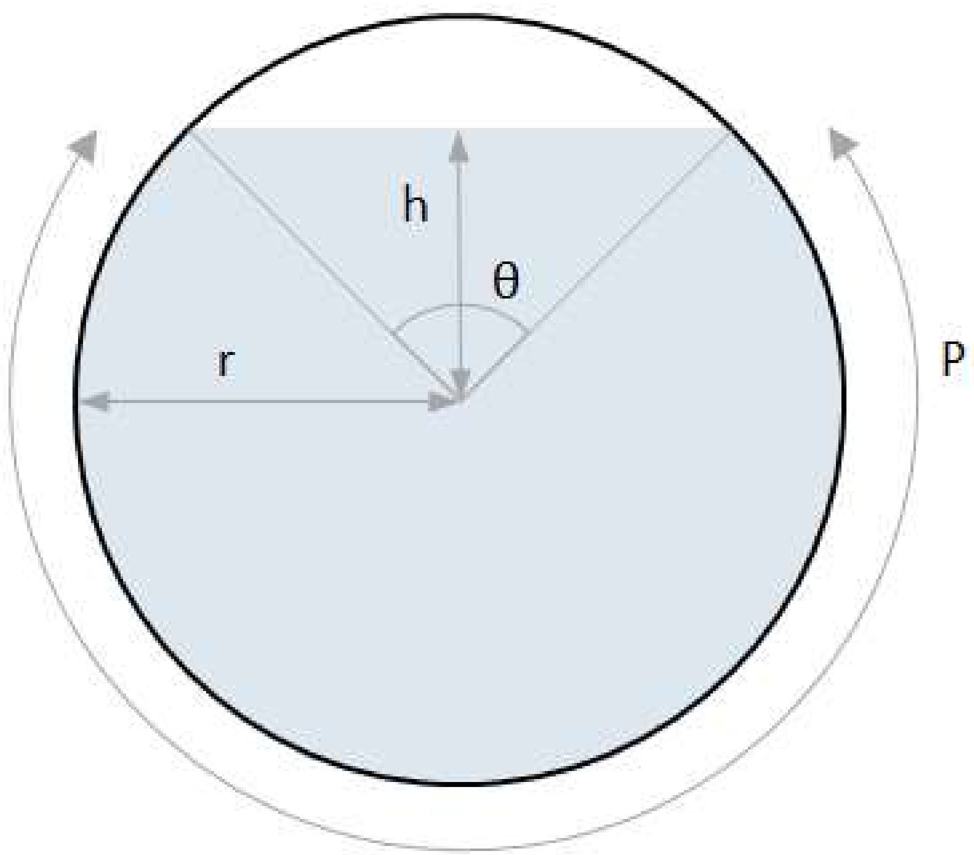
Maximum Flow Rate in Open-Channel Flow for a Circular Pipe

Introduction

This application determines the greatest attainable water flow rate in a partially filled circular pipe.



It uses the Manning formula to determine the flow rate in the open-channel flow of water:

$$Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}},$$

where:

- Q is the flow rate

- n is an empirical coefficient
- A is the cross-sectional area of flow
- R is the hydraulic radius
- S_0 is the incline of the channel

An equation that represents the hydraulic radius of a partially filled circular pipe is derived and substituted into the Manning formula. The resulting equation is then optimized to find the maximum flow rate.

Manning Formula for a Circular Pipe

> *restart* :

Manning formula:

$$> Q := \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} :$$

For a partially filled circular pipe, the flow area (the blue shaded area in the preceding diagram) is:

$$> A := \pi \cdot r^2 - r^2 \cdot \frac{(\theta - \sin(\theta))}{2} :$$

The wetted perimeter is given by the following formula.

$$> P := 2 \cdot \pi \cdot r - r \cdot \theta :$$

Hence, the hydraulic radius, *R*, is:

$$> R := \frac{A}{P}$$

$$R := \frac{\pi r^2 - r^2 \left(\frac{\theta}{2} - \frac{\sin(\theta)}{2}\right)}{2\pi r - r\theta} \qquad \textbf{(2.1)}$$

The Manning formula then becomes:

> *Q*

$$\frac{1.49 \left(\pi r^2 - r^2 \left(\frac{\theta}{2} - \frac{\sin(\theta)}{2}\right)\right) \left(\frac{\pi r^2 - r^2 \left(\frac{\theta}{2} - \frac{\sin(\theta)}{2}\right)}{2\pi r - r\theta}\right)^{2/3} \sqrt{S_0}}{n} \qquad \textbf{(2.2)}$$

Maximum Flow Rate for a Circular Pipe

Pipe radius:

> $r := 3:$

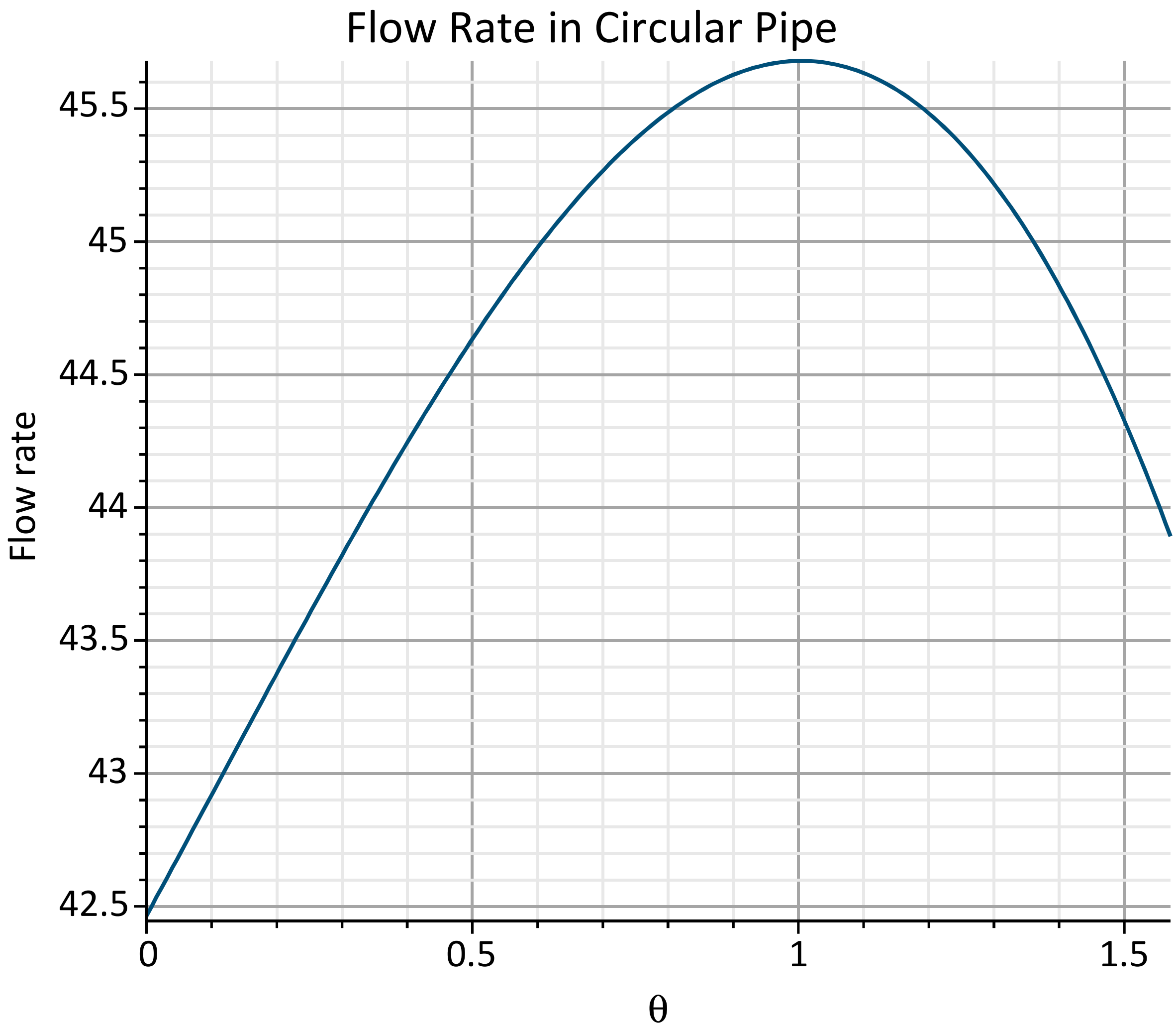
Incline of the channel:

> $S_0 := 0.0001:$

Roughness coefficient:

> $n := 0.013:$

> plot(Q, θ = 0 .. 0.5 π, labels = ["θ", "Flow rate"], labeldirections = [horizontal, vertical], labelfont = [Calibri], title = "Flow Rate in Circular Pipe", titlefont = [Calibri, 14], size = [600, 400], axesfont = [Calibri], gridlines, color = ColorTools:-Color("RGB", [0 / 255, 79 / 255, 121 / 255]))



> $res := Optimization:\text{-}Maximize(Q, \theta = 0 .. 0.5\pi)$

$$res := [45.6796864427174, [\theta = 1.00507814259573]] \quad (1)$$

The maximum flow rate is...

> $Q_{maxflow} := res[1]$

$$Q_{maxflow} := 45.6796864427174 \quad (2)$$

...when θ is ...

> $\theta_{maxflow} := rhs(res[2, 1]);$

$$\theta_{maxflow} := 1.00507814259573 \tag{3}$$

... and the flow depth is:

> $h := r \cdot \cos(0.5\,\theta_{maxflow}) :$
> $h + r$

$$5.62908731621774 \tag{4}$$