Escape-Time Fractals

Main Concept

Fractals are geometric shapes that exhibit self-similarity. That is, they have the same pattern at different scales. In fact, fractals continue to show intricate details at arbitrarily small scales.

One variety of fractals is the escape-time fractal. These fractals are generated by iterating a formula on each point in a given space. If a point diverges as the formula is iterated, it escapes; otherwise, it remains bounded. Three of the more well known escape-time fractals are the Mandelbrot set, Julia set, and Burning Ship fractal.

The Mandelbrot set uses complex values, and is generated by iterating the following formula on each point *c*:

$$z \to z^2 + c,\ z_0 = 0$$

The Julia set uses the same formula $z \to z^2 + c$, but the iteration is different. For a fixed parameter c, the Julia set is found by iterating the formula on each point z.

The Burning Ship fractal is again similar to the Mandelbrot set. It uses the same formula, but before squaring *z*, the absolute values of its real and imaginary components are taken and used in place of the original *z* point:

$$z \to (\ |\Re(z)| + I \cdot |\Im(z)|\ )^2 + c, z_0 = 0.$$

The Newton fractal uses the formula:

$$z \to z - \frac{p(z)}{p'(z)}.$$

This formula is somewhat different from the three others listed above, as it takes a function *p* as a parameter. The Newton fractal is also different in that instead of checking whether a point diverges, it checks whether a point converges to a root.

Despite their similar underlying mathematics, the stunning images of these fractals all have their own unique features.

Using the App

Specify the fractal and set parameters pertaining to the generation of the fractal in the first box on the right. Because it is not possible to iterate infinitely to see if a given point will remain bounded, a point is instead considered to have escaped after it passes a given cut-off value. If it has not passed this value after a specified number of iterations, it is assumed that the point will not escape. These parameters can be adjusted with the **cut-off** and **iterations** sliders. Note that changing these values will not always result in a change in the final image.

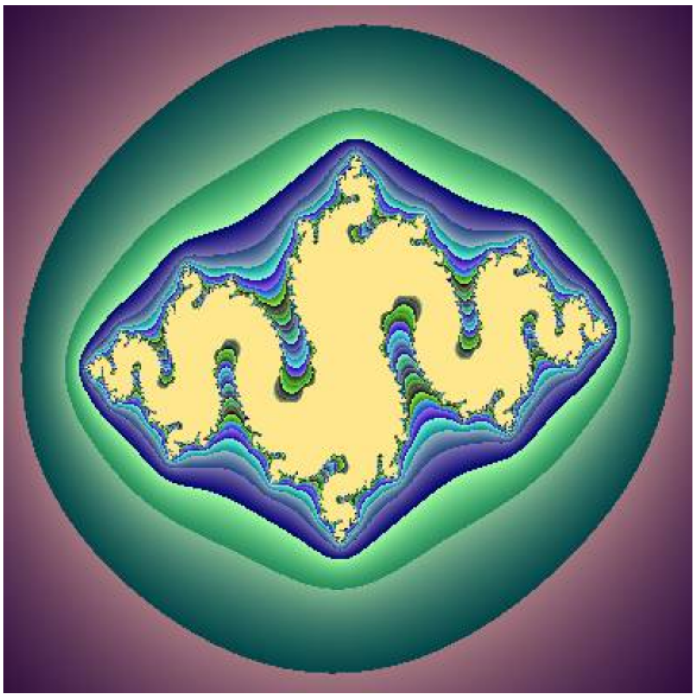
Julia set: The parameter specifying the *c* component of the Julia set formula can be changed by selecting a point on the plot in the center of the second box. At low resolutions, clicking and dragging this point will continuously update the image; however, at high resolutions, this will not occur.

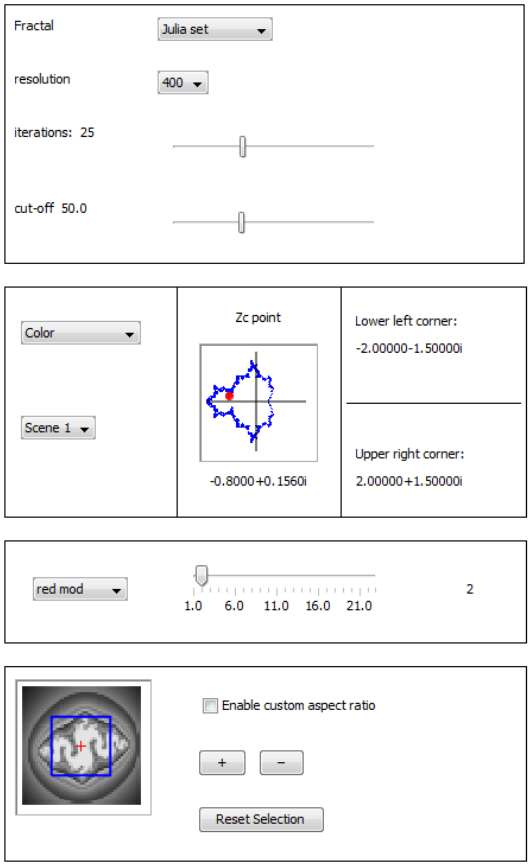
Newton fractal: The function used in the formula for the Newton fractal can be changed by entering it in the box provided. This function must be a function of *z* (and no other variables), and it must be written in correct Maple syntax. It may be easier to first enter the function in 2-D math mode in a separate worksheet, and then copy the function to the box. The image will not update upon changing the formula; click **Compute** to update the image.

Parameters can also be set to specify how the image is colored. *Black and White* mode gives a grayscale image. *Monochromatic* mode uses the slider in the third box to specify proportions of red, green, and blue included in the image. *Toggle RGB* mode is similar, but colors can only be set to fully or not at all present. Each color can be toggled with a check box. Selecting all the boxes results in a black and white image; clearing all the boxes causes no image to display. In *Color* mode, the relative proportions of each color are not locked. The slider in the third box can be used to adjust parameters to change the coloring of the image.

For each fractal, a number of preset scenes have been set at points of interest with interesting settings. Zoom functions are provided to allow viewing of any area.

Two zoom modes are available. In regular zoom mode, a rectangle can be dragged in the larger box. Clicking inside the square will zoom in to the corresponding portion of the image, and clicking outside of the square will zoom out. The edges and corners of the square can be dragged to resize and reshape the rectangle. In *custom* zoom mode, any rectangle can be drawn by clicking and dragging. The buttons at the right can be used to zoom in or out in either mode. The **Reset Selection** button sets the rectangle to its default position in regular zoom mode. It clears the selection in custom zoom mode.





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