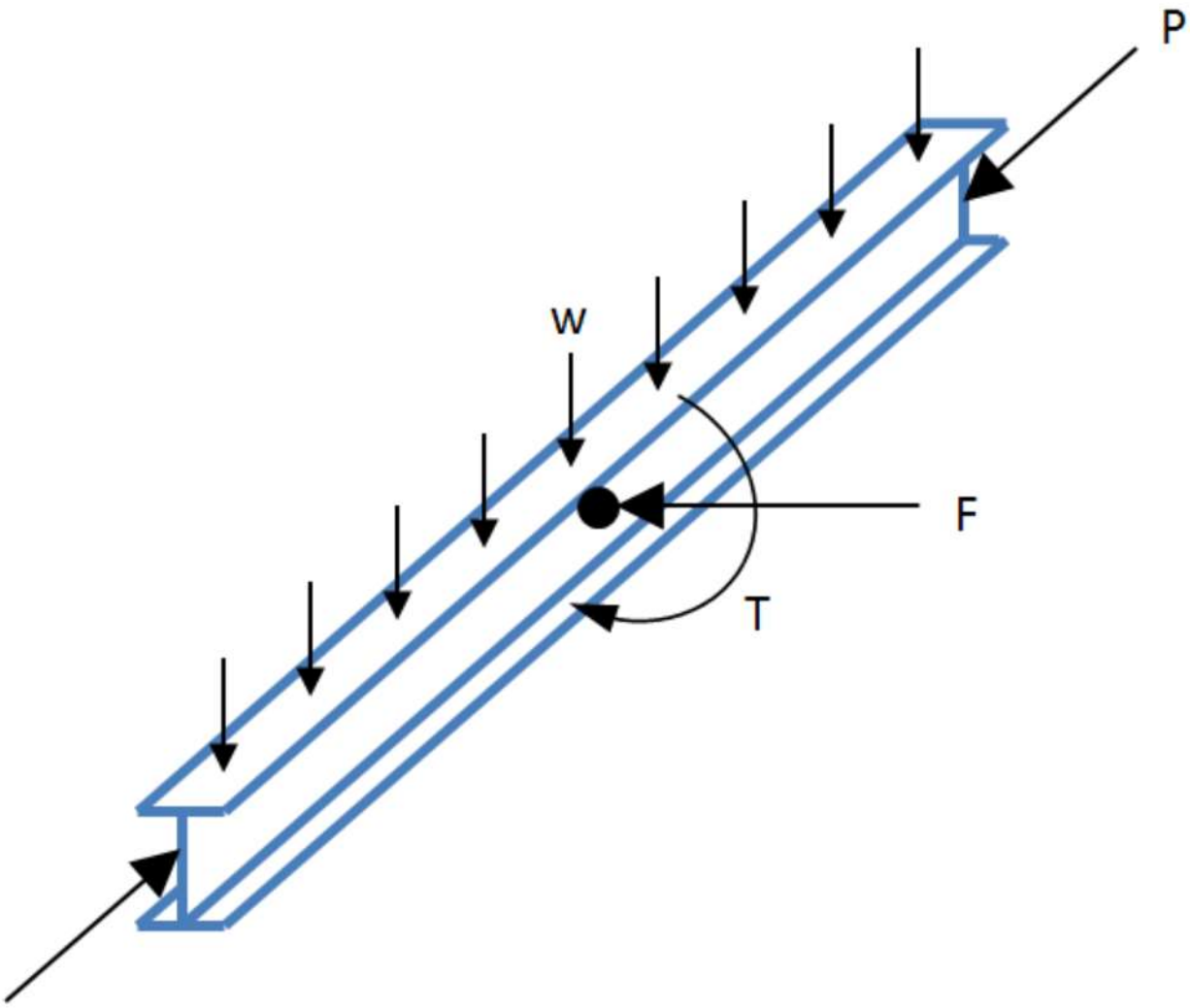
Simply Supported Beam Design with Torsional Loading

Using AISC Steel Shapes v14.1 Data

Introduction

This application performs a design analysis on a simply supported beam with torsional loading for a W10X54 steel beam (as defined by the AISC Steel Shapes Database). The application folows the design code and equations in AISC 360-10.



References:

- *Simplified Design for Torsional Loading of Rolled Steel Members*, Lin, P.H., Engineering Journal, AISC, 1977
- *2010 Specification for Structural Steel Buildings (ANSI/AISC 360-10)*, Fourth Printing (https://www.aisc.org/content.aspx?id=2884)

You will need to install the AISC Shapes Database package from the MapleCloud before you can use this application.

Load the AISC Package

```
> with(AISCShapes) :
> with(Units[Standard]) :
```

Data from the AISC Shapes Database for Steel Shape W10X54

> $Cw := Property("W10X54", "Cw");$
$Property("W10X54", "Cw", "metadata")$

$$Cw := 2320.0\ in^6$$

"Warping constant" **(3.1)**

> $JT := Property("W10X54", "J");$
$Property("W10X54", "J", "metadata")$

$$JT := 1.82\ in^4$$

"Torsional moment of inertia" **(3.2)**

> $d := Property("W10X54", "d");$
$Property("W10X54", "d", "metadata")$

$$d := 10.1\ in$$

"Overall depth of member, or width of shorter leg for angles, or width of the outstanding legs of long legs back-to-back double angles, or the width of the back-to-back legs of short legs back-to-back double angles" **(3.3)**

> $Sx := Property("W10X54", "Sx");$
$Property("W10X54", "Sx", "metadata")$

$$Sx := 60.0\ in^3$$

" Elastic section modulus about the x-axis" **(3.4)**

> $Sy := Property("W10X54", "Sy");$
$Property("W10X54", "Sy", "metadata")$

$$Sy := 20.6\ in^3$$

"Elastic section modulus about the y-axis" **(3.5)**

> $rx := Property("W10X54", "rx");$
$Property("W10X54", "rx", "metadata")$

$$rx := 4.37\ in$$

"Radius of gyration about the x-axis = sqrt(Ix/A)" **(3.6)**

> $A := Property("W10X54", "A");$
$Property("W10X54", "A", "metadata")$

$$A := 15.8\ in^2$$

"Cross-sectional area of member" **(3.7)**

> $Zx := Property("W10X54", "Zx");$
$Property("W10X54", "Zx", "metadata")$

$$Zx := 66.6\ in^3$$

"Plastic section modulus about the x-axis" **(3.8)**

> $Ix := Property("W10X54", "Ix");$

Property("W10X54", "Ix", "metadata")

$$Ix := 303.0\ \text{in}^4$$

"Moment of inertia about the x-axis" **(3.9)**

> *Iy* := *Property*("W10X54", "Iy");
Property("W10X54", "Iy", "metadata")

$$Iy := 103.0\ \text{in}^4$$

"Moment of inertia about the y-axis" **(3.10)**

Parameters

Gravity distributed load:

> $w := 1.15 \frac{\text{kipf}}{\text{ft}}$:

Lateral point load at the middle:

> $F := 5\,\text{kipf}$:

Torsion at mid-span:

> $T := 5.1\,\text{ft}\,\text{kipf}$:

Axial Load:

> $P := 96\,\text{kipf}$:

Beam length:

> $L := 15\,\text{ft}$:

Beam yield stress:

> $Fy := 50\,\text{ksi}$:

Vertical bending unbraced length:

> $Lb := 15\,\text{ft}$:

Axial vertical unbraced length:

> $Lx := 15\,\text{ft}$:

Axial horizontal unbraced length:

> $Ly := 7.5\,\text{ft}$:

Young's modulus and shear modulus:

> $E := 29000\,\text{ksi}$:

> $G := 11200\text{ksi}$:

Torsional property (Phillip, 1977):

> $\lambda := \sqrt{\frac{G \cdot JT}{E \cdot Cw}}$

$$0.01740610961 \ \frac{1}{\text{in}} \tag{4.1}$$

Determine Governing Moments at Middle of Span

Flexural moment:

> $Mx := \frac{w L^2}{8}$

$$32.34 \text{ foot kipf} \tag{5.1}$$

> $My := \frac{F L}{4.0}$

$$18.75 \text{ foot kipf} \tag{5.2}$$

> $M0 := \frac{T L}{4 d}$

$$22.72 \text{ foot kipf} \tag{5.3}$$

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> $\beta := \frac{4 \sinh\left(\frac{\lambda L}{2}\right)^2}{\lambda L \sinh(\lambda L)}$

$$\beta := 0.5850278056 \tag{5.4}$$

Torsional moment:

> $MT := \beta M0$

$$13.29 \text{ foot kipf} \tag{5.5}$$

Check Torsional Capacity (AISC 360-10 H3.3 & Philip page 100)

Maximum combined normal stress at the load point:

> $fbx := \frac{Mx}{Sx} + \frac{2\,MT}{Sy}$

$$21.96 \ \frac{\text{kipf}}{\text{inch}^2} \tag{6.1}$$

Safety factor for compression:

> $\Omega := 1.67$:

> $Fnx := \frac{Fy}{\Omega}$

29.94 ksi **(6.2)**

> $\frac{fbx}{Fnx}$

0.7333393767 **(6.3)**

This is less then 1, so it is satisfactory.

Check Combined Compression and Bending Capacity (AISC 360 -10, H1)

> $Mrx := \left(\frac{Mx}{Sx} + \frac{2\,MT}{Sy} \right) Sx$

109.78 foot kipf **(7.1)**

Effective length factor:

> $K := 0.85$:

Elastic bucking stress:

> $Fe := \frac{\pi^2 E}{\left(\frac{K\,L}{rx} \right)^2}$

233.50 ksi **(7.2)**

Critical stress:

> $Fcr := 0.658^{\frac{Fy}{Fe}}\, Fy$

45.71 ksi **(7.3)**

> $Pn := Fcr\,A$

722.27 kipf **(7.4)**

Allowable axial strength:

> $Pc := \frac{Pn}{\Omega}$

432.50 kipf **(7.5)**

This is greater than 3/4 Pr, so it is satisfactory.

Available flexural strength (Chapter F AISC 360-10):

> $Mn := \min(FyZx, FySx)$

250.00 foot kipf **(7.6)**

> $Mcx := \frac{Mn}{\Omega}$

$$149.70\ \text{foot kipf} \tag{7.7}$$

This is greater than Mrx, so it is satisfactory.

> $Mcy := \frac{Mn}{\Omega}$

$$149.70\ \text{foot kipf} \tag{7.8}$$

These should be below 1 for a satisfactory design.

> $\frac{P}{Pc} + \frac{8}{9} \cdot \left(\frac{Mrx}{Mcx} + \frac{My}{Mcy} \right)$

$$.99 \tag{7.9}$$

Determine Deflections

Max twist angle (Lin, p100 eq4) in degrees:

> $\phi := \frac{T}{2\,GJT\lambda} \cdot \left(\frac{\lambda \cdot L}{2} - \frac{2 \cdot \sinh\left(\frac{\lambda \cdot L}{2} \right)}{\sinh(\lambda \cdot L)} \right) \cdot \sinh\left(\frac{\lambda \cdot L}{2} \right)$

$$\phi := 0.2304416908 \tag{8.1}$$

> $I3 := Ix \sin\left(\frac{(90 - \phi)\,\pi}{180} \right)^2 + Iy \cos\left(\frac{(90 - \phi)\,\pi}{180} \right)^2$

$$303.00\ \text{in}^4 \tag{8.2}$$

> $I4 := Ix \cos\left(\frac{(90 - \phi)\,\pi}{180} \right)^2 + Iy \sin\left(\frac{(90 - \phi)\,\pi}{180} \right)^2$

$$103.00\ \text{in}^4 \tag{8.3}$$

Vertical deflection at the middle:

> $\Delta vert := \frac{5\,w\,L^4}{384\,E\,I3}$

$$.15\ \text{in} \tag{8.4}$$

Horizontal deflection at the middle:

> $\Delta horiz := \frac{F\,L^3}{48\,E\,I4}$

$$.20\ \text{in} \tag{8.5}$$