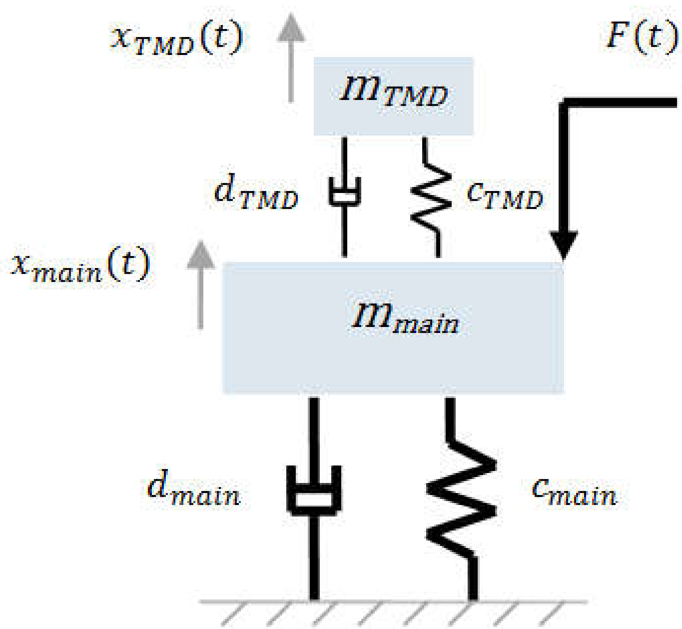
Tuned Mass Damper Design for Attenuating Vibration

Introduction

A mass-spring-damper is disturbed by a force that resonates at the natural frequency of the system.

This application calculates the optimum spring and damping constant of a parasitic tuned-mass damper that the minimizes the vibration of the system.

The vibration of system with and without the tuned mass-spring-damper is viewed as a frequency response, time-domain simulation and power spectrum.



> restart : with(DynamicSystems) : with(plots) : with(SignalProcessing) :

> params := $\left[m_1 = 1.764\,10^5, k_1 = 3.45\,10^7, b_1 = 1.531\,10^5, m_2 = 8165\right]$:

Derive Expressions for the Optimum Spring and Damping Constant of the Tuned Mass Damper

Mass ratio

> $\mu := \frac{m_2}{m_1}$:

Natural frequency of tuned mass damper

> $\omega_2 := \sqrt{\frac{k_{2\_calc}}{m_2}}$:

Natural frequency of main system

> $\omega_1 := \sqrt{\frac{k_1}{m_1}}$:

Hence the natural frequency in rad s^{-1} is

> $\mathrm{eval}(\omega_1, params)$

$$13.98492872$$

Ratio of natural frequencies

> $\alpha := \frac{\omega_2}{\omega_1}$:

Optimum ratio of natural frequencies

> $\alpha_{opt} := \frac{1}{1+\mu}$:

Hence the optimum spring constant of the tuned mass-spring-damper

> $k_{2\_calc} := \mathrm{solve}(\alpha = \alpha_{opt}, k_{2\_calc})$

$$k_{2\_calc} := \frac{m_1 k_1 m_2}{(m_1 + m_2)^2}$$

Damping Ratio

> $z := \frac{b_{2\_calc}}{2\, m_2 \omega_2}$:

Optimum damping ratio

> $z_{opt} := \sqrt{\frac{3\,\mu}{8\,(1+\mu)^3}}$:

Hence the optimum damping constant of the tuned mass-spring-damper

> $b_{2\_calc} := \mathrm{solve}(z = z_{opt}, b_{2\_calc})$

$$b_{2\_calc} := \frac{1}{2}\sqrt{6}\,\sqrt{\frac{m_2}{m_1\left(1+\frac{m_2}{m_1}\right)^3}}\,m_2\,\sqrt{\frac{m_1 k_1}{(m_1+m_2)^2}}$$

> $k_{2\_calc} := \text{eval}(k_{2\_calc}, \text{params});$

$$k_{2\_calc} := 1.45873086110^6$$

> $b_{2\_calc} := \text{evalf}(\text{eval}(b_{2\_calc}, \text{params}));$

$$b_{2\_calc} := 26869.77094$$

Full list of parameters

> $params_{tuned} := [m_1 = 1.764\,10^5, k_1 = 3.45\,10^7, b_1 = 1.531\,10^5, m_2 = 8165, k_2 = k_{2\_calc}, b_2 = b_{2\_calc}]:$

> $params_{nottuned} := [m_1 = 1.764\,10^5, k_1 = 3.45\,10^7, b_1 = 1.531\,10^5, m_2 = 0, k_2 = k_{2\_calc}, b_2 = b_{2\_calc}]:$

Equations of Motion for the Entire System

Equation of motion for the whole system

> $\text{de} := m_2\left(\frac{d^2}{dt^2}x_2(t)\right) = -k_2(x_2(t) - x_1(t)) - b_2\left(\frac{d}{dt}x_2(t) - \frac{d}{dt}x_1(t)\right),$
$m_1\left(\frac{d^2}{dt^2}x_1(t)\right) = -k_1 x_1(t) - b_1\left(\frac{d}{dt}x_1(t)\right) - k_2(x_1(t) - x_2(t)) - b_2\left(\frac{d}{dt}x_1(t) - \frac{d}{dt}x_2(t)\right)$
$+ F(t):$
$\text{ic} := x_1(0) = 0, D(x_1)(0) = 0, x_2(0) = 0, D(x_2)(0) = 0:$

> sys := DiffEquation([de], [F(t)], [x_1(t)]) :

Frequency Response

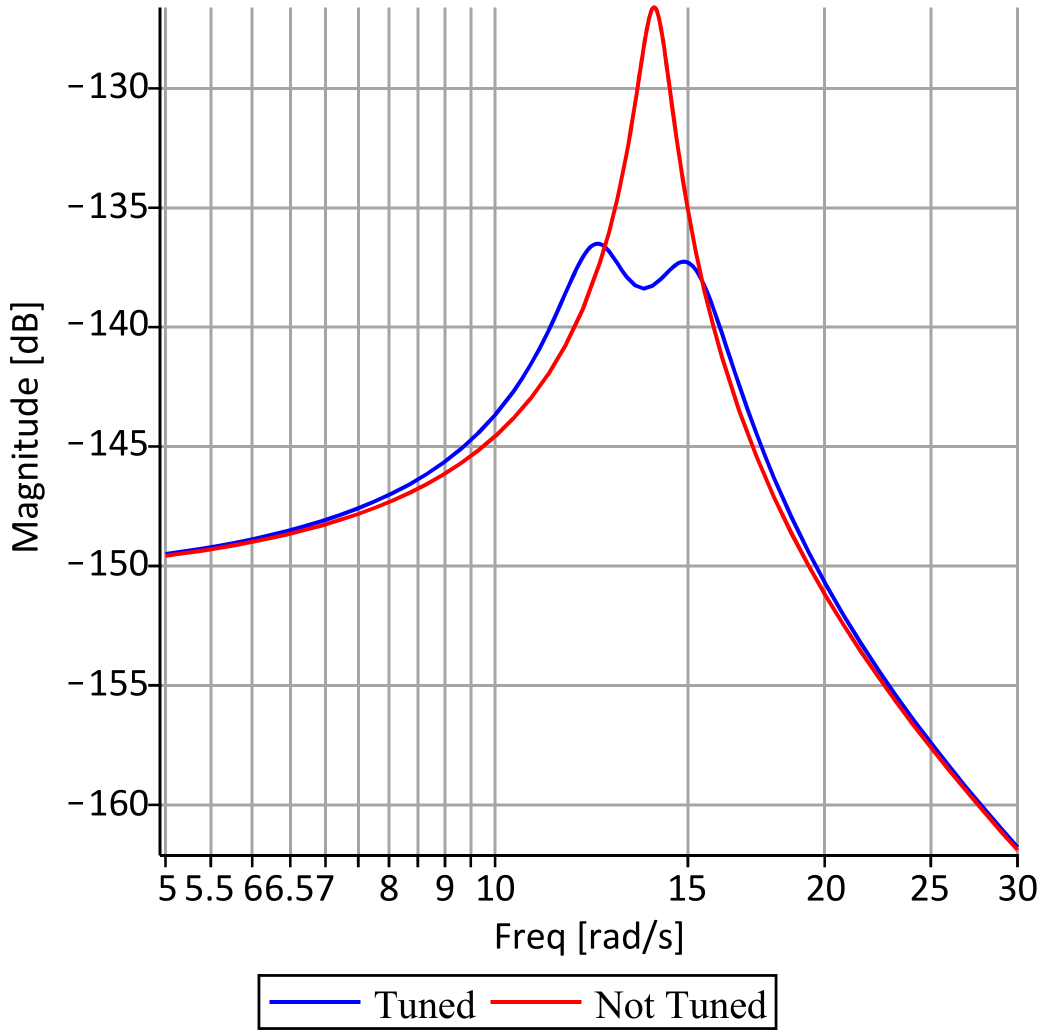
Response with Tuned Mass Damper

> p1 := MagnitudePlot(sys, range = 5 ..30, parameters = $params_{tuned}$, color = blue, legend = "Tuned") :

Response with no Tuned Mass Damper

> p2 := MagnitudePlot(sys, range = 5 ..30, parameters = $params_{nottuned}$, color = red, axesfont = [Calibri], labelfont = [Calibri], legend = "Not Tuned") :

> display(p1, p2)



Dynamic Response

Assuming that the main system is perturbed at its natural frequency

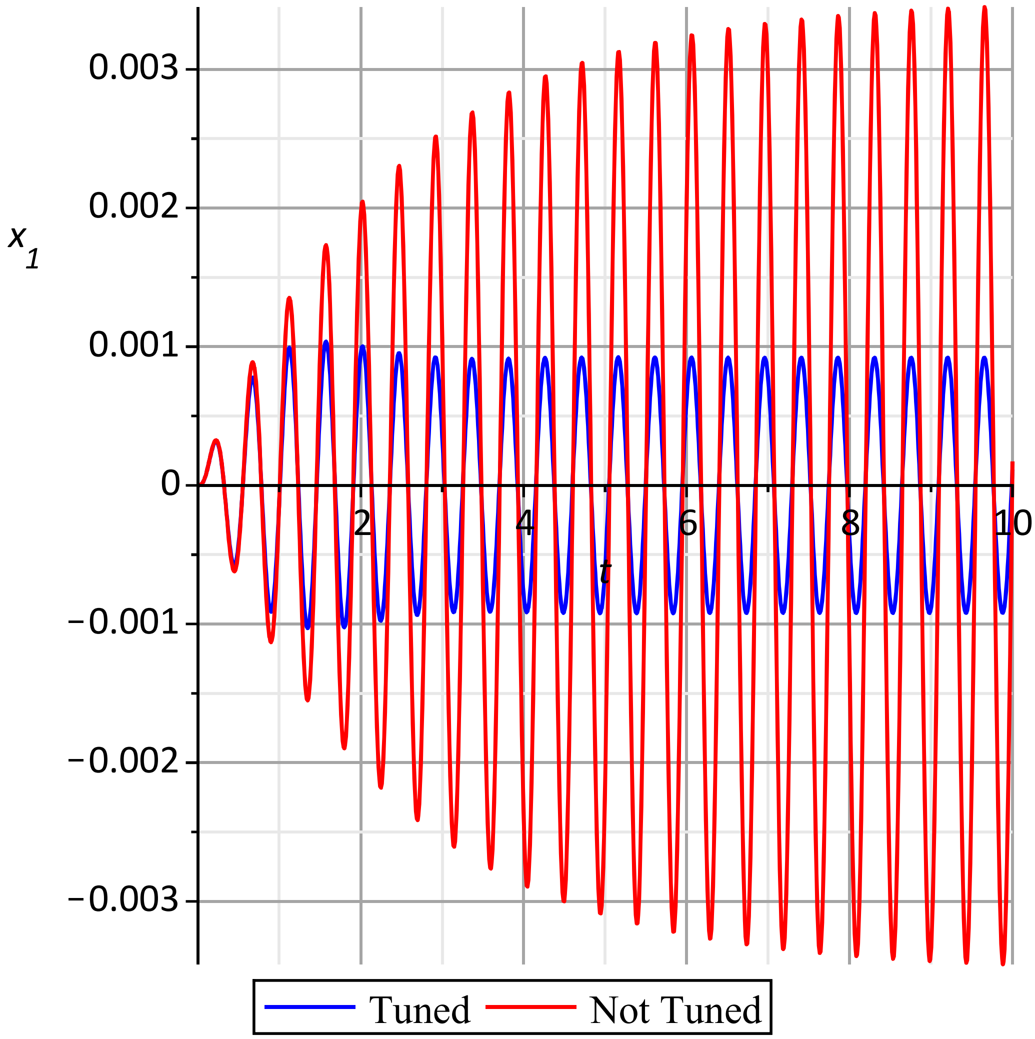
> $f := \text{eval}\left(\omega_1, \text{params}\right)$

$$f := 13.98492872$$

> $p1 := \text{ResponsePlot}\left(\text{sys}, 7500\sin(f \cdot t), \text{parameters} = params_{tuned}, \text{color} = \text{blue}, \text{numpoints} = 2^{10}, \text{legend} = \text{"Tuned"}\right):$

> $p2 := \text{ResponsePlot}\left(\text{sys}, 7500\sin(f\,t), \text{parameters} = params_{nottuned}, \text{color} = \text{red}, \text{numpoints} = 2^{10}, \text{legend} = \text{"Not Tuned"}\right):$

> display(p1, p2, axesfont = [Calibri], labelfont = [Calibri], axesfont = [Calibri], *gridlines*)



Power Spectrum

> tunedResponseData := plottools:-getdata(p1)[3] :

> notTunedResponseData := plottools:-getdata(p2)[3] :

> $samplingRate := \frac{1}{tunedResponseData[2, 1] - tunedResponseData[1, 1]}$:

> psTuned := PowerSpectrum(FFT(tunedResponseData[.., 2])) :

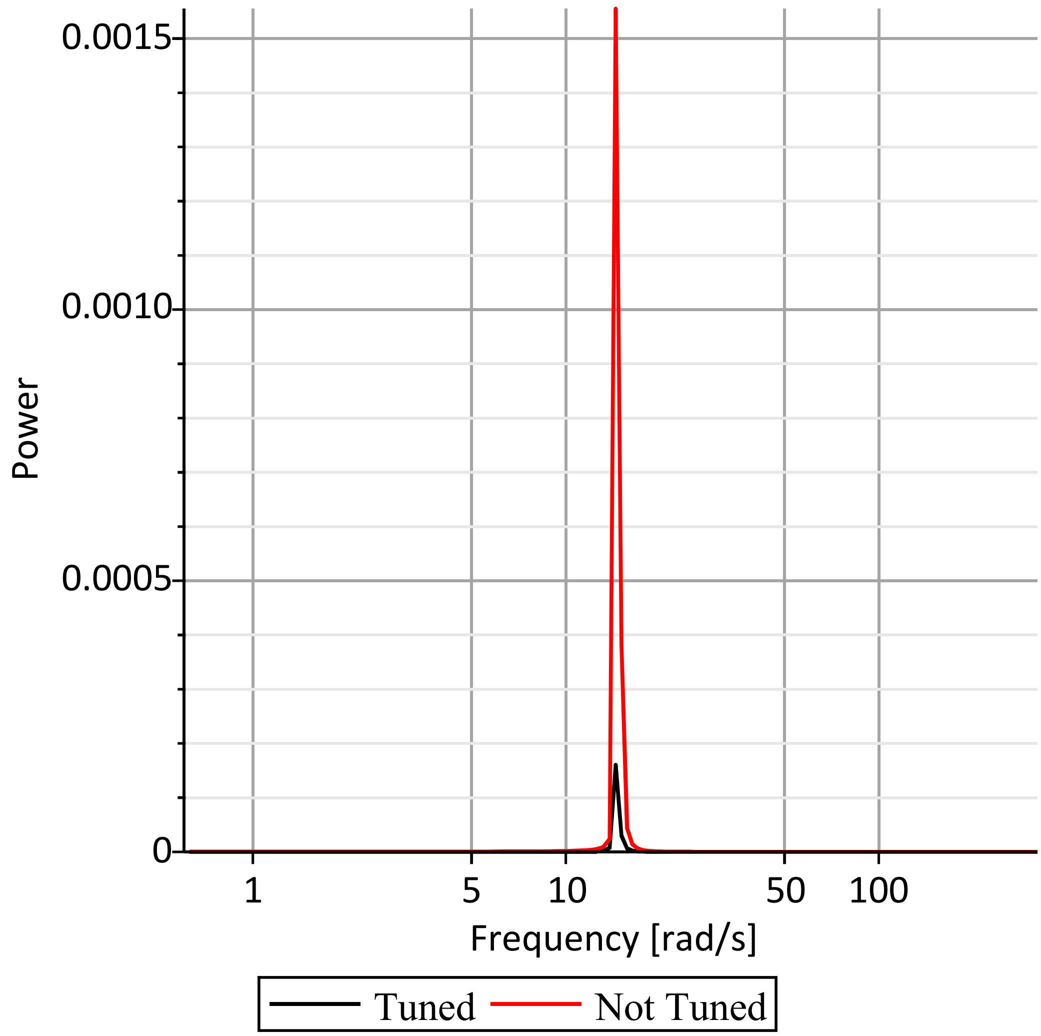
> psNotTuned := PowerSpectrum(FFT(notTunedResponseData[.., 2])) :

> $psPlot1 := pointplot\left(\left[seq\left(\left[\frac{i \cdot samplingRate}{2^{10}} \cdot 2 \cdot Pi, psTuned[i]\right], i = 1 .. \frac{2^{10}}{2}\right)\right], connect = true, legend = "Tuned", gridlines\right)$:

$psPlot2 := pointplot\left(\left[seq\left(\left[\frac{i \cdot samplingRate}{2^{10}} \cdot 2 \cdot Pi, psNotTuned[i]\right], i = 1 .. \frac{2^{10}}{2}\right)\right], connect = true, color = red, legend = "Not Tuned"\right)$:

> display(psPlot1, psPlot2, axis[1] = [mode = log], axesfont = [Calibri], labels = ["Frequency [rad/s]",

"Power"], labeldirections = [horizontal, vertical], labelfont = [Calibri])



>