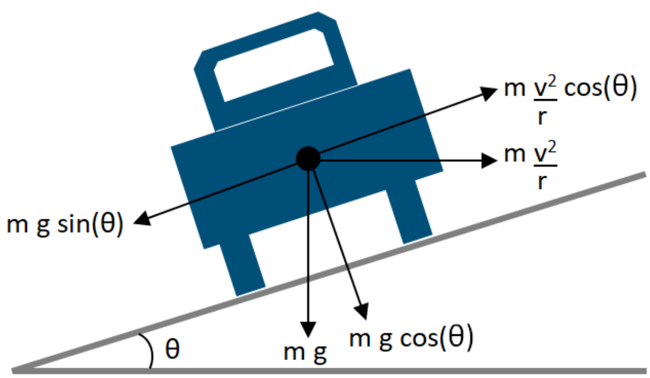
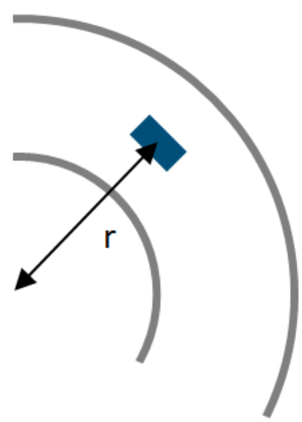
# Minimum Road Radius for Highway Superelevation

## Introduction

When a vehicle travels around a bend, a centripetal force is needed to keep the vehicle on the road. If the road is completely horizontal, this force is entirely provided by side friction (i.e the friction of the tires on the road).

However, road curves are usually banked - the outer edge is higher than the inner edge. This uses the weight of the vehicle to help keep the vehicle on the road, and is called superelevation.




Super elevation is usually measured in the number of meters (or feet) of vertical rise per 100 meters (or feet) of horizontal distance. This means that $\tan(\theta) = \frac{superelevation}{100}$.

The physics involve these parameters:

- v - vehicle velocity
- f - side friction
- θ - superelevation
- g - gravity
- r - road radius

Values of minimum road radius for specific values of superelevation, side friction, and velocity are tabulated in Table 3.7 (pp 3-34) of *Policy on Geometric Design of Highways and Street*, 7th Edition, 2018, American Association of State Highway Transportation Officials (also known as the AASHTO Green Book).

In this application, we derive an equation that describes the relationship between velocity, side friction, road radius and superelevation for a point-mass vehicle traveling around a bend. This relationship is then rearranged to give an explicit equation for the road radius.

This equation is then used to calculate the road radius required to maintain the trajectory of a vehicle as it travels around a bend. For a specific set of parameters, the calculated road radius matches the minimum road radius given in the AASHTO Green Book.

# Theory

*restart* :
*with*( *Units*[*Simple*]) :

Centripetal force

$$F_c := m \cdot \frac{v^2}{r} \cdot \cos(\theta) :$$

Force of the weight of the car contributing to the centripetal force

$$F_{se} := m \cdot g \cdot \sin(\theta) :$$

Force of the side friction contributing to the centripetal force (proportional to the normal force of the car)

$$F_{sf} := f \cdot m \cdot g \cdot \cos(\theta) :$$

Force balance

$$sum\_of\_forces := F_c = F_{se} + F_{sf} = \frac{m v^2 \cos(\theta)}{r} = m g \sin(\theta) + f m g \cos(\theta)$$

Solve for the road radius

$$R := solve(sum\_of\_forces, r) = \frac{\cos(\theta) v^2}{g (f \cos(\theta) + \sin(\theta))}$$

# Comparison with the Minimum Road Radius Given in the AASHTO Green Book

For given values of v, f and θ, the road radius given predicted by this equation match those tabulated in the AASHTO Green Book.

At a speed of 30 km hr$^{-1}$, superelevation of 6, side friction of 0.28, the AASHTO Green Book quotes a radius of 20.8 m

$$eval\left(R, \left[v = 30\ \text{km hour}^{-1}, \theta = \frac{6.0}{100}, f = 0.28, g = 9.81\ \text{m s}^{-2}\right]\right) = 20.8\ \text{m}$$

At a speed of 40 km hr$^{-1}$, superelevation of 4, side friction of 0.23, the AASHTO Green Book quotes a radius of 46.7 m

$$eval\left(R, \left[v = 40\ \text{km hour}^{-1}, \theta = \frac{4.0}{100}, f = 0.23, g = 9.81\ \text{m s}^{-2}\right]\right) = 46.6\ \text{m}$$

At a speed of 80 mph, superelevation of 8, side friction of 0.08, the AASHTO Green Book quotes a radius of 2266.7 ft

$$eval\left(R, \left[v = 80\ \text{mph}, \theta = \frac{8.0}{100}, f = 0.08, g = 32.2\ \text{ft s}^{-2}\right]\right) = 2669.3\ \text{ft}$$

At a speed of 10 mph, superelevation of 12, side friction of 0.38, the AASHTO Green Book quotes a radius of 13.3 ft

$$eval\left(R, \left[v = 10\ \text{mph}, \theta = \frac{12.0}{100}, f = 0.38, g = 32.2\ \text{ft s}^{-2}\right]\right) = 13.3\ \text{ft}$$