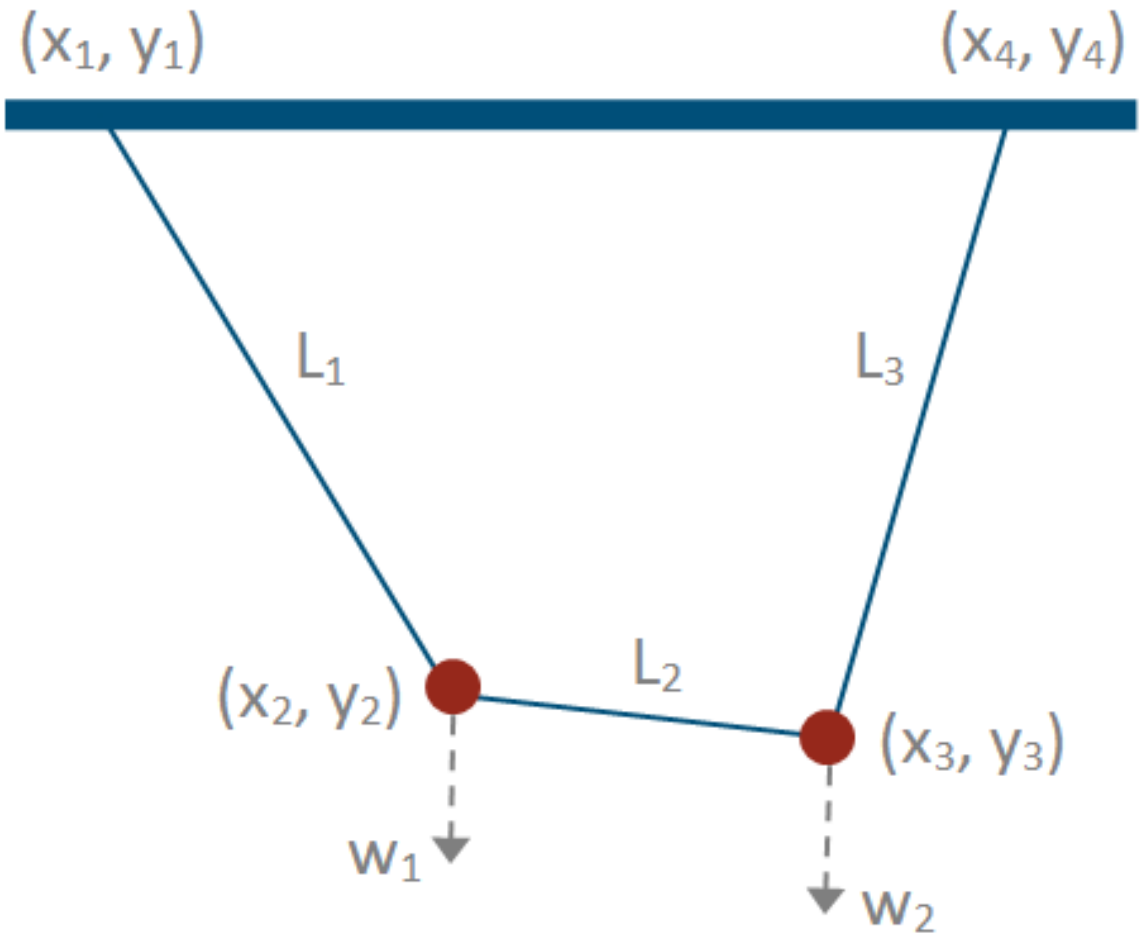
Equilibrium Positions of Two Objects Connected by Ropes

Introduction

Two objects are connected and suspended by a system of ropes, as illustrated below.



These parameters are known

- The object weights, w_1 and w_2
- the length of the ropes L_1, L_2, and L_3
- the anchor points (x_1, y_1) and (x_4, y_4)

The objects positions and the rope tensions are calculated from a numeric solution of the equations that describe

- horizontal and vertical force balances
- and the constraints imposed by the lengths of the ropes

```
> restart :
  with( plots ) :
  with( plottools ) :
  with( ColorTools ) :
```

Parameter and Support Functions

Length of a rope section given the coordinates of the endpoints (x_a, y_a) and (x_b, y_b)

```
> d := (x_a, y_a, x_b, y_b) → sqrt((x_a - x_b)^2 + (y_a - y_b)^2) :
```

Angle of the rope with respect to the x-axis

```
> a := (x_a, y_a, x_b, y_b) → piecewise(y_a > y_b, arctan(y_a - y_b, x_a - x_b), 2 π + arctan(y_a
      - y_b, x_a - x_b)) :
```

Rope lengths

```
> L_1, L_2, L_3 := 1.2, 1.5, 2 :
```

Object weights

```
> w_1, w_2 := 10, 1 :
```

Anchor positions

```
> x_1, y_1 := 0, 2 :
  x_4, y_4 := 3, 2 :
```

Equations

Length of rope sections

```
> lengths :=
    d(x_1, y_1, x_2, y_2) = L_1
  , d(x_2, y_2, x_3, y_3) = L_2
  , d(x_3, y_3, x_4, y_4) = L_3 :
```

The system is at an equilibrium when the net horizontal and vertical forces on each object are zero.

Vertical forces

> vF :=

$$T_1 \cdot \sin(a(x_1, y_1, x_2, y_2)) - T_2 \cdot \sin(a(x_2, y_2, x_3, y_3)) = w_1$$
$$, T_2 \cdot \sin(a(x_2, y_2, x_3, y_3)) - T_3 \cdot \sin(a(x_3, y_3, x_4, y_4)) = w_2 :$$

Horizontal forces

> hF :=

$$T_1 \cdot \cos(a(x_1, y_1, x_2, y_2)) = T_2 \cdot \cos(a(x_2, y_2, x_3, y_3))$$
$$, T_2 \cdot \cos(a(x_2, y_2, x_3, y_3)) = T_3 \cdot \cos(a(x_3, y_3, x_4, y_4)) :$$

Numeric Solution and Visualization

> res := fsolve({lengths, vF, hF}, {$x_2 = 1, y_2 = 2, x_3 = 2, y_3 = 2, T_1 = 1, T_2 = 1, T_3 = 1$})

$$\{T_1 = 10.17, T_2 = 879.77 \times 10^{-3}, T_3 = 1.23, x_2 = 102.73 \times 10^{-3}, x_3 = 1.59, y_2 = 804.41 \times 10^{-3}, y_3 = 584.98 \times 10^{-3}\} \quad (4.1)$$

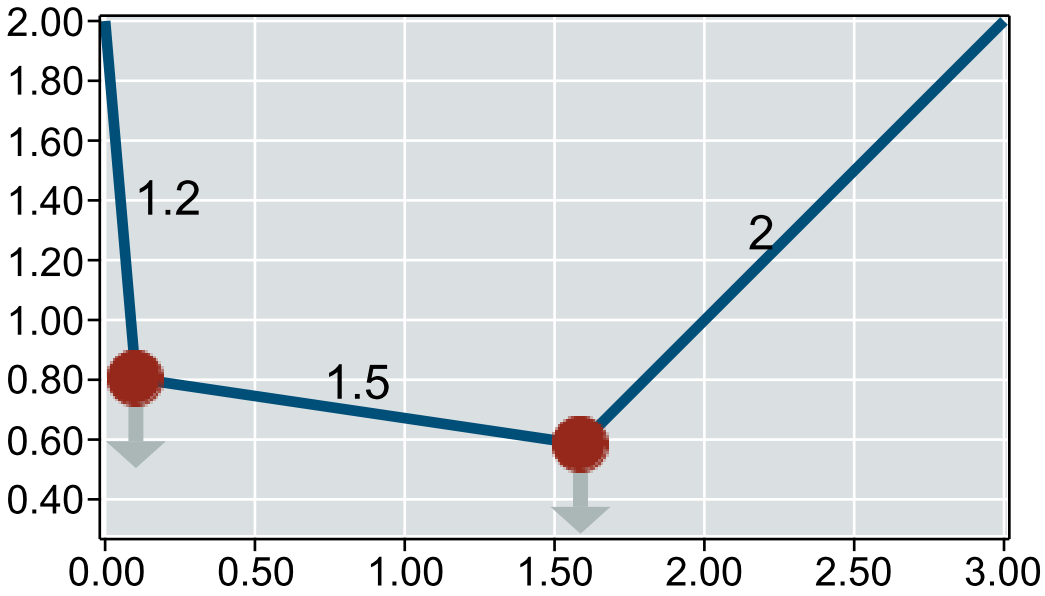
> p1 :=

pointplot(eval([[x_1, y_1], [x_2, y_2]], res), connect = true, thickness = 4, color = Color("RGB", [0, 79/255, 121/255]))
, pointplot(eval([[x_2, y_2], [x_3, y_3]], res), connect = true, thickness = 4, color = Color("RGB", [0, 79/255, 121/255]))
, pointplot(eval([[x_3, y_3], [x_4, y_4]], res), connect = true, thickness = 4, color = Color("RGB", [0, 79/255, 121/255]))
, pointplot(eval([x_2, y_2], res), symbol = solidcircle, symbolsize = 40, color = Color("RGB", [150/255, 40/255, 27/255]))
, pointplot(eval([x_3, y_3], res), symbol = solidcircle, symbolsize = 40, color = Color("RGB", [150/255, 40/255, 27/255]))
, textplot(eval([$(x_1 + x_2) \cdot 0.5, (y_1 + y_2) \cdot 0.5, L_1$], res), font = [Arial, 14], align = {right})
, textplot(eval([$(x_2 + x_3) \cdot 0.5, (y_2 + y_3) \cdot 0.5, L_2$], res), font = [Arial, 14], align = {above})
, textplot(eval([$(x_3 + x_4) \cdot 0.5, (y_3 + y_4) \cdot 0.5, L_3$], res), font = [Arial, 14], align = {left}) :

p2 :=

arrow(eval([[x_2, y_2], [$x_2, y_2 - 0.3$]], res)[ ], 0.05, 0.2, 0.3, color = Color("RGB", [171/255, 183/255, 183/255]), border = false)
, arrow(eval([[x_3, y_3], [$x_3, y_3 - 0.3$]], res)[ ], 0.05, 0.2, 0.3, color = Color("RGB", [171/255, 183/255, 183/255]), border = false) :

```
> plots:-display( p2, p1, scaling = constrained, size = [800, 400], axesfont = [Arial], axes
      = box, background = Color( "RGB", [218 / 255, 223 / 255, 225 / 255]), axis
      = [gridlines = [10, color = Color( "RGB", [1, 1, 1]) ]])
```




```
>
```