Newton's Method

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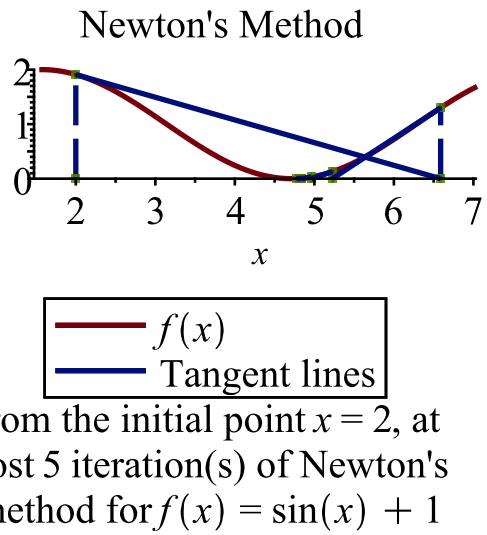
Newton's Method is a method of successive iteration that helps us to find the roots of an algebraic function f(x).
Given an initial guess, x0, we can compute an x1, then an x2 and so on.
The general formula to compute the (n+1) iteration in terms of the n-th iteration is the following: $x_{n+1} = x_n - \frac{f(x_n)}{\frac{df}{dx}(x_n)}$.

Comments	Calculations
To the right is a simple Maple script that does this in a loop. First we define the function and compute its derivative.	$restart$: $f := x \to \sin(x) + 1$: $df := \frac{d}{dx} f(x)$:
Second, we define the initial guess and store it as the first element of a vector.	$x0 := 2$: $N := 5$: $X := Vector(N+1)$: $X[1] := x0$:
Third, we begin a loop and compute Newton's function at every step to find our better guess.	**for** n **from** 1 **to** N **do** $X[n+1] := evalf\left(X[n] - \frac{f(X[n])}{eval(df, x = X[n])}\right)$: $printf$("For n = %d Newton's approximation is %f.\n", $n, X[n+1]$); **end do**: For n = 1 Newton's

This works very well but not necessarily something you want to do for every problem.	approximation is 6.588038. For n = 2 Newton's approximation is 5.225039. For n = 3 Newton's approximation is 4.962949. For n = 4 Newton's approximation is 4.837009. For n = 5 Newton's approximation is 4.774618.
Instead, we can use Maple's built in function that does all the work for us. To the right is an example of how you do just that. Also, it gives a nice plot to show you graphically how Newton's method eventually gets to the solution.	*with*(*Student*[*Calculus1*]) : *NewtonsMethod*($f(x)$, $x = x0$, *iterations* = *N*, *output* = *sequence*) 2, 6.588037826, 5.225039135, 4.962948756, 4.837009302, 4.774618375 **(1)** *NewtonsMethod*($f(x)$, $x = x0$, *iterations* = *N*, *output* = *plot*, *thickness* = 2)  From the initial point $x = 2$, at most 5 iteration(s) of Newton's method for $f(x) = \sin(x) + 1$