# ▼ The classic SIR model

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## ▼ Background

The classical SIR model assumes that a population can be divided into three distinct compartments (see e.g. Bloomfiled[2009, pp 149]): $S$ is the proportion of susceptibles, $I$ is the proportion of infected persons and $R$ is the proportion of persons that have recovered from infection and are now immune against the disease. This model was originally developed in 1927 by Kermack and McKendrik.



**Figure 1**

The transmission of persons from the compartment of susceptibles to the compartment of infected persons and from the compartment of infected persons to the compartment of recovered persons can be described by a set of three differential equations with two parameters:

$S(t)$ : fraction of susceptibles

$I(t)$ : fraction of infected persons

$R(t)$ : fraction of recovered persons

$\beta > 0$ : infection (transmission) rate

$\gamma \ge 0$ : removal rate

(1) $\dfrac{dS(t)}{t} = -\beta \cdot S(t) \cdot I(t)$

(2) $\dfrac{dI(t)}{t} = \beta \cdot S(t) \cdot I(t) - \gamma \cdot I(t)$

(3) $\dfrac{(}{t} = \gamma \cdot I(t)$

Initial conditions:

$S(0) \in (0, 1]$

$I(0) \in (0, 1]$

$R(0) = 0$

$S(0) + I(0) = 1$

The characteristc spread of an epidemic disease is described by additional parameters which for the SIR model as decribed above are as follows:

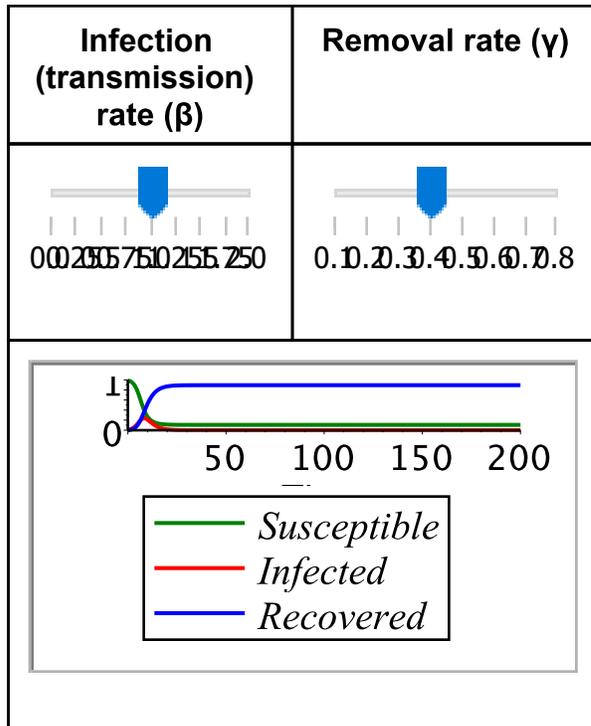The basic reproduction ratio $R_0$ is defined as:

$$R_0 = \frac{\beta}{\gamma}$$

$R_0$ is the number of secondary infections that is produced by one primary infection in a wholly susceptible population. Only when $R_0 > 1$, an epidemic occurs. The parameter $\rho = \gamma/\beta$ is the relative removal rate. $1/\gamma$ is the average infectious period.

For $I(t) = 0$, the model is in equilibrium.

Warnings: (1) The basic reproduction ratio and the recovered persons use unfortunately the same capital letter $R$ as symbol but one has to be aware that these are different entities. (2) Not all combinations of infection and removal rates within the ranges of the interactive model below will match a real existing infectious disease.

The SIR model exists in different flavors and parametrizations, the model as presented here follows Bloomfield[2009].

# ▼ Interactive SIR model

| Infection (transmission) rate (β) | Removal rate (γ) |
|---|---|
| 0.00 0.250.50.751.01.251.51.752.0 | 0.10.20.30.40.50.60.70.8 |
|  | |

| Model summary | | |
|---|---|---|
| **Parameter name** | **Symbol** | **Value** |
| Susceptibles, fraction at t=0 | $S(0)$ | 0.99 |
| Infected, fraction at t=0 | $I(0)$ | 0.01 |
| Recovered, fraction at t=0 | $R(0)$ | 0.00 |
| Infection (transmission) rate | β | 1.00 |

| | | |
|---|---|---|
| Removal rate | $\gamma$ | 0.40 |
| Basic reproduction ratio | $R_0 = \dfrac{\beta}{\gamma}$ | 2.50 |
| Relative removal rate | $\rho = \dfrac{\gamma}{\beta}$ | 0.40 |
| Average infectious period | $\dfrac{1}{\gamma}$ | 2.50 |

## References

Bloomfield  V [2009]: *Computer Simulation and Data Analysis in Molecular Biology and Biophysics. An Introduction Using R.* Springer, Dordrecht, Heidelberg, London, NewYork

Kermack WO, McKendrick AG [1927]: *A contribution to the Mathematical Theory of Epidemics.* Proc. Roy. Soc. A 115, 700-711 (Download link)