\rangle, \langle 0.24733e\text{-}1\rangle, \langle 0.27469e\text{-}1\rangle, \langle 0.23670e\text{-}1\rangle, \langle 0.25492e\text{-}1\rangle, \langle 0.24688e\text{-}1\rangle\rangle);$

$$\left[\begin{array}{c} \textit{26 x 1 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{array}\right] \quad \textbf{(12)}$$

$convert(z, list)$

$[0.025174, 0.019364, 0.025701, 0.025862, 0.016531, 0.017451, 0.016531, 0.024774, 0.028142, 0.024401, 0.024220, 0.024668, 0.020405, 0.026002, 0.018531, 0.016179, 0.025140, 0.019228, 0.027747, 0.023010, 0.020916, 0.024733, 0.027469, 0.023670, 0.025492, 0.024688]$ **(13)**

Calculating the average redshift of the galaxies in the cluster:

$zave := Mean(z);$

$$\left[\; 0.0229241923076923 \;\right] \tag{14}$$

Convert this value to a scalar for computational ease:

$zave := SelectFirst(zave)$

$$0.0229241923076923 \tag{15}$$

Using the speed of light in km/s, (ckms) an estimate of the Hubble Constant (H0) can be calculated:

$ckms := 3e5;$

$$3.\ 10^{5} \tag{16}$$

$H0 := \frac{ckms \cdot zave}{dMpc};$

$$74.7528010033445 \tag{17}$$

The currently accepted value is 70±7 km/s/Mpc (NASA, 2014).

Calculating the standard deviation of the redshifts:

$sigz := StandardDeviation(z);$

$$\left[\; 0.00370946964424006 \;\right] \tag{18}$$

Calculating the 1sd velocity dispersion of the galaxies:

$sigvkms := sigz \cdot ckms;$

$$\left[\; 1112.84089327202 \;\right] \tag{19}$$

Converting from a vector element to a scalar for easier calculation:

$sigvkms := SelectFirst(sigvkms)$

$$1112.84089327202 \tag{20}$$

Converting to metres/sec:

$sigv := sigvkms \cdot 1000 \frac{\mathrm{m}}{\mathrm{s}};$

$$\frac{1.11284089327202\ 10^{6}\ \mathrm{m}}{\mathrm{s}} \tag{21}$$

Calculating the characteristic radius in Mpc:

$$rMpc := \frac{dMpc \cdot rarcsec}{206265.};$$

$$1.454040334 \tag{22}$$

The Coma cluster has a greater radius than this, but we are looking only at selected galaxies near the centre. Converting this figure to metres:

$$R := rMpc \cdot Mpc;$$

$$4.487168471\ 10^{22}\ \mathrm{m} \tag{23}$$

Calculating the total mass of the cluster via the virial theorem, based on the assumption that most of the mass is located near the centre:

$$M := simplify\left(\frac{5 \cdot sigv^2 \cdot R}{G}\right)$$

$$4.16564923966514\ 10^{45}\ \mathrm{kg} \tag{24}$$

Expressing the total mass of the cluster in solar masses:

$$Mtot := \frac{\%}{Msun};$$

$$2.08282461983257\ 10^{15} \tag{25}$$

Calculating the baryonic mass of the Coma cluster, assuming roughly 1000 galaxies, each with 10^{11} solar-type stars:

$$Mbar := 1000 \cdot 1e11;$$

$$1.000\ 10^{14} \tag{26}$$

Calculating the percentage of dark matter to baryonic matter:

$$fDM := 1. - \frac{Mbar}{Mtot};$$

$$0.951988276378239 \tag{27}$$

This figure of 95% is the maximum found in a number of galaxy clusters (Schombert, 2015).

References

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gsfc.nasa.gov/features/cosmic/nearest_superclusters_info.html (Accessed 2015-12-17).

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