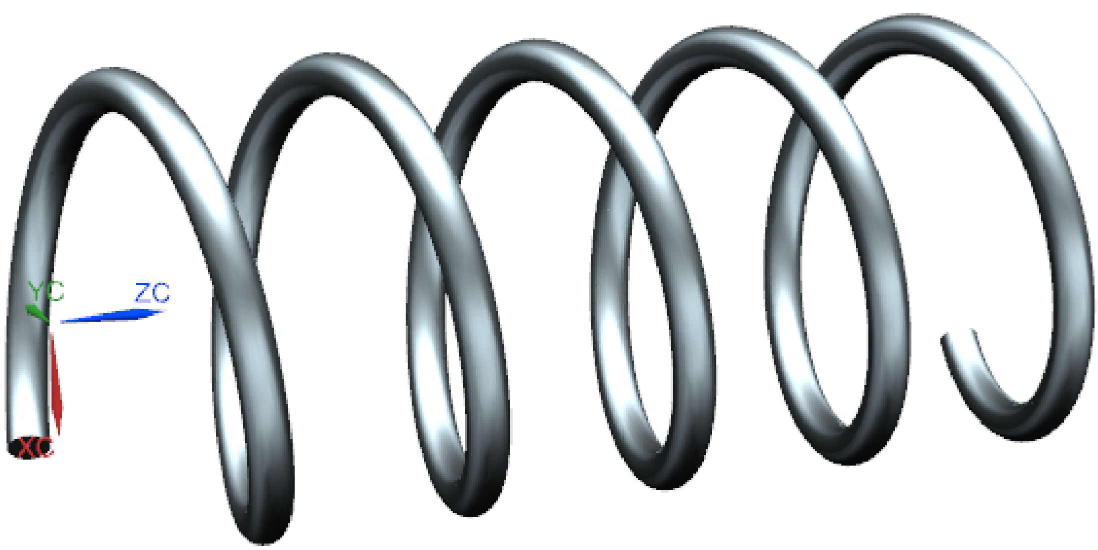
Optimizing the Design of a Helical Spring

Introduction

The design optimization of helical springs is of considerable engineering interest, and demands strong solvers. While the number of constraints is small, the coil and wire diameters are raised to higher powers; this makes the optimization difficult for gradient-based solvers working in standard floating-point precision; a larger number of working digits is needed.

Maple lets you increase the number of digits used in calculations; hence numerically difficult problems like this can be solved.

This application minimizes the mass of a helical spring. The constraints include the minimum deflection, the minimum surge wave frequency, the maximum stress, and a loading condition.



The design variables are the diameter of the wire d, the outside diameter of the spring D, and the number of coils N.

Reference: Arora, Jasbir S. Introduction to Optimum Design. 3rd edition. Massachusetts: Academic Press, 2011.

```
> restart:
  local γ:
  with(Units[Simple]):
```

Parameters

Gravitational constant

> $g := 386\ \mathrm{inch}\ \mathrm{s}^{-2}$:

Weight density of spring material

> $\gamma := 0.285\ \mathrm{lbf}\ \mathrm{inch}^{-3}$:

Shear modulus

> $G := 1.15 \cdot 10^{7}\ \mathrm{lbf}\ \mathrm{inch}^{-2}$:

Mass density of material

> $\rho := \frac{\gamma}{g}$

$$7890.583227\ \frac{\mathrm{kg}}{\mathrm{m}^{3}}$$

Allowable shear stress

> $\tau_{a} := 80000\ \mathrm{lbf}\ \mathrm{inch}^{-2}$:

Number of inactive coils

> $Q := 2$:

Applied Load

> $P := 10\ \mathrm{lbf}$:

Minimum spring deflection

> $\Delta := 0.5\ \mathrm{inch}$:

Lower limit of surge wave frequency

> $\omega_{0} := 100\ \mathrm{Hz}$:

Limit on outer diameter of coil

> $D_{0} := 1.5\ \mathrm{inch}$:

Engineering Relationships

Spring Constant

$$K := \frac{d^4 \cdot G}{8 \cdot D^3 \cdot N}$$

$$\frac{1.44 \times 10^6 \, d^4}{D^3 \, N} \, \frac{\text{lbf}}{\text{in}^2}$$

Shear stress

$$\tau := \frac{8 \cdot k \cdot P \cdot D}{\pi \cdot d^3}$$

$$\tau := \frac{80 \, k \, D}{\pi \, d^3} \, \text{lbf}$$

Wahl stress concentration factor

$$k := \frac{4\,D - d}{4 \cdot (D - d)} + \frac{0.615 \cdot d}{D}$$

$$k := \frac{4\,D - d}{4\,D - 4\,d} + \frac{0.615\,d}{D}$$

Frequency of surge waves

$$\omega := \frac{d}{2 \cdot Pi \cdot N \cdot D^2} \cdot \sqrt{\frac{G}{2 \cdot \rho}}$$

$$\frac{356.746\,d}{N\,D^2} \, \frac{\text{m}}{\text{s}}$$

Constraints

Minimum deflection

$$cons1 := \frac{P}{K} \geq \Delta$$

$$500.00 \times 10^{-3} \leq \frac{176.70 \times 10^{-9} \, D^3 \, N}{d^4} \, \text{m}$$

The outer diameter of the spring should be smaller than or equal to D_0.

$$cons2 := D + d \leq D_0$$

$$D + d \leq 38.10 \times 10^{-3} \, \text{m}$$

Avoid resonance by making the frequency of surge waves along a spring greater than a minimum defined value.

> $cons3 := \omega \geq \omega_0$

$$100.00 \leq \frac{356.75\, d}{N\, \mathrm{D}^2}\ \mathrm{m}$$

The shear stress cannot exceed the allowable shear stress.

> $cons4 := \tau \leq \tau_a$

$$\frac{80.00 \left(\frac{4.00\,\mathrm{D} - d}{4.00\,\mathrm{D} - 4.00\, d} + \frac{615.00 \times 10^{-3}\, d}{\mathrm{D}} \right) \mathrm{D}}{\pi\, d^3} \leq 124.00 \times 10^{6}\ \frac{1}{\mathrm{m}^2}$$

Collect all the constraints

> $cons := \{cons1, cons2, cons3, cons4\}:$

Objective function

Mass of spring

> $mass := \frac{1}{4} \cdot (\mathrm{N} + \mathrm{Q}) \cdot \pi^2 \cdot \mathrm{D} \cdot \mathrm{d}^2 \cdot \rho$

$$77876.93497 \left(\frac{N}{4} + \frac{1}{2} \right) \mathrm{D}\, d^2\ \frac{\mathrm{kg}}{\mathrm{m}^3}$$

Optimization

> $bounds := \mathrm{N} = 2..15,\ \mathrm{d} = 0.05\,\mathrm{inch}..2\,\mathrm{inch},\ \mathrm{D} = 0.25\,\mathrm{inch}..\mathrm{D}_0:$

Hence the optimized design variables are

> $Digits := 20:$
> $results := Optimization\text{:-}Minimize(mass, cons, bounds, iterationlimit = 10^5):$

The optimized spring has a weight of

> $results_1$

$$8.92 \times 10^{-3}\ \mathrm{lb}$$

and dimensions of

> $results_2$

$$[\mathrm{D} = 356.88 \times 10^{-3}\ \mathrm{in},\ N = 11.29,\ d = 51.70 \times 10^{-3}\ \mathrm{in}]$$