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Classroom Tips and Techniques: Slider-Control of Parameters in Numeric Solutions of ODEs

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Introduction

In the article "Sliders for Parameter-Dependent Curves," and again in the article "Caustics for a Plane Curve," the use of sliders to control parameters was explored. This month's article explores the use of sliders to control parameters in a differential equation that must be solved numerically.

The differential equation governing the motion of a damped linear oscillator contains at least two parameters, the damping coefficient and the spring constant. Sliders are easily implemented for visualizing the effect of these parameters because the equation can be solved analytically. The essence of month's article is the implementation of sliders to control parameters in an ODE that must be solved numerically.

Initializations

The following modifications to the input/output protocol for differential equations allow a more natural notation. For a full discussion of these devices, see the Classroom Tips and Techniques article Notational Devices for ODEs.

```
interface(typesetting = extended) :
Typesetting[Settings](useprime, prime = t) :
Typesetting[Suppress](y(t)) :
```

The Damped Oscillator

The ODE governing the motion of a damped (and driven) linear oscillator is taken as

$$y'' + a\,y' + b\,y = \cos(t)$$

and the inert initial conditions

$$y(0) = 0, y'(0) = 0$$

are assumed. Note that the driving term $\cos(t)$ has been included, making the differential equation nonhomogeneous. This choice for the driving term implies that the solution becomes unbounded when the parameters assume the values *a* = 0, *b* = 1. Resonance also occurs when $a^2 = 4b > 0$. However, the solution remains bounded in this case.

Analytic Solution

Maple represents the analytic solution of the initial value problem comprised of and as

$$Y := -\frac{1}{2\left(a^2+b^2-2b+1\right)\left(a^2-4b\right)}\left(e^{\left(-\frac{a}{2}+\frac{\sqrt{a^2-4b}}{2}\right)t}\left(-a^2+4b+a\sqrt{a^2-4b}\,b+a\sqrt{a^2-4b}+a^2 b-4b^2\right)\right)$$
$$-\frac{1}{2\left(a^2+b^2-2b+1\right)\left(a^2-4b\right)}\left(e^{\left(-\frac{a}{2}-\frac{\sqrt{a^2-4b}}{2}\right)t}\left(a^2 b-a^2-a\sqrt{a^2-4b}-a\sqrt{a^2-4b}\,b-4b^2+4b\right)\right)$$
$$+\frac{\sin(t)\,a+(b-1)\cos(t)}{a^2+b^2-2b+1}$$

Table 1 lists the fundamental set (solutions of the homogeneous ODE) for different regimes for the parameters *a* and *b*.

Fundamental Set	**Parameter Regime**
$\left[e^{-a\,t/2}\cos(\omega\,t),\, e^{-a\,t/2}\sin(\omega\,t)\right]$	$a^2 < 4b,\ \omega = \sqrt{4b-a^2}\,/2$
$\left[e^{\lambda_+ t}, e^{\lambda_- t}\right]$	$a^2 > 4b,\ \lambda_\pm = \frac{-a \pm \sqrt{a^2-4b}}{2}$
$\left[e^{-a\,t/2}, t\,e^{-a\,t/2}\right]$	$a^2 = 4b > 0$
$[1, t]$	$a^2 = 4b = 0$

Table 1 Fundamental set for different parameter-value regimes

The parameter-dependence of the particular solution is given by

$$y_p = \begin{cases} A\cos(t) + B\sin(t) & (a, b) \neq (0, 1) \\ t\,(A\cos(t) + B\sin(t)) & (a, b) = (0, 1) \end{cases}$$

It is no surprise, then, that the analytic solution Y is undefined if (a, b) is one of the pairs $(0, 1)$, $(2\sqrt{s}, s), s \geq 0$. Where the "form" of the solution changes, division-by-zero errors are generated under simple evaluation in Maple.

Slider-Control of Parameters for the Analytic Solution

The following code will launch the Interactive Parameter Maplet in which the graph of the solution Y is under the control of two sliders that vary the values of the parameters *a* and *b*.

```
YY := evalc(Y) :
plots[interactiveparams](plot, [YY, t = 0 ..20, -5 ..5], a = 0 ..5, b = 0 ..5)
```

Figure 1 is a screen-shot of the Interactive Parameter Maplet.

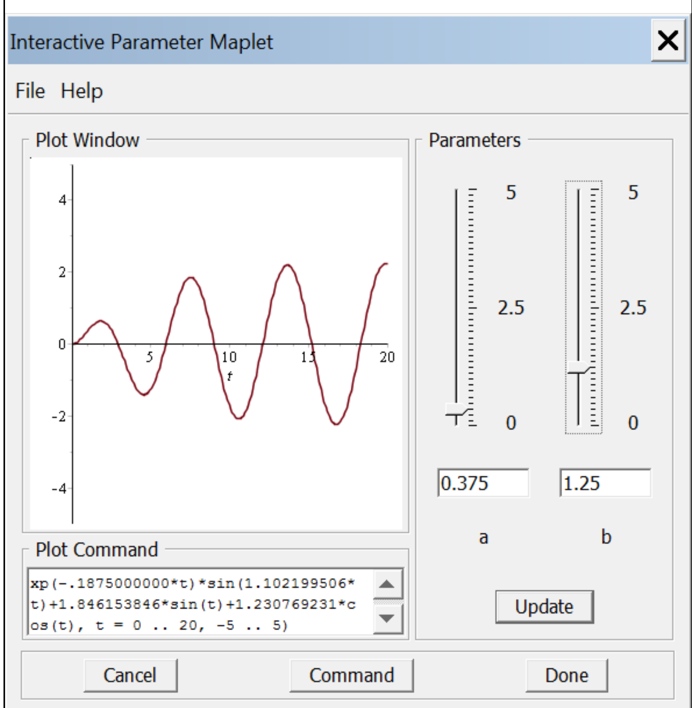


Figure 1 Solution of the damped oscillator for one set of parameter values

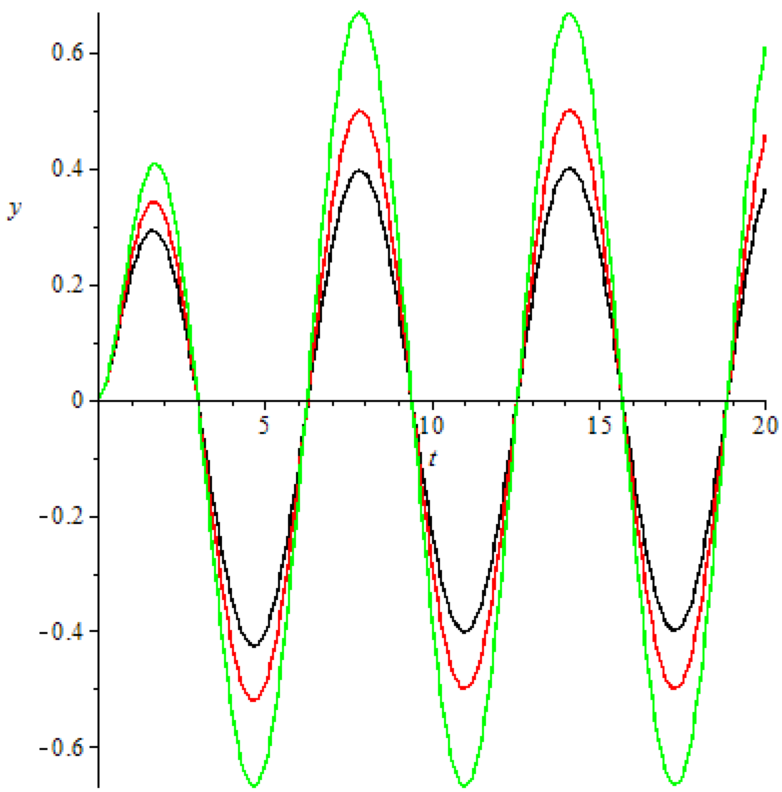
Applying the **evalc** command to Y dramatically improves the performance of the Interactive Parameter Maplet. Without it, the performance of the Maplet degrades significantly.

Had the Plot Builder been invoked on the expression for YY, the same Interactive Parameter Maplet could have been been generated "interactively." However, the expression for YY, is very large, expanded in part because of the many instances of the **signum** function applied to $a^2 - 4b$.

Slider-Control of Parameters for a Numeric Solution

Maple's **dsolve**/**numeric** command writes a procedure that only initiates computations when called. This procedure can have provision for symbolic parameters. For example, graphs of the numeric solution for several sets of parameter values can be generated by the following code.

$Q := dsolve(\{\mathbf{(1)}, \mathbf{(2)}\}, y(t), numeric, parameters = [a, b], compile = true) :$
$Q(parameters = [2.5, 1]) :$
$p_1 := plots[odeplot](Q, [t, y(t)], 0\,..20, color = black) :$
$Q(parameters = [2, 1]) :$
$p_2 := plots[odeplot](Q, [t, y(t)], 0\,..20, color = red) :$
$Q(parameters = [1.5, 1]) :$
$p_3 := plots[odeplot](Q, [t, y(t)], 0\,..20, color = green) :$
$plots[display](p_1, p_2, p_3)$



The inclusion of the *compile* option considerably speeds up the ensuing numeric calculations. Additionally, it is possible to write a procedure that passes parameter values to the numeric procedure in a "continuous" fashion. The following code shows how this might be done.

```
F := proc(a :: numeric, b :: numeric)
Q(parameters = [a, b]) :
plots[odeplot](Q, [t, y(t)], 0 ..20, view = [0 ..20, -10 ..10]);
end proc:
```

The **interactiveparams** command can now be applied to the function F with the same output that was obtained for the analytic solution.

```
plots[interactiveparams](F, [a, b], a = 0 ..5, b = 0 ..5)
```

Figure 2 contains a screen-shot of the resulting Interactive Parameter Maplet.

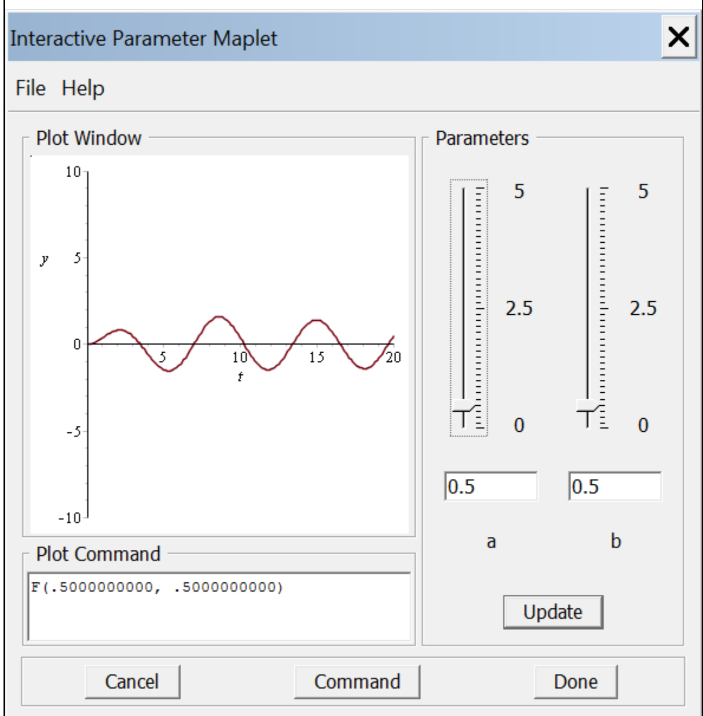


Figure 2 Interactive Parameter Maplet serving a numeric solution of the initial value problem containing two parameters.

Parameter Control with a 2D-Slider

Since there are only two parameters, it is possible to control the graph of the solution with a "2D" slider. The graph on the left in Figure 2 is a representation of the *ab*-plane. Dragging across this plane sends values of the pair (a, b) to code that, on the right, graphs a numerical solution of the initial value problem and . Be sure to begin by pressing the Initialize button under the graph on the right.

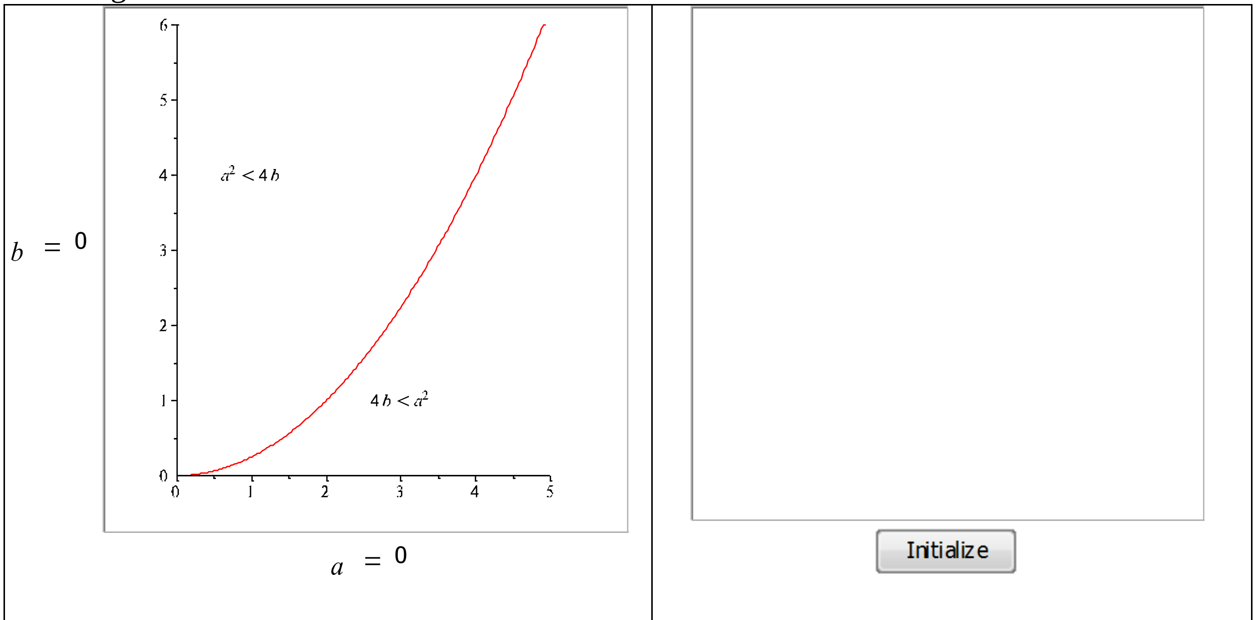


Figure 3 Parameter-control via a "2D" slider

To see the codes that drive the application in Figure 3, right-click (or its equivalent) on the Initialize button and select "Component Properties." In the dialog that opens, click "Edit". Then, right-click on the left-hand graph, select "Component/Component Properties" and in the dialog that opens, click "Edit" opposite "Action When Dragged."