Application Demonstration

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Fractal Teapot from Utah

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(Abstract)

In this worksheet we demonstrate fractally generated the famous teapot from Utah. Additionally fractal teacup and teaspoon are also presented. Theory that stands behind fractal modeling of 3D shapes is described in our previous worksheet "Fractal rendering of 3D patches" and the references given there. Data describing control points of bicubic Bezier patches that are used for modeling of the teapot, teacup and teaspoon can be found at ftp://ftp.funet.fi/pub/sci/graphics/packages/objects/teaset.tar.Z .

Remark
Calculations take time. So then, be patient if you want to see rendering results. Sometimes, if the matrix of control points P is near singular, we do not get the image of a modeled 3D shape. Especially, it occurs for the "teacup". In that case the suitable part of the worksheet should be recalculated.

1. Subdivision Matrices

```
> restart;
> with(plots):
> with(linalg):
> B1:=matrix([[1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0],
  [1/2,1/2,0,0,0,0,0,0,0,0,0,0,0,0,0,0],
  [1/4,1/2,1/4,0,0,0,0,0,0,0,0,0,0,0,0,0],
  [1/8,3/8,3/8,1/8,0,0,0,0,0,0,0,0,0,0,0,0],

  [1/2,0,0,0,1/2,0,0,0,0,0,0,0,0,0,0,0],
  [1/4,1/4,0,0,1/4,1/4,0,0,0,0,0,0,0,0,0,0],
  [1/8,1/4,1/8,0,1/8,1/4,1/8,0,0,0,0,0,0,0,0,0],
```

```
  [1/16,3/16,3/16,1/16,1/16,3/16,3/16,1/16,0,0,0,0,0,0,0,0],

  [1/4,0,0,0,1/2,0,0,0,1/4,0,0,0,0,0,0,0],
  [1/8,1/8,0,0,1/4,1/4,0,0,1/8,1/8,0,0,0,0,0,0],
  [1/16,1/8,1/16,0,1/8,1/4,1/8,0,1/16,1/8,1/16,0,0,0,0,0],
  [1/32,3/32,3/32,1/32,1/16,3/16,3/16,1/16,1/32,3/32,3/32,1/32,0,
  0,0,0],

  [1/8,0,0,0,3/8,0,0,0,3/8,0,0,0,1/8,0,0,0],
  [1/16,1/16,0,0,3/16,3/16,0,0,3/16,3/16,0,0,1/16,1/16,0,0],
  [1/32,1/16,1/32,0,3/32,3/16,3/32,0,3/32,3/16,3/32,0,1/32,1/16,
  1/32,0],
  [1/64,3/64,3/64,1/64,3/64,9/64,9/64,3/64,3/64,9/64,9/64,3/64,
  1/64,3/64,3/64,1/64]]):
> B2:=matrix([
> [1/8,0,0,0,3/8,0,0,0,3/8,0,0,0,1/8,0,0,0],
  [1/16,1/16,0,0,3/16,3/16,0,0,3/16,3/16,0,0,1/16,1/16,0,0],
  [1/32,1/16,1/32,0,3/32,3/16,3/32,0,3/32,3/16,3/32,0,1/32,1/16,
  1/32,0],
  [1/64,3/64,3/64,1/64,3/64,9/64,9/64,3/64,3/64,9/64,9/64,3/64,
  1/64,3/64,3/64,1/64],

  [0,0,0,0,1/4,0,0,0,1/2,0,0,0,1/4,0,0,0],
  [0,0,0,0,1/8,1/8,0,0,1/4,1/4,0,0,1/8,1/8,0,0],
  [0,0,0,0,1/16,1/8,1/16,0,1/8,1/4,1/8,0,1/16,1/8,1/16,0],
  [0,0,0,0,1/32,3/32,3/32,1/32,1/16,3/16,3/16,1/16,1/32,3/32,3/32,
  1/32],

  [0,0,0,0,0,0,0,0,1/2,0,0,0,1/2,0,0,0],
  [0,0,0,0,0,0,0,0,1/4,1/4,0,0,1/4,1/4,0,0],
  [0,0,0,0,0,0,0,0,1/8,1/4,1/8,0,1/8,1/4,1/8,0],
  [0,0,0,0,0,0,0,0,1/16,3/16,3/16,1/16,1/16,3/16,3/16,1/16],

  [0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0],
  [0,0,0,0,0,0,0,0,0,0,0,0,1/2,1/2,0,0],
  [0,0,0,0,0,0,0,0,0,0,0,0,1/4,1/2,1/4,0],
  [0,0,0,0,0,0,0,0,0,0,0,0,1/8,3/8,3/8,1/8]]):
> B3:=matrix([
> [1/8,3/8,3/8,1/8,0,0,0,0,0,0,0,0,0,0,0,0],
  [0,1/4,1/2,1/4,0,0,0,0,0,0,0,0,0,0,0,0],
  [0,0,1/2,1/2,0,0,0,0,0,0,0,0,0,0,0,0],
  [0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0],
```

```
  [1/16,3/16,3/16,1/16,1/16,3/16,3/16,1/16,0,0,0,0,0,0,0,0],
  [0,1/8,1/4,1/8,0,1/8,1/4,1/8,0,0,0,0,0,0,0,0],
  [0,0,1/4,1/4,0,0,1/4,1/4,0,0,0,0,0,0,0,0],
  [0,0,0,1/2,0,0,0,1/2,0,0,0,0,0,0,0,0],

  [1/32,3/32,3/32,1/32,1/16,3/16,3/16,1/16,1/32,3/32,3/32,1/32,0,
  0,0,0],
  [0,1/16,1/8,1/16,0,1/8,1/4,1/8,0,1/16,1/8,1/16,0,0,0,0],
  [0,0,1/8,1/8,0,0,1/4,1/4,0,0,1/8,1/8,0,0,0,0],
  [0,0,0,1/4,0,0,0,1/2,0,0,0,1/4,0,0,0,0],

  [1/64,3/64,3/64,1/64,3/64,9/64,9/64,3/64,3/64,9/64,9/64,3/64,
  1/64,3/64,3/64,1/64],
  [0,1/32,1/16,1/32,0,3/32,3/16,3/32,0,3/32,3/16,3/32,0,1/32,1/16,
  1/32],
  [0,0,1/16,1/16,0,0,3/16,3/16,0,0,3/16,3/16,0,0,1/16,1/16],
  [0,0,0,1/8,0,0,0,3/8,0,0,0,3/8,0,0,0,1/8]]):
> B4:=matrix([
> [1/64,3/64,3/64,1/64,3/64,9/64,9/64,3/64,3/64,9/64,9/64,3/64,
  1/64,3/64,3/64,1/64],
  [0,1/32,1/16,1/32,0,3/32,3/16,3/32,0,3/32,3/16,3/32,0,1/32,1/16,
  1/32],
  [0,0,1/16,1/16,0,0,3/16,3/16,0,0,3/16,3/16,0,0,1/16,1/16],
  [0,0,0,1/8,0,0,0,3/8,0,0,0,3/8,0,0,0,1/8],

  [0,0,0,0,1/32,3/32,3/32,1/32,1/16,3/16,3/16,1/16,1/32,3/32,3/32,
  1/32],
  [0,0,0,0,0,1/16,1/8,1/16,0,1/8,1/4,1/8,0,1/16,1/8,1/16],
  [0,0,0,0,0,0,1/8,1/8,0,0,1/4,1/4,0,0,1/8,1/8],
  [0,0,0,0,0,0,0,1/4,0,0,0,1/2,0,0,0,1/4],

  [0,0,0,0,0,0,0,0,1/16,3/16,3/16,1/16,1/16,3/16,3/16,1/16],
  [0,0,0,0,0,0,0,0,0,1/8,1/4,1/8,0,1/8,1/4,1/8],
  [0,0,0,0,0,0,0,0,0,0,1/4,1/4,0,0,1/4,1/4],
  [0,0,0,0,0,0,0,0,0,0,0,1/2,0,0,0,1/2],

  [0,0,0,0,0,0,0,0,0,0,0,0,1/8,3/8,3/8,1/8],
  [0,0,0,0,0,0,0,0,0,0,0,0,0,1/4,1/2,1/4],
  [0,0,0,0,0,0,0,0,0,0,0,0,0,0,1/2,1/2],
  [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1]]):
```

2. Rendering Procedure of Bicubic Bezier Patch

(n - number of iterations, p - collection of 16 points defining bicubic Bezier patch)

```
> BicuBezPatch := proc(n,p)
      local j, l, s, seqpoints,f,f1,f2,f3,f4,points,d,P,deter:

  d:=rand(0..1):

  P:=matrix([
  [p[1][1],p[1][2],p[1][3],1,1,0,0,0,0,0,0,0,0,0,0,0],
  [p[2][1],p[2][2],p[2][3],1,0,1,0,0,0,0,0,0,0,0,0,0],
  [p[3][1],p[3][2],p[3][3],1,0,0,1,0,0,0,0,0,0,0,0,0],
  [p[4][1],p[4][2],p[4][3],1,0,0,0,1,0,0,0,0,0,0,0,0],
  [p[5][1],p[5][2],p[5][3],1,0,0,0,0,1,0,0,0,0,0,0,0],
  [p[6][1],p[6][2],p[6][3],1,0,0,0,0,0,1,0,0,0,0,0,0],
  [p[7][1],p[7][2],p[7][3],1,0,0,0,0,0,0,1,0,0,0,0,0],
  [p[8][1],p[8][2],p[8][3],1,0,0,0,0,0,0,0,1,0,0,0,0],
  [p[9][1],p[9][2],p[9][3],1,0,0,0,0,0,0,0,0,1,0,0,0],
  [p[10][1],p[10][2],p[10][3],1,0,0,0,0,0,0,0,0,0,1,0,0],
  [p[11][1],p[11][2],p[11][3],1,0,0,0,0,0,0,0,0,0,0,1,0],
  [p[12][1],p[12][2],p[12][3],1,0,0,0,0,0,0,0,0,0,0,0,1],
  [p[13][1],p[13][2],p[13][3],1,seq(d(),i=1..12)],
  [p[14][1],p[14][2],p[14][3],1,seq(d(),i=1..12)],
  [p[15][1],p[15][2],p[15][3],1,seq(d(),i=1..12)],
  [p[16][1],p[16][2],p[16][3],1,seq(d(),i=1..12)]]);

   deter:=det(P):

  while deter = 0 do
     P:=matrix([
            [p[1][1],p[1][2],p[1][3],1,1,0,0,0,0,0,0,0,0,0,0,0],
            [p[2][1],p[2][2],p[2][3],1,0,1,0,0,0,0,0,0,0,0,0,0],
            [p[3][1],p[3][2],p[3][3],1,0,0,1,0,0,0,0,0,0,0,0,0],
            [p[4][1],p[4][2],p[4][3],1,0,0,0,1,0,0,0,0,0,0,0,0],
            [p[5][1],p[5][2],p[5][3],1,0,0,0,0,1,0,0,0,0,0,0,0],
            [p[6][1],p[6][2],p[6][3],1,0,0,0,0,0,1,0,0,0,0,0,0],
            [p[7][1],p[7][2],p[7][3],1,0,0,0,0,0,0,1,0,0,0,0,0],
            [p[8][1],p[8][2],p[8][3],1,0,0,0,0,0,0,0,1,0,0,0,0],
            [p[9][1],p[9][2],p[9][3],1,0,0,0,0,0,0,0,0,1,0,0,0],
            [p[10][1],p[10][2],p[10][3],1,0,0,0,0,0,0,0,0,0,1,0,0],
            [p[11][1],p[11][2],p[11][3],1,0,0,0,0,0,0,0,0,0,0,1,0],
            [p[12][1],p[12][2],p[12][3],1,0,0,0,0,0,0,0,0,0,0,0,1],
            [p[13][1],p[13][2],p[13][3],1,seq(d(),i=1..12)],
```

```
            [p[14][1],p[14][2],p[14][3],1,seq(d(),i=1..12)],
            [p[15][1],p[15][2],p[15][3],1,seq(d(),i=1..12)],
            [p[16][1],p[16][2],p[16][3],1,seq(d(),i=1..12)]]);

            deter:=det(P):
    end do:

       f1:=evalm(inverse(P)&*B1&*P):
       f2:=evalm(inverse(P)&*B2&*P):
       f3:=evalm(inverse(P)&*B3&*P):
       f4:=evalm(inverse(P)&*B4&*P):

       f:=[f1,f2,f3,f4]:
       points:=[[seq(P[16,i],i=1..16)]]:
       seqpoints:=points:

       s := NULL;

       for j to n do
               seqpoints:=[s,seq(evalm(seqpoints[ii]&*f[1]),ii=1..
    nops(seqpoints)),
               seq(evalm(seqpoints[ii]&*f[2]),ii=1..nops(seqpoints)
    ),
               seq(evalm(seqpoints[ii]&*f[3]),ii=1..nops(seqpoints)
    ),
               seq(evalm(seqpoints[ii]&*f[4]),ii=1..nops(seqpoints))
    ]:
       od;

        l:=seqpoints;
        pointplot3d([seq([l[i][1],l[i][2],l[i][3]],i=1..nops(l))],
    scaling=constrained,
        axes=FRAMED,symbolsize=10,symbol=circle,labels=[x,y,z]);
    end:
```

3. Data for rendering

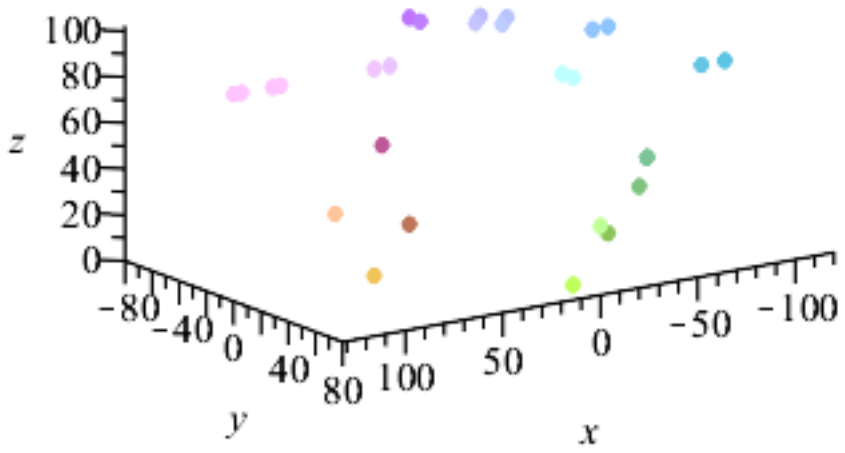
Data for teapot (32 cubic Bezier patches), teacup (26 cubic Bezier patches) and teaspoon (16 cubic Bezier patches)

```
> read "teapot.txt":
```

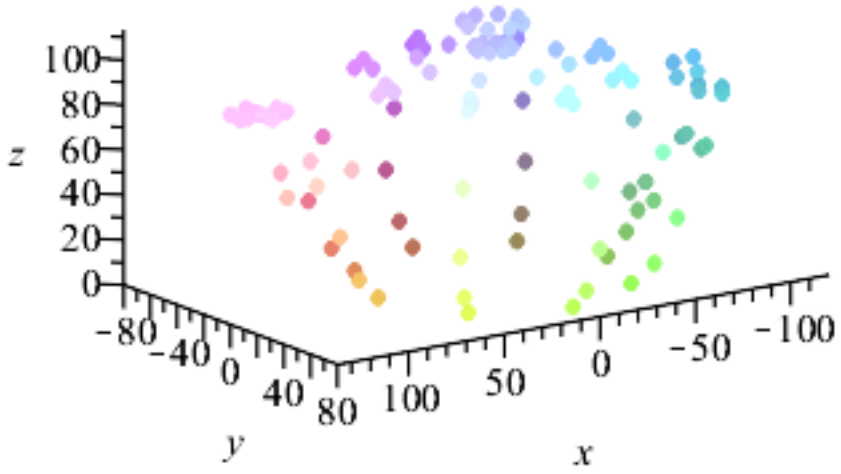
```
> read "teacup.txt":
> read "teaspoon.txt":
```

4. Fractal rendering of the teapot from Utah

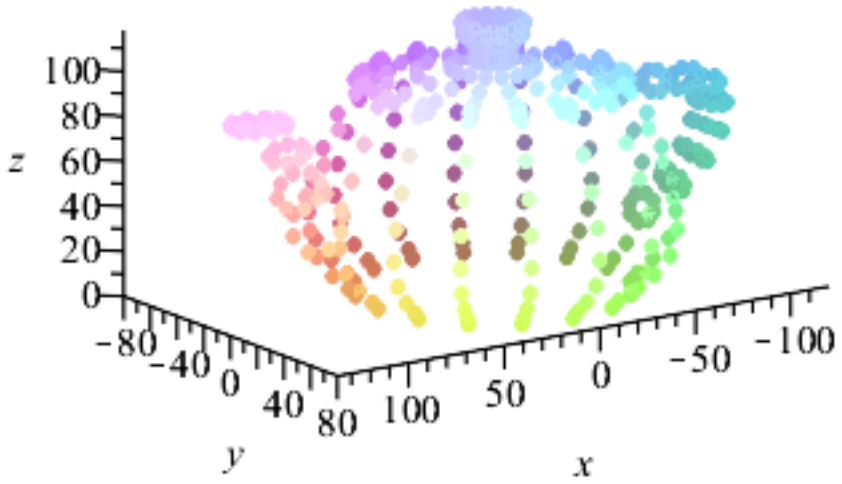
```
> for n from 0 to 4 do
  display(BicuBezPatch(n,p1),BicuBezPatch(n,p2),BicuBezPatch(n,
  p3),BicuBezPatch(n,p4),BicuBezPatch(n,p5),BicuBezPatch(n,p6),
  BicuBezPatch(n,p7),BicuBezPatch(n,p8),BicuBezPatch(n,p9),
  BicuBezPatch(n,p10),BicuBezPatch(n,p11),BicuBezPatch(n,p12),
  BicuBezPatch(n,p13),BicuBezPatch(n,p14),BicuBezPatch(n,p15),
  BicuBezPatch(n,p16),BicuBezPatch(n,p17),BicuBezPatch(n,p18),
  BicuBezPatch(n,p19),BicuBezPatch(n,p20),BicuBezPatch(n,p25),
  BicuBezPatch(n,p26),BicuBezPatch(n,p27),BicuBezPatch(n,p28),
  BicuBezPatch(n,p29),BicuBezPatch(n,p30),BicuBezPatch(n,p31),
  BicuBezPatch(n,p32));
  print('Iteration'=n);
  od;
```



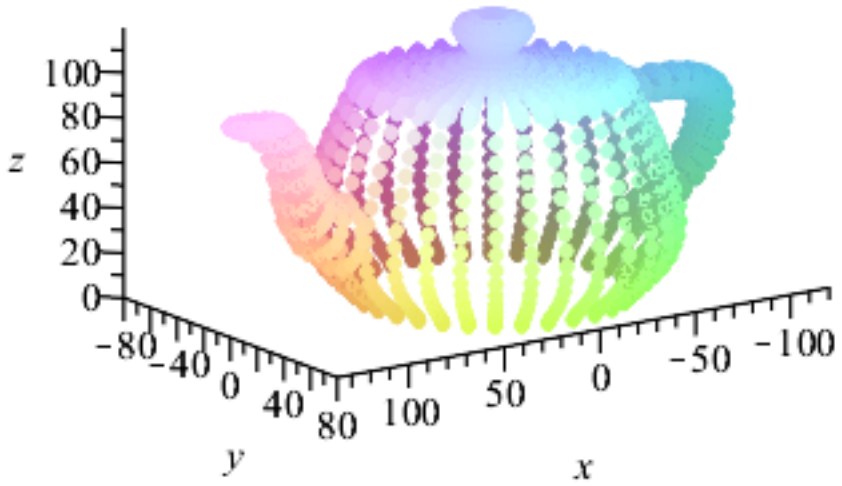
Iteration = 0



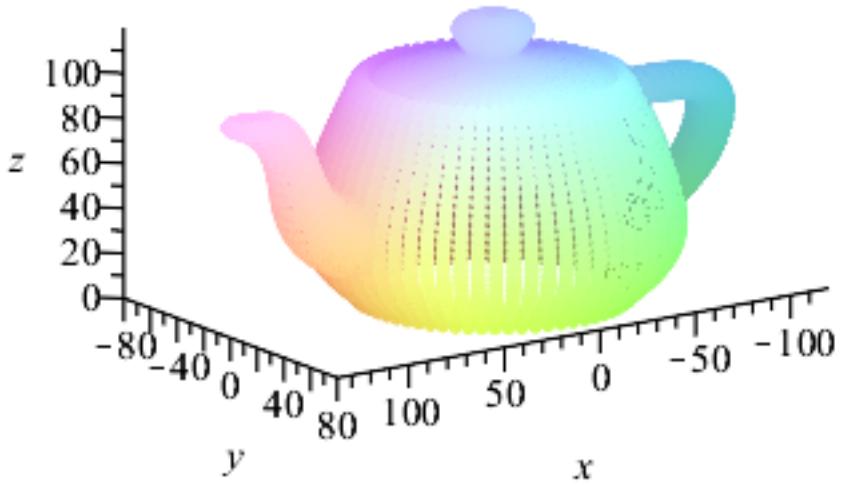
Iteration = 1



Iteration = 2



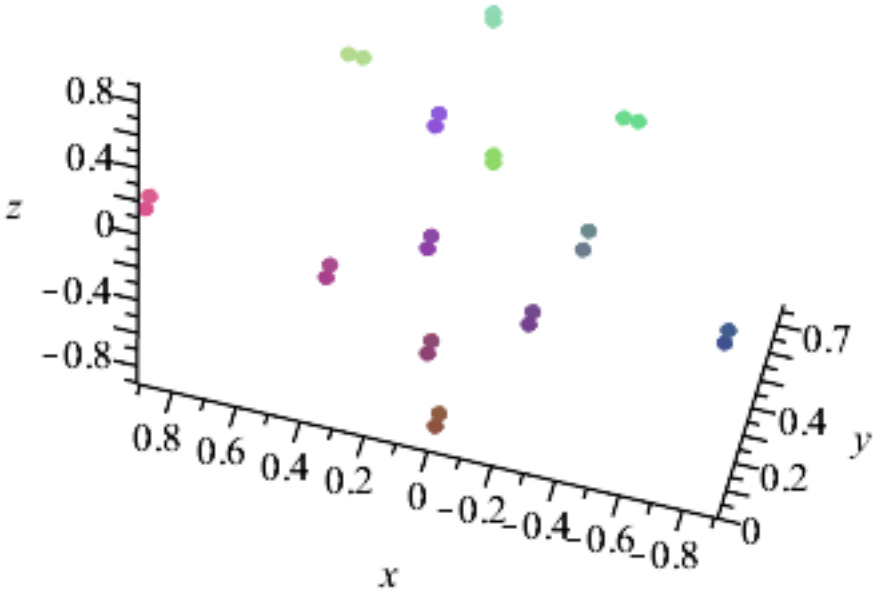
Iteration = 3



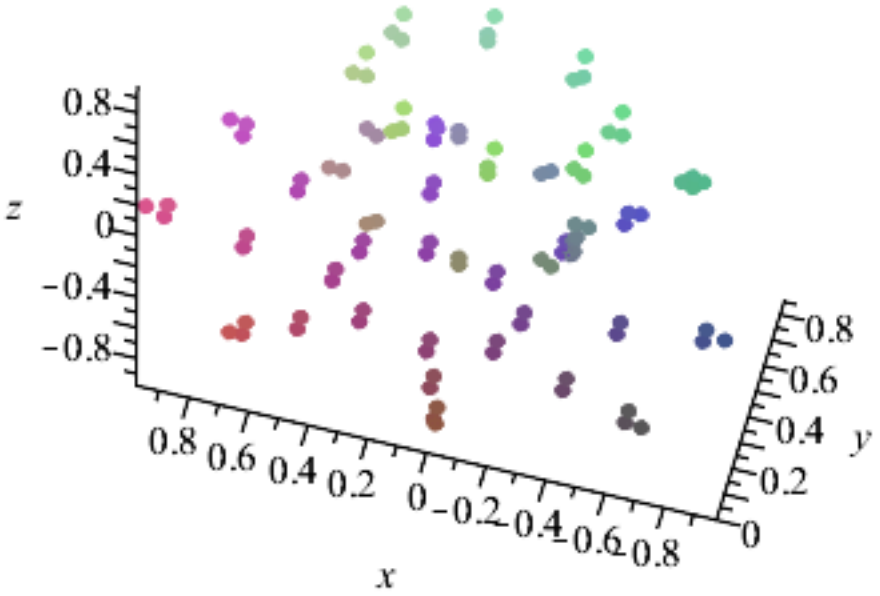
Iteration = 4 **(4.1)**

5. Fractal rendering of the teacup

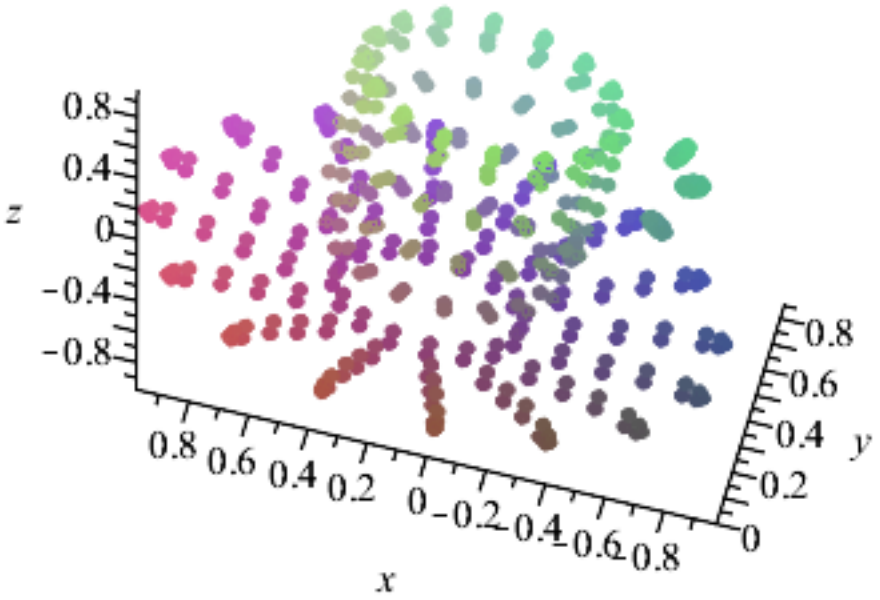
```
> for n from 0 to 4 do
  display({BicuBezPatch(n,f1),BicuBezPatch(n,f2),BicuBezPatch(n,
  f3),BicuBezPatch(n,f4),BicuBezPatch(n,f5),BicuBezPatch(n,f6),
  BicuBezPatch(n,f7),BicuBezPatch(n,f8),BicuBezPatch(n,f9),
  BicuBezPatch(n,f10),BicuBezPatch(n,f11),BicuBezPatch(n,f12),
  BicuBezPatch(n,f13),BicuBezPatch(n,f14),BicuBezPatch(n,f15),
  BicuBezPatch(n,f16),BicuBezPatch(n,f17),BicuBezPatch(n,f18),
  BicuBezPatch(n,f19),BicuBezPatch(n,f20),BicuBezPatch(n,f21),
  BicuBezPatch(n,f22),BicuBezPatch(n,f23),BicuBezPatch(n,f24),
  BicuBezPatch(n,f25),BicuBezPatch(n,f26)},orientation=[75,150]);
  print('Iteration'=n);
  od;
```



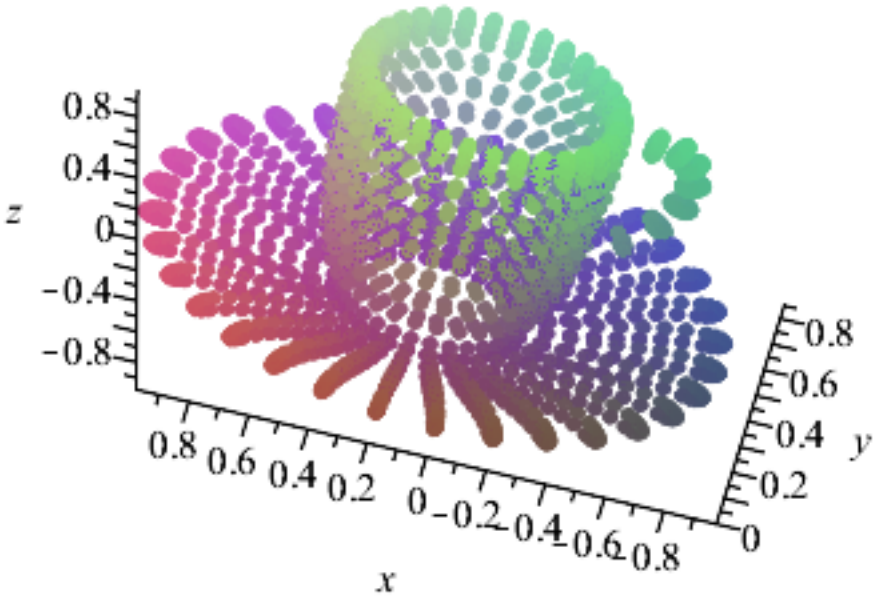
Iteration = 0



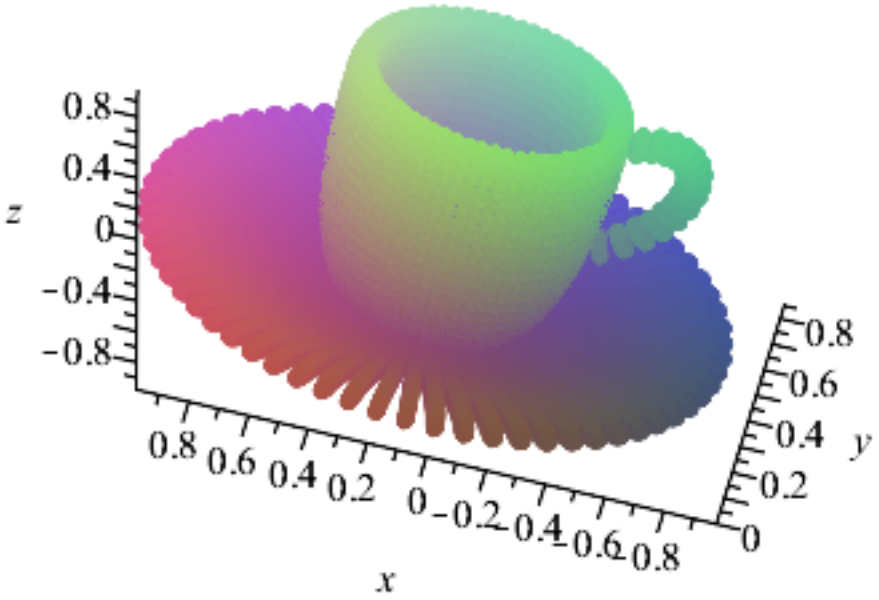
Iteration = 1



Iteration = 2



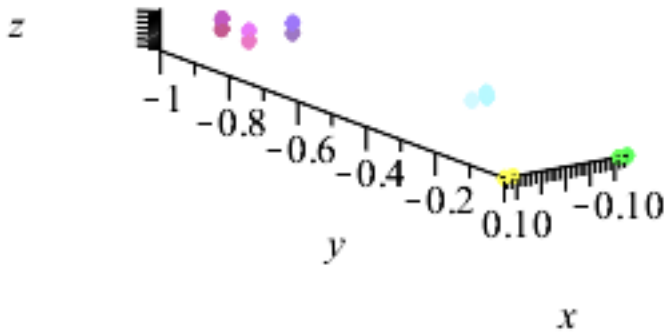
Iteration = 3



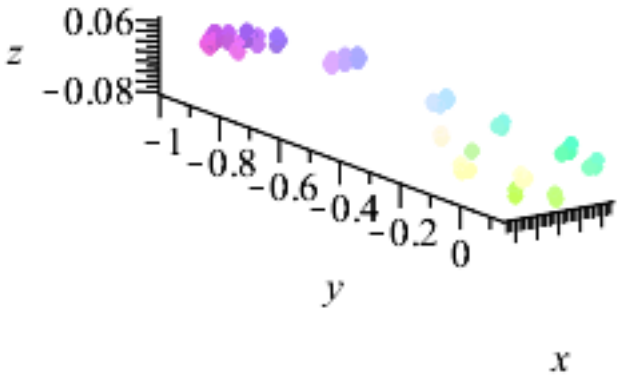
Iteration = 4 **(5.1)**

6. Fractal rendering of the teaspoon

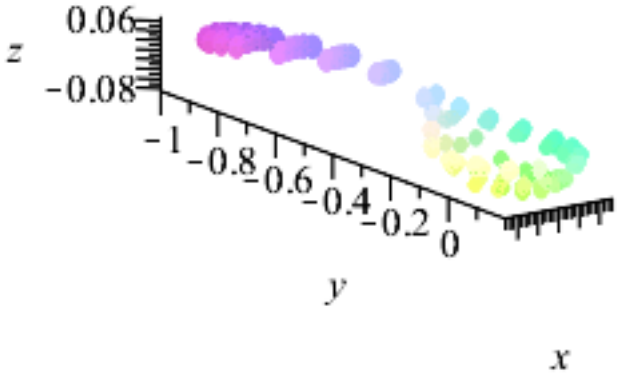
```
> for n from 0 to 4 do
  display(BicuBezPatch(n,g1),BicuBezPatch(n,g2),BicuBezPatch(n,
  g3),BicuBezPatch(n,g4),BicuBezPatch(n,g6),BicuBezPatch(n,g7),
  BicuBezPatch(n,g8),BicuBezPatch(n,g9),BicuBezPatch(n,g10),
  BicuBezPatch(n,g11),BicuBezPatch(n,g12),BicuBezPatch(n,g13),
  BicuBezPatch(n,g14),BicuBezPatch(n,g15),BicuBezPatch(n,g16));
  print('Iteration'=n);
  od;
```



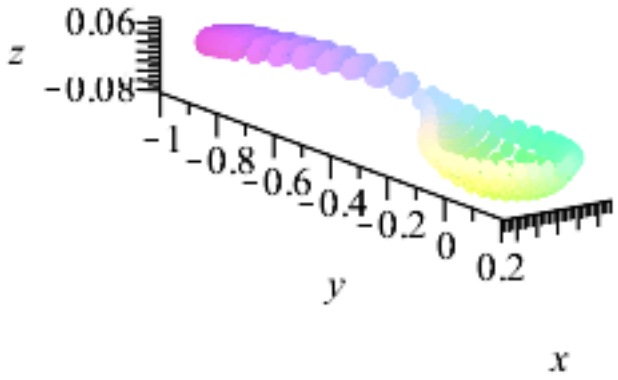
Iteration = 0



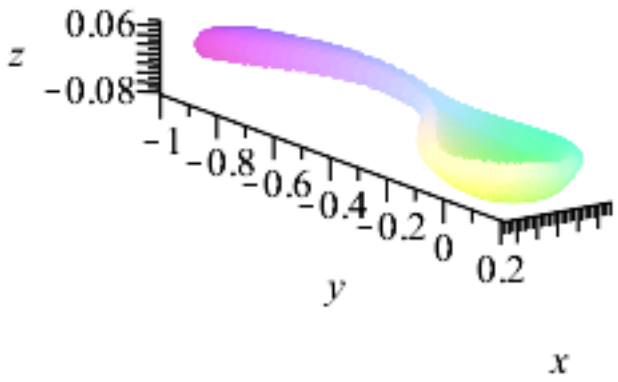
Iteration = 1



Iteration = 2



Iteration = 3



$Iteration = 4$ **(6.1)**