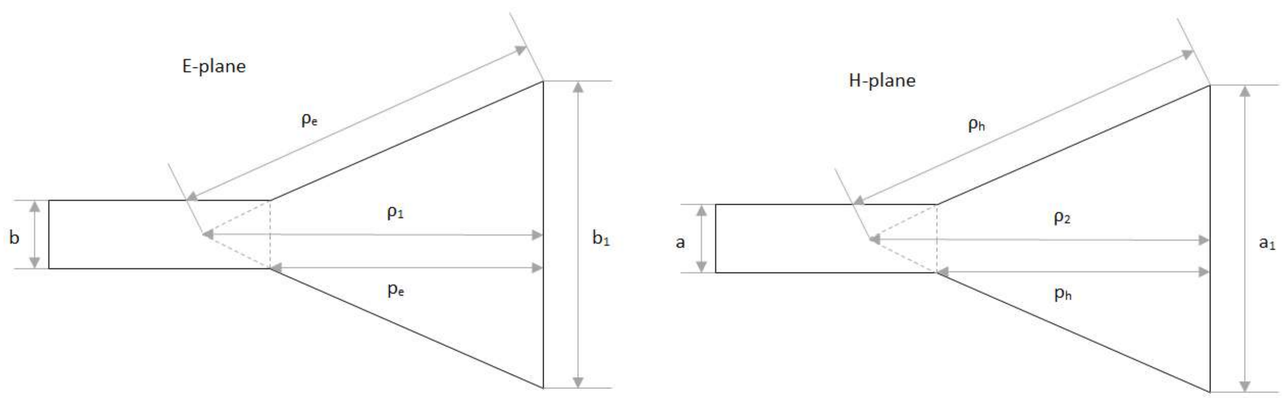
Pyramidal Horn Design

Introduction

This application calculates the optimum design parameters for an X-band pyramidal horn.



Reference:
Based on example 13.6, page 782 of *Antenna Theory, Analysis and Design*, Constantine A. Balanis, 3rd Edition.

> *restart* :
> *with*(*plots*) :
> *with*(*ColorTools*) :

Parameters

The design equations require that the gain be unitless and that the wavelength, l, be in cm. So our first equations which address design criteria (1) and (2), are

Gain in dB at design frequency:

> $G_{odB} := 22.6$:

Hence

> $G_o := 10^{\frac{G_{odB}}{10}}$

(2.1)

$$G_o := 181.9700859 \quad \textbf{(2.1)}$$

Frequency (s^{-1}):

> $f := 11.0 \cdot 10^{9.}$:

Geometrical constraints (cm):

> $a := 2.286$:
> $b := 1.016$:

Speed of light (cm/s):

> $c := 3 \cdot 10^{10}$:

Wavelength (in cm):

> $\lambda := \frac{c}{f}$

$$\lambda := 2.727272727 \quad \textbf{(2.2)}$$

Governing Equations

These equations are extracted from the reference, and are derived therein.

We require the following for optimum directivity.

> $cons1 := G_o = \frac{2 \cdot \pi}{\lambda^2} \cdot a_1 \cdot b_1$:

> $cons2 := a_1 = \sqrt{3 \cdot \lambda \cdot \rho_h}$:

> $cons3 := b_1 = \sqrt{2 \cdot \lambda \cdot \rho_e}$:

The dimensions p_e and p_h are equal.

> $cons4 := p_e = (b_1 - b) \cdot \sqrt{\left(\frac{\rho_e}{b_1}\right)^2 - \frac{1}{4}}$:

> $cons5 := p_h = (a_1 - a) \cdot \sqrt{\left(\frac{\rho_h}{a_1}\right)^2 - \frac{1}{4}}$:

> $cons6 := p_e = p_h$:

Numerically Solve the Governing Equations

> $res := fsolve(\{cons1, cons2, cons3, cons4, cons5, cons6\})$

(4.1)

$$res := \{a_1 = 16.55573668, b_1 = 13.01154178, p_e = 27.97911838, p_h = 27.97911838, \rho_e = 31.03837357, \rho_h = 33.50018428\} \quad (4.1)$$

Plot the E-Plane Radiation Pattern

> $assign(res)$

> $\rho_1 := \sqrt{\rho_e^{\,2} - \left(\frac{b_1}{2}\right)^2}:$

> $t_1 := \theta \rightarrow \sqrt{\frac{2}{\lambda \cdot \rho_1}} \cdot \left(-\frac{b_1}{2} - \rho_1 \cdot \sin(\theta)\right):$

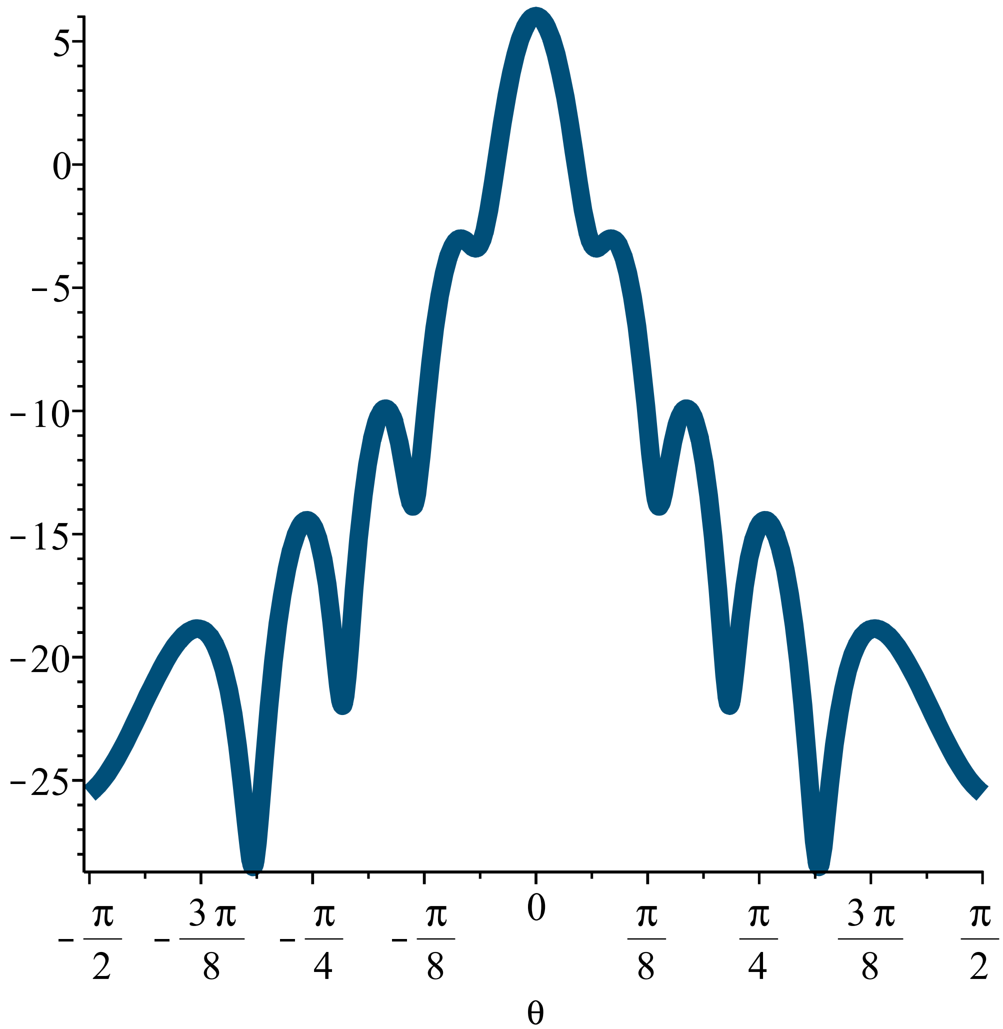
> $t_2 := \theta \rightarrow \sqrt{\frac{2}{\lambda \cdot \rho_1}} \cdot \left(\frac{b_1}{2} - \rho_1 \cdot \sin(\theta)\right):$

> $F := \theta \rightarrow \mathrm{FresnelC}(t_2(\theta)) - \mathrm{FresnelC}(t_1(\theta)) - I \cdot (\mathrm{FresnelS}(t_2(\theta)) - \mathrm{FresnelS}(t_1(\theta))):$

Radiation Pattern

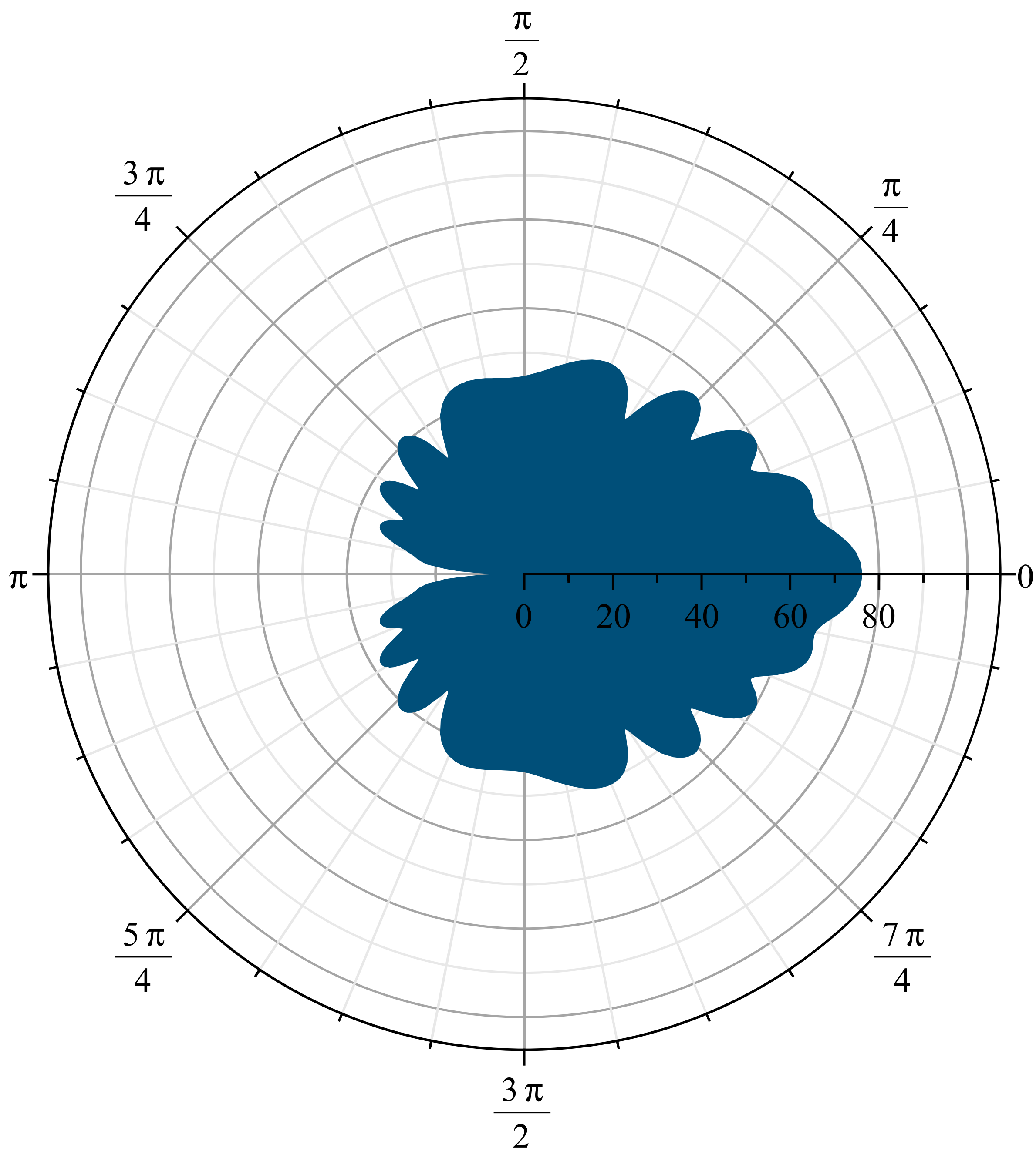
> $E_\theta := \theta \rightarrow 20 \cdot \log 10\left(|1 + \cos(\theta)| \cdot \frac{|F(\theta)|}{|F(0)|}\right):$

> $plot\left(E_\theta(\theta), \theta = -\frac{\pi}{2} .. \frac{\pi}{2}, thickness = 7, color = Color(\text{"RGB"}, [0, 79/255, 121/255]), axes = frame\right)$



> *polarplot*$\left(E_\theta(\theta) + 70, \theta = 0..2\pi\right.$, *thickness* = 0, *color* = *Color*("RGB", [0, 79 / 255, 121 / 255]), *filled* = *true*, *transparency* = 0, *title* = "E-Plane Radiation Pattern", *size* = [800, 800])

E-Plane Radiation Pattern



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