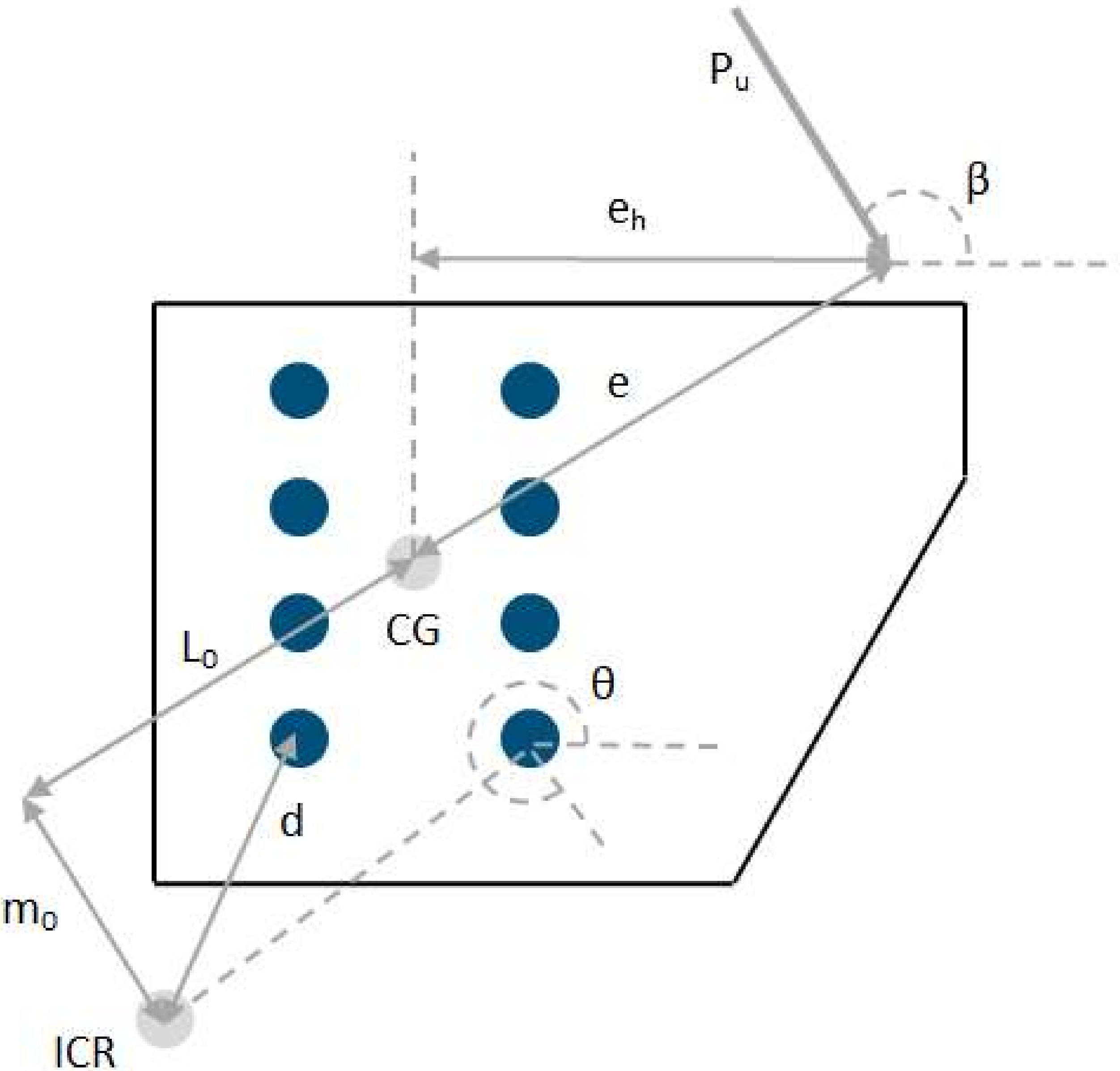
# Bolt Group Coefficient for Eccentric Loads

## Introduction

This application calculates the bolt coefficient for eccentrically loaded bolt groups using the Instantaneous Center of Rotation method (also known as the Ultimate Strength method).




The bolt coefficient C is the ratio of the factored force (or available strength) of the bolt group $P_u$

and the shear capacity of a single bolt $\phi r_n$,

$$C = \frac{P_u}{\phi r_n}$$

Once the coefficient is known, a bolt group can be designed for any load.

Traditionally, bolt group coefficients are extracted by using tabulated values in the AISC Steel Construction Manual. However, these tables are limited to common bolt patterns, and specific load eccentricities and angles. Non-tabulated values must be extracted by using linear interpolation.

This Maple worksheet, however, calculates the bolt group coefficient for any bolt and load configuration by implementing the theory used to generate the tables.

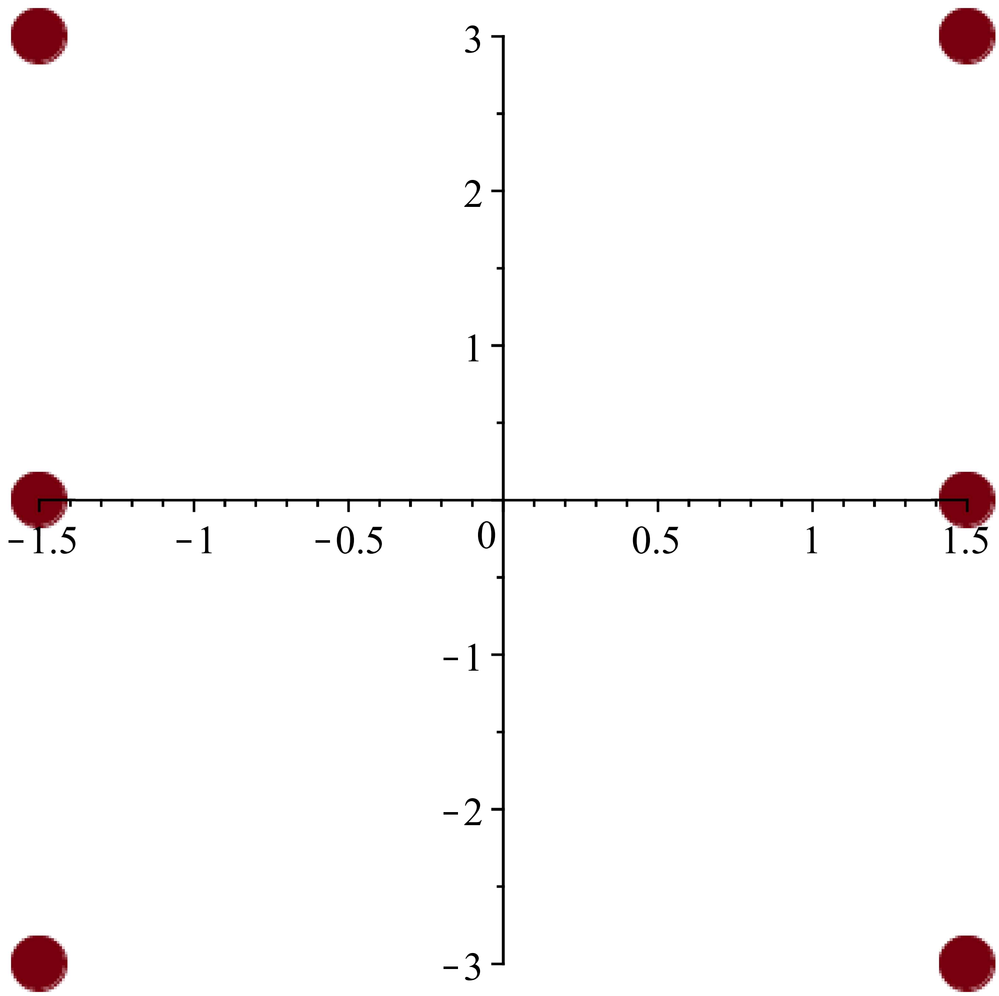
The results agree with those presented in AISC Manual of Steel Construction: Load and Resistance Factor Design, 2nd Edition.

```
> restart :
```

# Parameters

The following commands define the bolt locations and plot them.

```
> boltLoc := ⟨⟨-1.5|-3⟩, ⟨1.5|-3⟩, ⟨-1.5|0⟩, ⟨1.5|0⟩, ⟨-1.5|3⟩, ⟨1.5|3⟩⟩ :
> plot( boltLoc, style = point, symbol = solidcircle, symbolsize = 40 )
```




The shear strength of a single bolt (kip):

> $\phi r_n := 21.60 :$

The horizontal component of force eccentricity with respect to the centroid of bolt group (in):

> $e_h := 2 :$

Force angle to horizontal axis (deg):

> $\beta := \frac{75\pi}{180.0}$

$$\beta := 1.308996939 \tag{2.1}$$

# Calculations

Translate the bolt locations so that the centroid is at the origin.

> $X := boltLoc_{.., 1} - \sim Statistics{:-}Mean\left(boltLoc_{.., 1}\right) :$
> $Y := boltLoc_{.., 2} - \sim Statistics{:-}Mean\left(boltLoc_{.., 2}\right) :$

The eccentricity:

> $e := e_h \sin(\beta)$

$$e := 1.931851653 \tag{3.1}$$

Adjusted beta:

> $\beta := \left| \begin{cases} \beta & e > 0 \\ \beta + 0.5\pi & \text{otherwise} \end{cases} \right|$

$$\beta := 1.308996939 \tag{3.2}$$

Instantaneous center of rotation (ICR):

> $X_0 := (L_0, m_0) \mapsto -L_0 \sin(\beta) - m_0 \cos(\beta)$ :

> $Y_0 := (L_0, m_0) \mapsto L_0 \cos(\beta) - m_0 \sin(\beta)$ :

Bolt angle to ICR (rad):

> $\theta := (L_0, m_0) \rightarrow \text{arctan~}(\, Y\text{-~}Y_0(L_0, m_0), X\text{-~}X_0(L_0, m_0)\,)\, \text{-~Pi}/2$ :

Bolt distance to ICR (inches):

> $d := (L_0, m_0) \rightarrow \left( \left( X - \text{~}X_0(L_0, m_0) \right)^{\sim 2} + \left( Y - \text{~}Y_0(L_0, m_0) \right)^{\sim 2} \right)^{\sim 0.5}$ :
> $\text{dmax} := (L_0, m) \rightarrow \max(d(L_0, m_0))$ :

Bolt displacement (inches) from Crawford and Kulak (1971):

> $\text{Delta} := (L_0, m_0) \rightarrow \frac{d(L_0, m_0)}{\text{~dmax}(L_0, m_0)} \cdot 0.34$ :

Load-deformation relationship from Crawford and Kulak (1971):

> $R := (L_0, m_0) \rightarrow \phi r_n \cdot \left( 1 - \text{~}e^{-10 \cdot \Delta(L_0, m_0)} \right)^{0.55}$ :

> $Rn := (L_0, m_0) \rightarrow \left( 1 - \text{~}e^{\sim\left( -\frac{10 \cdot d(L_0, m_0)}{\text{~dmax}(L_0, m_0)} \cdot 0.34 \right)} \right)^{\sim 0.55}$ :

# Optimization

The sums of the bolt forces in the vertical and horizontal directions are equal to the applied shear and axial loads.

> $\text{forceX} := (P_0, m_0, L_0) \rightarrow P_0 \cdot \sin(\text{beta}) + \text{add}\left(i, i\ \mathbf{in}\ Rn(L_0, m_0) \cdot \text{~sin~}(\theta(L_0, m_0))\right)$ :
> $\text{forceY} := (P_0, m_0, L_0) \rightarrow P_0 \cdot \cos(\text{beta}) + \text{add}\left(i, i\ \mathbf{in}\ Rn(L_0, m_0) \cdot \text{~cos~}(\theta(L_0, m_0))\right)$ :

The moment of the bolt forces about the ICR is equal to the moment of the applied load.

> $moment := (P_0, m_0, L_0) \rightarrow P_0 \cdot (L_0 + e) - add(i, i \textbf{ in } Rn(L_0, m_0) \cdot \sim d(L_0, m_0))$ :

The least squares solution to the three balance equations:

> $res := Optimization\text{:-}Minimize\left(forceX(P_0, m_0, L_0)^2 + forceY(P_0, m_0, L_0)^2 + moment(P_0, m_0, L_0)^2, \{0 \le P_0 - e \le L_0\}, iterationlimit = 200\right)$

$$res := [9.75024564308638301\ 10^{-15}, [L_0 = 3.58864965454563, P_0 = 4.46665769665432, m_0 = -0.183532617971935]] \quad \textbf{(4.1)}$$

## Results

Factored force on bolt group (kips):

> $P_u := eval(P_0\, \phi r_n, res_2)$

$$P_u := 96.4798062477334 \quad \textbf{(5.1)}$$

Bolt coefficient:

> $C := \frac{P_u}{\phi r_n}$

$$C := 4.46665769665432 \quad \textbf{(5.2)}$$

## References

Behavior of eccentrically loaded bolted connections, Crawford S. F., Kulak, G. L. (1968)
AISC Manual of Steel Construction: Load and Resistance Factor Design 2nd Edition
http://www.bgstructuralengineering.com/BGSCM14/BGSCM004/BGSCM00403.htm
https://engineering.purdue.edu/~jliu/courses/CE591/PDF/CE591eccentric_shear_F13.pdf