The Comet 67P/Churyumov-Gerasimenko
Rosetta & Philae

A. C. Baroudy
acbaroudy@yahoo.com

Abstract

The Rosetta space probe launched 10 years ago by the European Space Agency (ESA) arrived recently (November 12, 2014) at the site of the comet known as 67P/Churyumov-Gerasimenco after a trip of 4 billions miles from Earth. After circling the comet, Rosetta released its precious load : the lander Philae packed with 21 different scientific instruments for the study of the comet with the main purpose : the origin of our solar system and possibly origin of life on our planet.

Our plan is rather a modest one since all we want is to get , by calculation, specific data concerning the comet and its lander.
We shall take a simplified model and consider the comet as a perfect solid sphere to which we can apply Newton's laws.

We want to find:

I- the acceleration on the comet surface ,
II- its radius,
III- its density,
IV- the velocity of Philae just after the 1st bounce off the comet (it has bounced twice),
V- the time for Philae to reach altitude of 1000 m above the comet .

We shall compare our findings with the already known data to see how close our simplified mathematical model findings are to the duck-shaped comet already known results.
It turned out that our calculations for a sphere shaped comet are very close to the already known data.

Conclusion

Even with a shape that defies the application of any mechanical laws we can always get very close to reality by adopting a simplified mathematical model in any preliminary study of a complicated problem.

Data recently collected from different sources concerning the comet and its lander Philae

The limited data we have collected concerning this endeavor are as follows:

1**- Rosetta** was orbiting the comet at a distance of 22.5 km when it released **Philae,**
2- the **lander Philae** weighs 100 kg and was the size of a washing machine,
3- **touchdown velocity** of **Philae** on the comet was 1m/s (3.6 km/hr),
4- upon a **first landing** it **bounced twice** from its landing site "**Agilkia**" on the comet then:
    a- after the **first bounce** it reached an altitude of of **1 km (1000 m) during 2 hrs**

above the comet surface and **landed back for the 2d tim**e ,

b- then a **2d bounce** which took it to a shorter distance away from the comet then a **final landing.**

5- **Philae's escape velocity** from the comet equals 0.5 m/s (1.8 km/s) hence:

$$v_{escape} = 0.5 = \sqrt{\frac{2\,G\,m_{comet}}{r_{comet}}} = 0.5 = \sqrt{2\,g_{comet} \cdot r_{comet}}, \qquad (1)$$

6- the comet is a solid object with an **irregular duck- shape** outline to which we can not apply Newton's laws hence our model,

7- It has a maximum length of **4.100 km**,

8- if we use a spring balance to weigh the lander, while on the comet, we would find that **its weight** is only **1 gram** instead of 100,000 grams on Earth.

From these data we propose to find
1- the acceleration on the comet surface
2- comet radius ,
3- comet density,
4- Philae velocity just at the 1st bounce off the comet,
5- time for Philae to reach altitude of 1000 m above the comet.
We shall see how close our simplified model results to the duck-shaped comet known data.

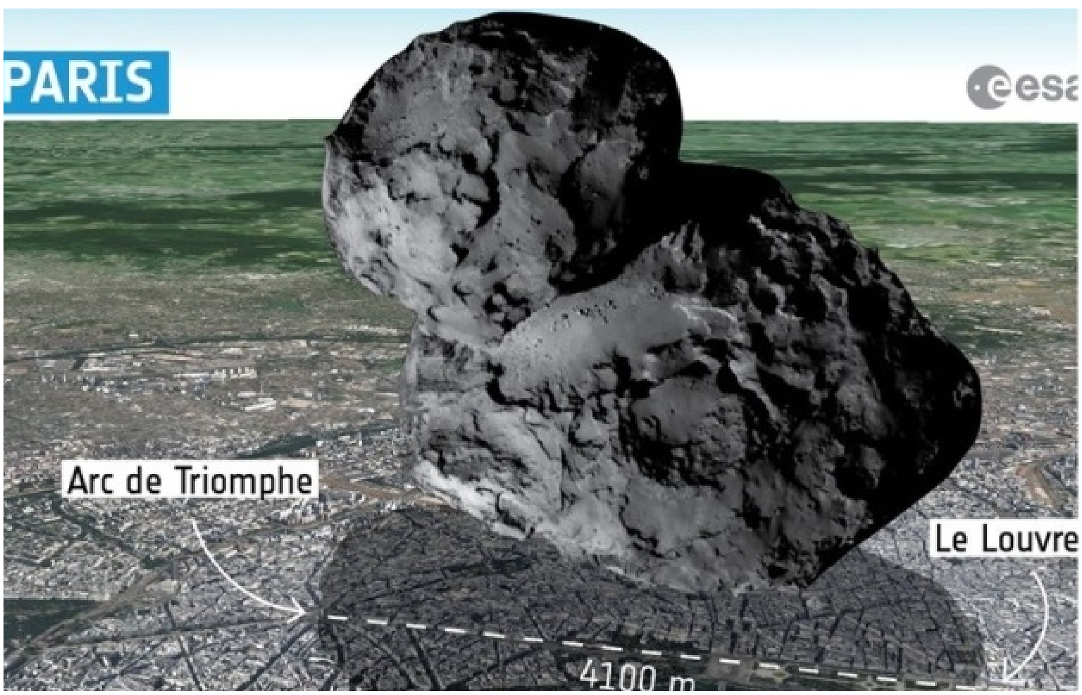

Photo:ESA

Before getting our problem solved we shall first apply our knowledge to Earth & its Moon.

Here are the data for both Earth & Moon

$$\begin{bmatrix} & \boldsymbol{Mass : kg} & \boldsymbol{Radius : m} \\ \boldsymbol{Earth} & \mathbf{5.98 \cdot 10^{24}} & \mathbf{6.38 \cdot 10^{6}} \\ \boldsymbol{Moon} & \mathbf{7.36 \cdot 10^{22}} & \mathbf{1.74 \cdot 10^{6}} \end{bmatrix},$$

Universal Constant G: $\boldsymbol{G = 6.67 \cdot 10^{-11}}$ **.**

PART ONE

DATA RELATED TO EARTH & MOON

Finding the acceleration g on Earth and g_{moon} on the Moon using:
1- Newton Universal Gravitation equation,
2- Newton equation of motion.

I- Finding g on Earth

From Newton law of gravitation we have:

$$F = \frac{G \cdot m \cdot M)}{r^2}, \qquad (2)$$

Where F is the **force of attraction** between body of mass **m** and body of mass **M** while r is the distance separating their center of mass. This formula is applicable when both m & M are perfect spheres.
We are interested in finding the **acceleration g which is the force of attraction on a unit mass**.
Hence equation (2), with m = 1 kg, becomes :

$$F = \frac{G \cdot M}{r^2}. \qquad (3)$$

For an object of **unit mass on Earth's surface** we also have the equation:

$$F = 1 \cdot g. \qquad (4)$$

Hence equating (3) & (4) we get the relation:

$$F = \frac{G \cdot M}{r^2} = g \quad \rightarrow \quad GM = gr^2.$$

Applying this formula to Earth surface we get:

$$g := \frac{G \cdot M}{R^2} = \frac{(6.670000000\ 10^{-11} \cdot 5.98 \cdot 10^{24})}{(6.38 \cdot 10^{6})^2} = 9.799088059 \simeq \frac{9.8\ m}{s^2}.$$

Which is what we expect.

II- Finding g_{moon} on the Moon

Now we can apply the same formula to the **surface of the Moon**:

$$g_{moon} := \frac{G \cdot m_{moon}}{r_{moon}^2} = \frac{(6.670000000\ 10^{-11} \cdot 7.36 \cdot 10^{22})}{(1.74 \cdot 10^{6})^2} = \frac{1.621455939\ m}{s^2}.$$

III- Finding the density on Earth's surface & on the Moon's

Let us find Earth density from equation (3):

$$F = \frac{G \cdot M}{R^2} = 9.8 == \frac{6.670000000\ 10^{-11} \cdot 4 \cdot \pi \cdot R^3 \cdot d}{3 \cdot R^2} = \frac{6.670000000\ 10^{-11} \cdot 4 \cdot \pi \cdot R \cdot d}{3},$$

which gives for

$$1\ m^3 = 5497.81267\ kg,$$

hence the density of the Earth

$$\mathbf{d} = \frac{5497.81267}{1000} \simeq \mathbf{5.5}\ .$$

As for the Moon we have:

$$f = \frac{G \cdot m_{moon}}{r_{moon}^2} = \frac{9.8}{6} = \frac{6.670000000\ 10^{-11} \cdot 4 \cdot \pi \cdot r_{moon}^3 \cdot d}{3 \cdot r_{moon}^2} = \frac{6.670000000\ 10^{-11} \cdot 4 \cdot \pi \cdot r_{moon} \cdot d_{moon}}{3},$$

which gives for density of the Moon:

$$d_{moon} = \frac{3359.774412}{1000} \simeq \mathbf{3.36}\ .$$

IV- Finding the acceleration ratio $\frac{g_{moon}}{g_{earth}}$

The ratio $\frac{g_{moon}}{g_{earth}}$ of the accelerations is

$$\frac{g_{moon}}{g_{earth}} = \frac{1.621455939}{9.8} = \frac{\frac{7.36 \cdot 10^{22}}{(1.74 \cdot 10^6)^2}}{\frac{5.98 \cdot 10^{24}}{(6.38 \cdot 10^6)^2}} \quad \frac{\frac{m_{moon}}{r_{moon}^2}}{\frac{M_{earth}}{R_{earth}^2}} = 0.1654546877 \simeq \frac{1}{6},$$

thus the force of attraction on the Moon is 1/6 of that on Earth.
Hence if we take a mass of 1kg to the Moon and suspend it at the end of a spring balance, say a dynamometer similar to the one depicted below, it will show that its weight on the Moon is only $165.4546877 \simeq 166.6$ *grams*.

Photo : Shanghai MCP Corp

We can simplify the above acceleration ratio formula using the equivalence mass = volume . density:

$$\frac{g_{moon}}{g_{earth}} = \frac{\dfrac{(7.36\cdot 10^{22})}{(1.74\cdot 10^{6})^2}}{\dfrac{(5.98\cdot 10^{24})}{(6.38\cdot 10^{6})^2}},$$

$$\frac{\dfrac{\frac{4}{3}\cdot\pi\cdot r^3_{moon}\cdot d_{moon\,density}}{r^2_{moon}}}{\dfrac{\frac{4}{3}\cdot\pi\cdot R^3_{earth}\cdot d_{earth\,density}}{R^2_{earth}}},$$

$$\mathbf{\frac{r_{moon}\cdot 3.360}{R_{earth}\cdot 5.500} = 0.1654700854}\ . \qquad (5)$$

PART TWO

DATA RELATED TO THE COMET AND ITS LANDER PHILAE FOUND BY CALCULATION

I- Finding the comet acceleration

1- a mass of **1 kg on Earth** attached to a dynamometer, the dynamometer reads 1000 grams i.e.

$$9.8\ Newtons = 1\ kg\cdot g_{earth} = 1\ kg\cdot\frac{9.8\ m}{s^2},$$

2- a mass of **1 kg on the Moon** where acceleration equals $\frac{g_{earth}}{6} = \frac{9.8}{6}$, the dynamometer reads 166.666 grams = 1000/6 since:

$$\frac{9.8}{6} = 1\ kg\cdot g_{moon} = 1\ kg\cdot\frac{9.8}{6} = \frac{1\ kg}{6}\cdot 9.8 = 0.166666\cdot 9.8,$$

i.e. a mass of 1000 grams taken to the Moon would weigh only $\frac{1\ kg}{6} = 166.666\ grams$.

By comparison we can also say that since a mass of 100 kg on the comet weighs only 1 gram then 1 kg should weigh on the comet 1/100 of a gram i.e. $\frac{1}{100}gram = \frac{1000}{100,000}gram = \frac{1\ kg}{10^5}$ i.e.

$$\frac{9.8}{10^5} = 1\ kg\cdot\frac{9.8}{10^5}.$$

Hence the acceleration on the comet must be equal to $\mathbf{\frac{9.8}{10^5}}$.

II- Finding the radius R_{comet} of the comet assuming it is a sphere

We can now apply (1) to the comet to get its radius:

$$v_{escape} = 0.5 = \sqrt{2\, g_{comet} \cdot R_{comet}},$$

$$0.5 = \sqrt{\frac{2 \cdot 9.8}{10^5} \cdot R_{comet}}, \rightarrow \; \boldsymbol{R_{comet} = 1275.510204 \; meters}$$

Since the comet is not a perfect sphere but an **irregular duck-shaped comet** with the largest dimension being 4100 meters then when **compacted into a perfect sphere** it would have a ***diameter of 2550 meters which is very plausible result.***

III- Finding the density d_{comet} of the comet assuming it is a sphere

From (1) we also have

$$v_{escape} = 0.5 = \sqrt{\frac{2\, G\, m_{comet}}{r_{comet}}},$$

$$0.5 = \sqrt{\frac{2\, G\, \frac{4}{3} \cdot \pi \cdot R_{comet}^3 \cdot d_{comet}}{R_{comet}}} \quad \text{where } R_{comet} \text{ \underline{is expressed in decimeter}},$$

$$0.5 = \sqrt{2\, G\, \frac{4}{3} \cdot \pi \cdot R_{comet}^2 \cdot d_{comet}},$$

$$\sqrt{\frac{2 \cdot 6.67 \cdot 10^{-11} \cdot 4}{3} \cdot 3.1416 \cdot (12750.510204)^2 \cdot d_{comet}} = 0.5,$$

we get : $\boldsymbol{comet\ density = d_{comet} = 2.751942961}$,

which is about half the density of the Earth's and so it is a ***very plausible result.***

<u>Commentary:</u>

The reported comet mean density was 0.4 which is less than half of water's density! If we take this for granted as well as the reported mass of $10^{13} kg = 10^{10} tons$ then a homogeneous sphere of the same volume would have for radius:

$$\frac{4}{3} \cdot 3.1416 \cdot r_{comet}^3 = \frac{m_{comet}}{comet_{density}} = \frac{10^{10}}{0.4} = 2.500000000\ 10^{10},$$

r_{comet}= 1814 m,

which is still in the same range as our finding above of 1275 m.

On the other hand if we consider the radius of the comet as R_{comet}= 1840 m =18400 decimeters and use escape velocity $v_{escape} = 1$ as reported by some to find the density from the relation:

$$v_{escape} = 1 = \sqrt{\frac{2\, G\, \frac{4}{3} \cdot \pi \cdot R_{comet}^3 \cdot d_{comet}}{R_{comet}}} \quad \text{where } R_{comet} \text{ \underline{is expressed in decimeter}}$$

then we would find the comet density = 5.438510607! which is the same as that of the Earth. This result is a far cry from the reported 0.4.

Conclusion :

either $v_{escape} = 1$ is not correct and $v_{escape} = 0.5$ is more likely or the radius should be 1840 m. In this last case with $v_{escape} = 0.5$ the density = 1.359627652.

IV- Velocity of Philae at the first bounce off the comet as a sphere

Using the equation of kinetic energy and potential energy and **taking into consideration the variation of the comet's gravitational field with altitude** we shall find the velocity of Philae at the moment of the 1st bounce off the comet.

We know that Philae with bounce velocity v_0 reached an altitude of 1000 m.

We take x-axis positive upward and the origin at the surface of the comet, the only force acting on Philae is its own weight given by:

$$-\frac{G\,M_{comet}\,m_{Philae}}{(x+R_{comet})^2},$$

hence the equation of motion becomes:

$$m_{Philae}\cdot\frac{dv}{dt} = -\frac{G\cdot M_{comet}\cdot m_{Philae}}{(x+R_{comet})^2} \quad \text{with initial velocity } v_0 .$$

Replacing $G\cdot M_{comet}$ with $g_{comet}\cdot R^2_{comet}$ and simplifying we get:

$$\frac{dv}{dt} = -\frac{g_{comet}\cdot R^2_{comet}}{(x+R_{comet})^2}. \qquad (6)$$

To solve (6) we have to make v as a function of x using the chain rule of differentiation:

$$\frac{dv}{dt} = \frac{dv}{dx}\,\frac{dx}{dt} = v\,\frac{dv}{dx},$$

hence (6) becomes:

$$v\,\frac{dv}{dx} = -\frac{g_{comet}\cdot R_{comet}}{(x+R_{comet})^2},$$

$$v\;dv = -\frac{(g\,R^2)_{comet}}{(x+R_{comet})^2}\,dx . \qquad (7)$$

Integrating (7) we get:

$$\frac{1}{2}v^2 = \frac{(g\,R^2)_{comet}}{(x+R_{comet})} + c . \qquad (8)$$

at t = 0, v = v_0 and x = 0 then equation (8) becomes:

$$\boldsymbol{v^2 = v_0^2 - 2\,(g\,R)_{comet} + 2\,\frac{(g\,R^2)_{comet}}{(x+R_{comet})}} . \qquad (9)$$

Philae reaches altitude **x =1000 m** when its velocity **v = 0**.

We plug these data into equation (9) along with $\boldsymbol{g_{comet} = \frac{9.8}{10^5}}$, $\boldsymbol{R_{comet} = 1275}$ then solve for $\boldsymbol{v_0}$ we get:

$$\boldsymbol{v_0 = 0.3314304660 \text{ m/s}} .$$

This result is about $\frac{2}{3}v_{escape\,velocity}$ on the comet and is very likely.

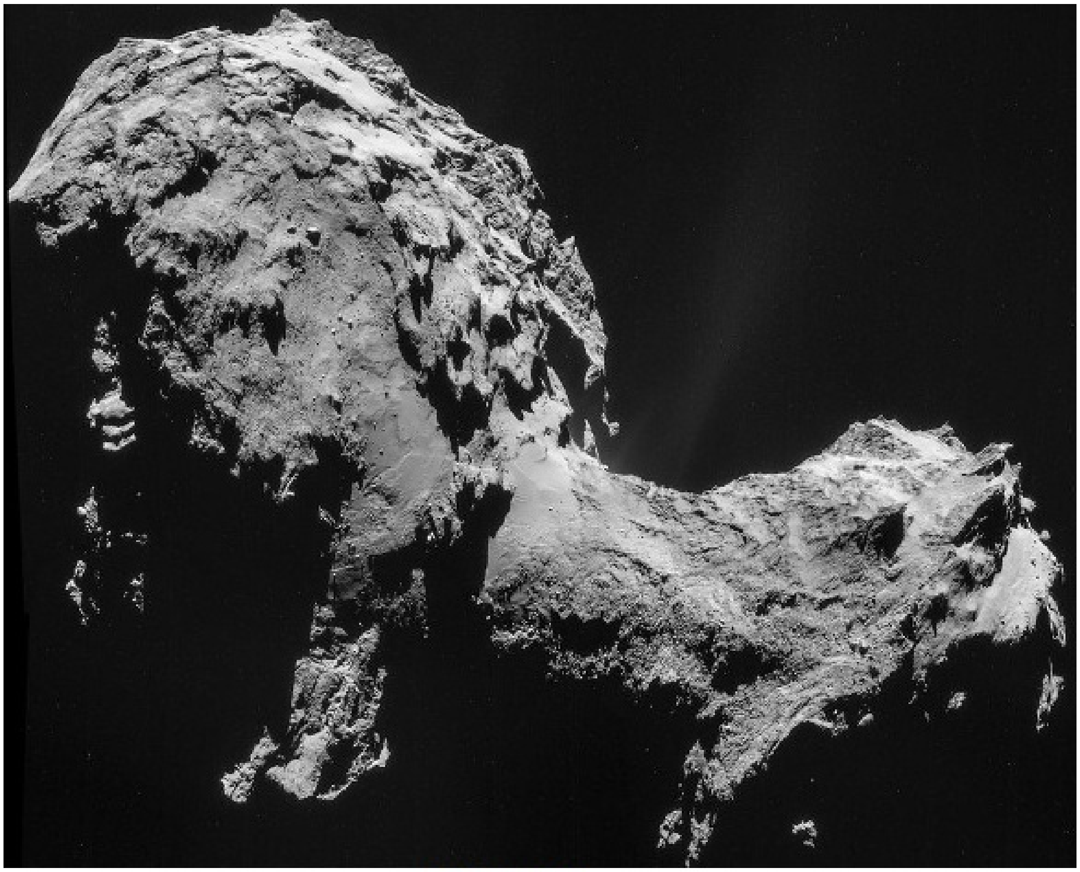

Photo: ESA

V- Time for the lander Philae to reach altitude of 1000 m after the first bounce off the comet as a sphere

Here we use equation (9) to get the time for the lander to reach altitude 1000 m above the comet when its initial velocity is

$$v_0 = 0.3314304660 \text{ m/s} ,$$

$$v^2 = \left(\frac{dx}{dt}\right)^2 = v_0^2 - 2\,(g\,R)_{comet} + 2\,\frac{(g\,R^2)_{comet}}{(x + R_{comet})} \ .$$

hence

$$\frac{dx}{\sqrt{v_0^2 - 2(g\,R)_{comet} + 2\cdot\frac{(g\,R^2)_{comet}}{(x + R_{comet})}}} = dt\,. \qquad (10)$$

Integrating (10) for x = 0 to x = 1000 gives the time of ascent of the lander.
This time is found to be:

$$t = 2\ hrs\ \text{ and } \frac{6}{100}\ hr,$$

which is pretty much the one reported above .

As we can see all 5 results we arrived at are very close to what was known from the published data except for the comet density.

Conclusion

Even with a shape that defies the application of any mechanical laws we can always get very close to reality by adopting a simplified mathematical model in any preliminary study of a complicated problem.

> *restart* :

$G := 6.67 \cdot 10^{-11}; M := 5.98 \cdot 10^{24}; r := 6.38 \cdot 10^{6}; d := 5.5; R_{moon} := 1.74 \cdot 10^{6}; dd_{moon} := 3.36; g := 9.8; \alpha := \frac{9.8}{6}$

$$G := 6.670000000\ 10^{-11}$$
$$M := 5.980000000\ 10^{24}$$
$$r := 6.38000000\ 10^{6}$$
$$d := 5.5$$
$$R_{moon} := 1.74000000\ 10^{6}$$
$$dd_{moon} := 3.36$$
$$g := 9.8$$
$$\alpha := 1.633333333 \quad \textbf{(1)}$$

Getting the radius of the comet from the equation of the escape velocity

$v_{escape} = \sqrt{2 \cdot g_{comet} \cdot r_{comet radius}} = \mathbf{1275.510204}\ m$

> $0.5 = \sqrt{\frac{2 \cdot 9.8}{10^{5}} \cdot R_{comet}}$; $solve(\%, R_{comet})$

$$0.5 = 0.01400000000\ \sqrt{R_{comet}}$$
$$1275.510204 \quad \textbf{(2)}$$

Getting density from

$v_{escape} = \sqrt{\frac{2\ G \cdot m_{comet} kg}{r_{comet}\ meters}} = \sqrt{\frac{\frac{2\ G \cdot 4}{3} \cdot 3.1416 \cdot r_{comet}^{3} (decimeters) \cdot density}{r_{comet}\ meters}}$, since the radius = 1275.510204 meters then we use it in decimeter i.e. we use it as 12750.510204 *decimeters.*

We get for the density of the comet 2.75 which is exactly half that of the Earth (5.5).

> $\sqrt{\frac{2 \cdot 6.67 \cdot 10^{-11} \cdot 4}{3} \cdot 3.1416 \cdot (12750.510204)^{2} \cdot \text{dens}} = 0.5$; $solve(\%, dens)$

$$0.3014048874\ \sqrt{dens} = 0.5$$
$$2.751942961 \quad \textbf{(3)}$$

Getting the velocity of the lander Philae at the first bounce off the comet.

> $\mathbf{v_0^2 - \frac{2 \cdot 9.8}{10^5} \cdot 1275 + \frac{2 \cdot \frac{9.8}{10^5} \cdot 1275^2}{1000 + 1275} = 0}$; $solve(\%, v_0)$;

$$v_0^2 - 0.1098461538 = 0$$

$$0.3314304660, \ -0.3314304660 \qquad \textbf{(4)}$$

Integrating to get the time of ascent.

We use either int(f,x=0..1000) since Maple can get the integration or use the form $evalf\left(\int_0^{1000} \mathrm{f}\, dx\right)$ for numeric integration in case Maple doesn't have a formula for the solution of the integral.

> $restart : f := \frac{1}{\sqrt{\left(v_0^2 - 2 \cdot g \cdot R + \frac{2 \cdot g \cdot R^2}{x + R}\right)}}$

$$f := \frac{1}{\sqrt{v_0^2 - 2\, g\, R + \frac{2\, g\, R^2}{x + R}}} \qquad \textbf{(5)}$$

> $v_0 := 0.3314304660$; $g := \frac{9.8}{10^5}$; $R := 1275$

$$v_0 := 0.3314304660$$

$$g := 0.00009800000000$$

$$R := 1275 \qquad \textbf{(6)}$$

> $int(f, x = 0 ..999.99999)$; $evalf\left(\int_0^{1000} \mathrm{f}\, dx\right)$

$$7422.589145$$

$$7423.364412 \qquad \textbf{(7)}$$

> $\frac{7422.58914}{3600}$

$$2.061830317 \qquad \textbf{(8)}$$