A hybrid discrete-polynomial dynamical system modeling board positions in the game of go

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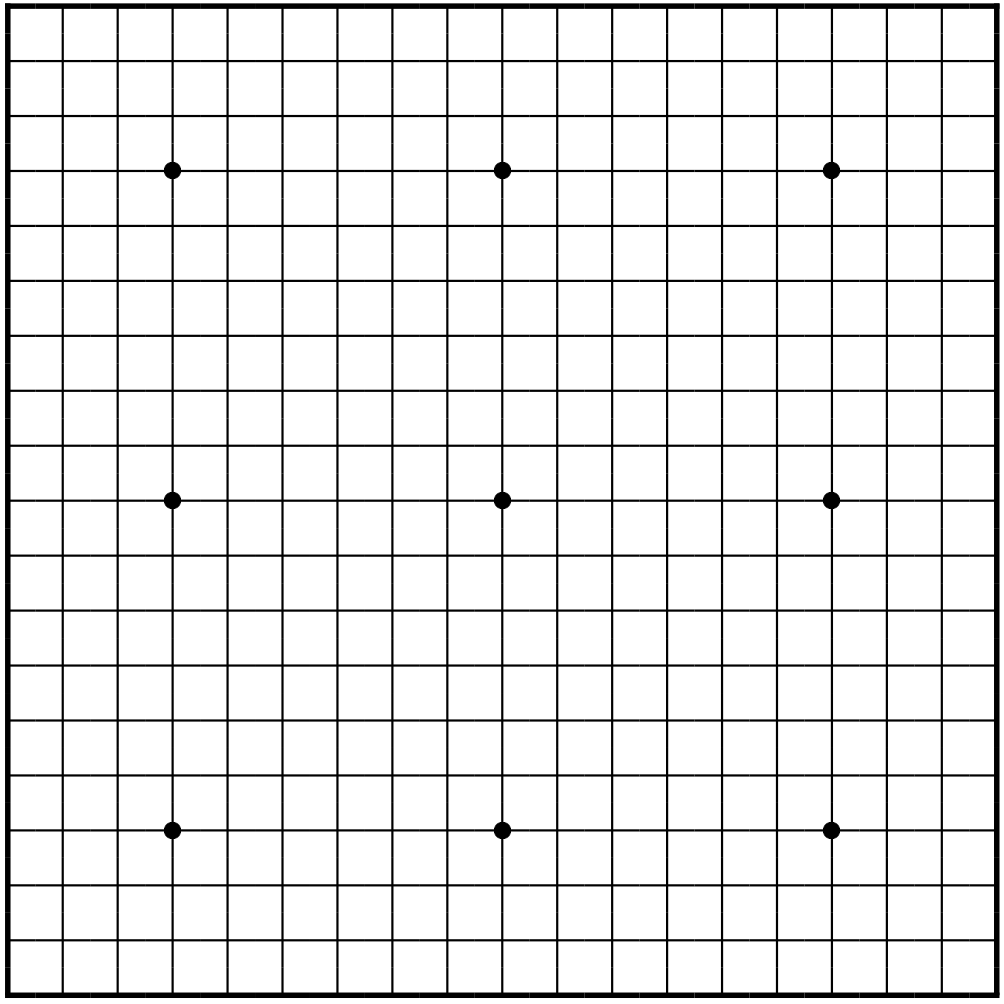
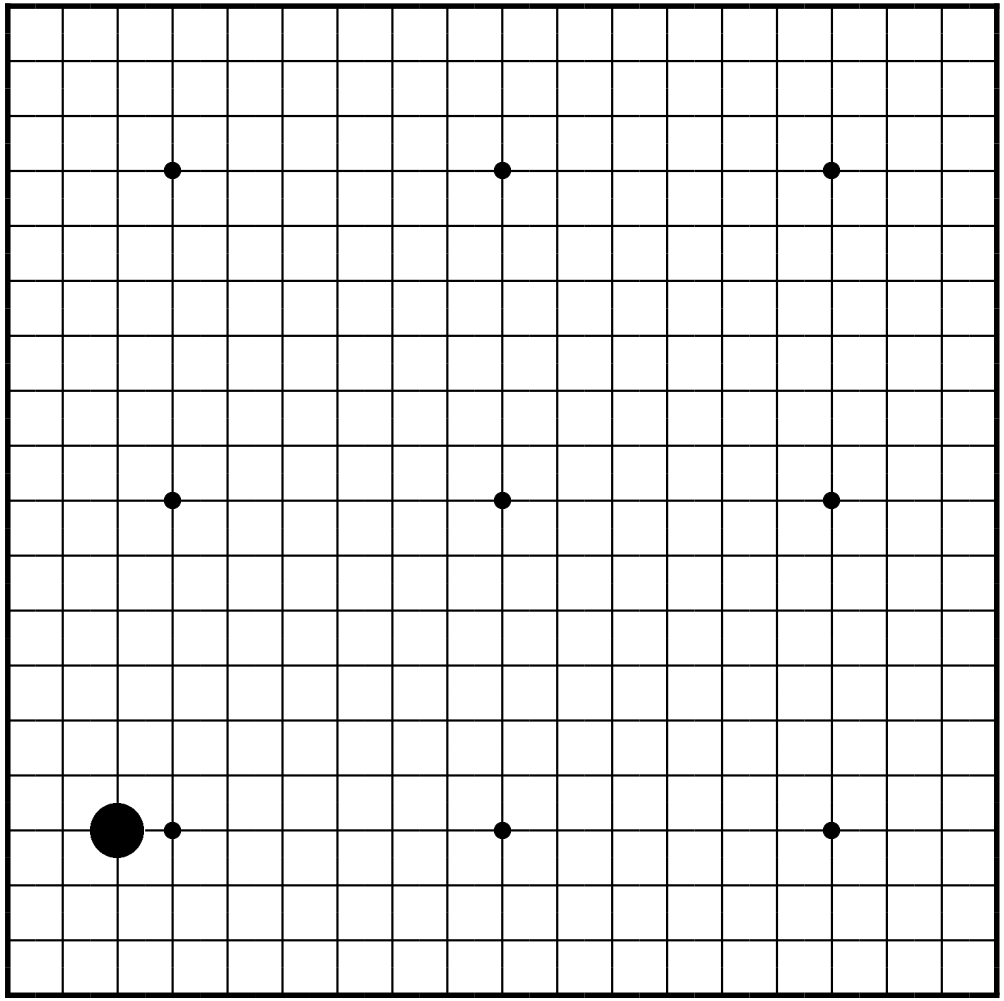
Outline

The Board



The normal board size is 19×19 but 9×9 and 13×13 are common too.

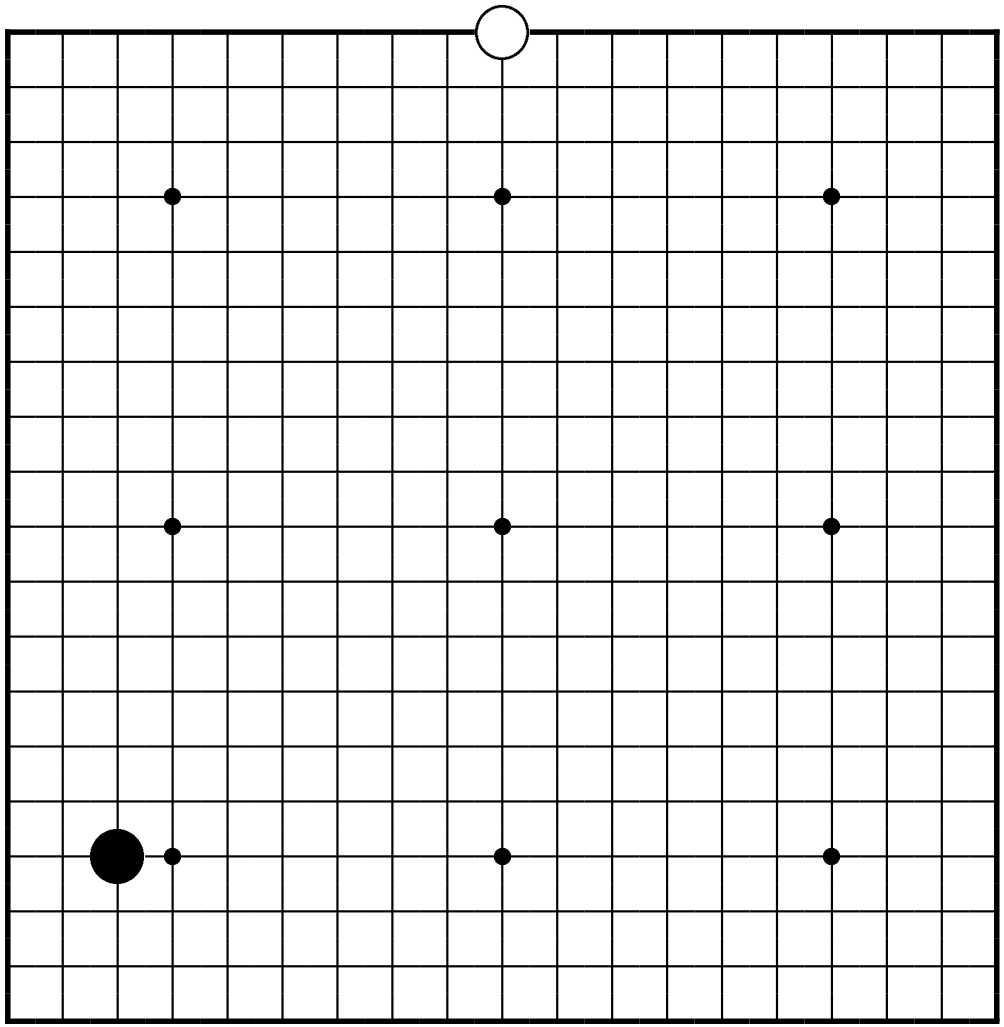
Making Moves



Two players ● and ○ alternate in putting a stone on an empty intersection of the grid (called *points*).

● starts.

Making Moves



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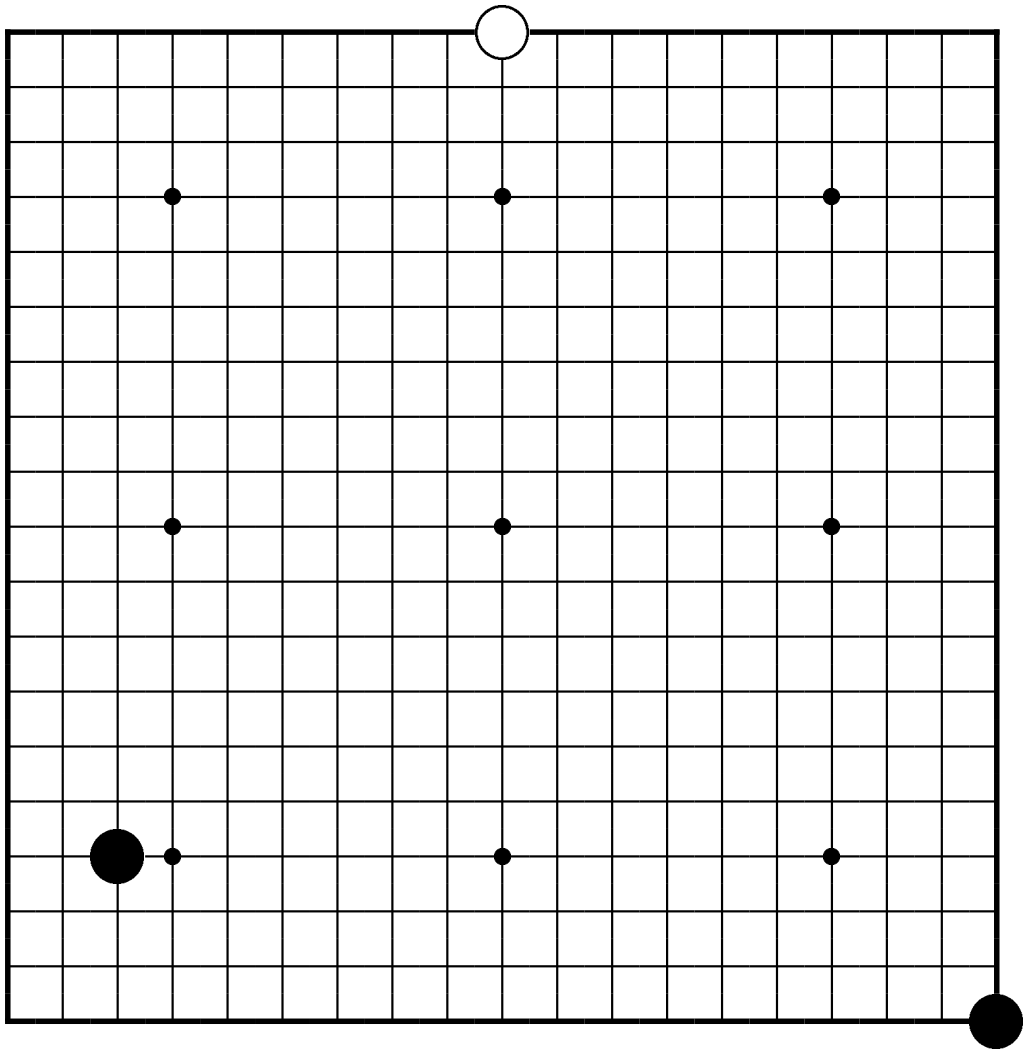
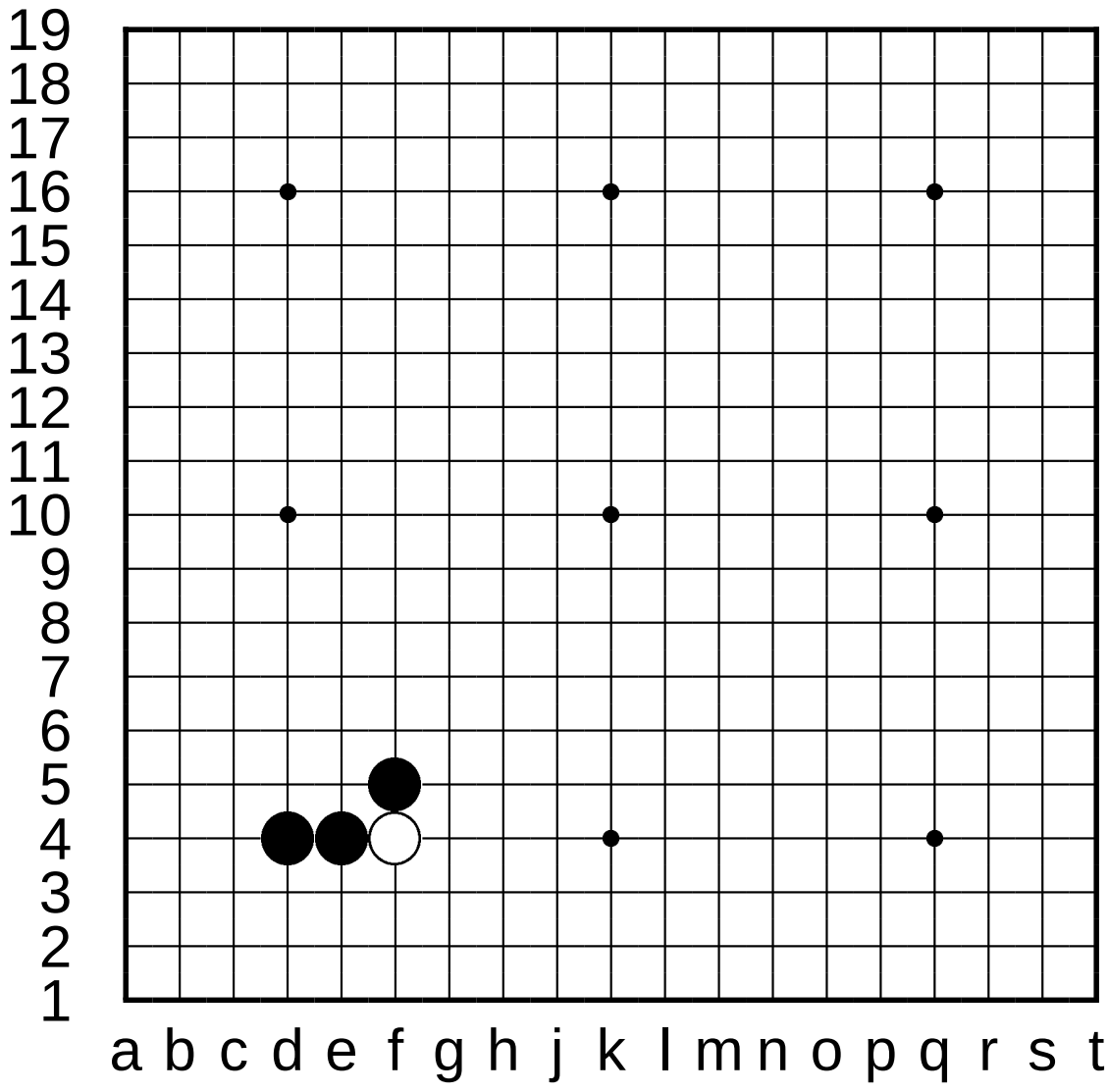


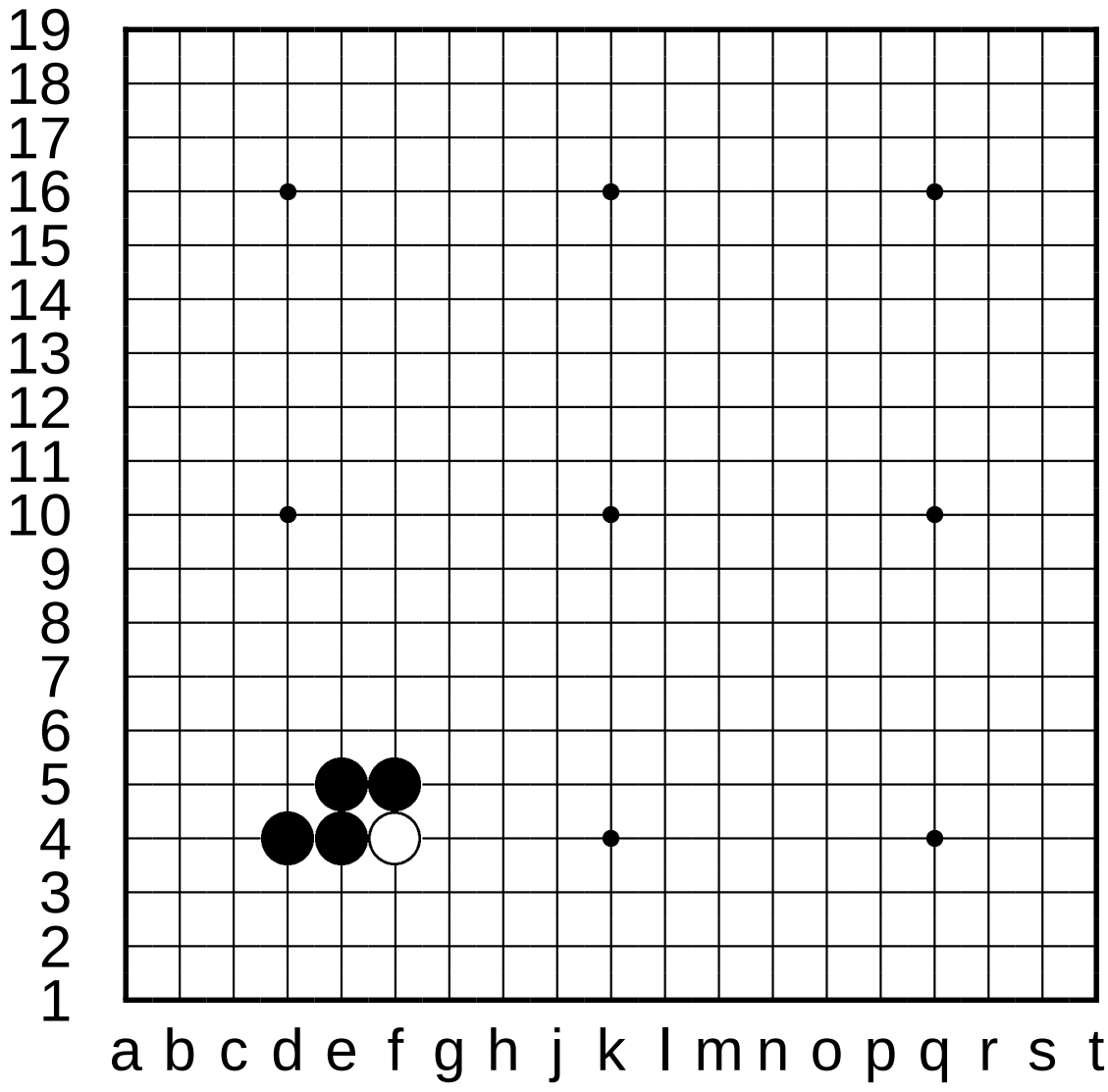
The number of stones available to both players is unlimited, in practice each side has $19^2/2 = 180$ stones.
Players may pass.

Chains and Liberties



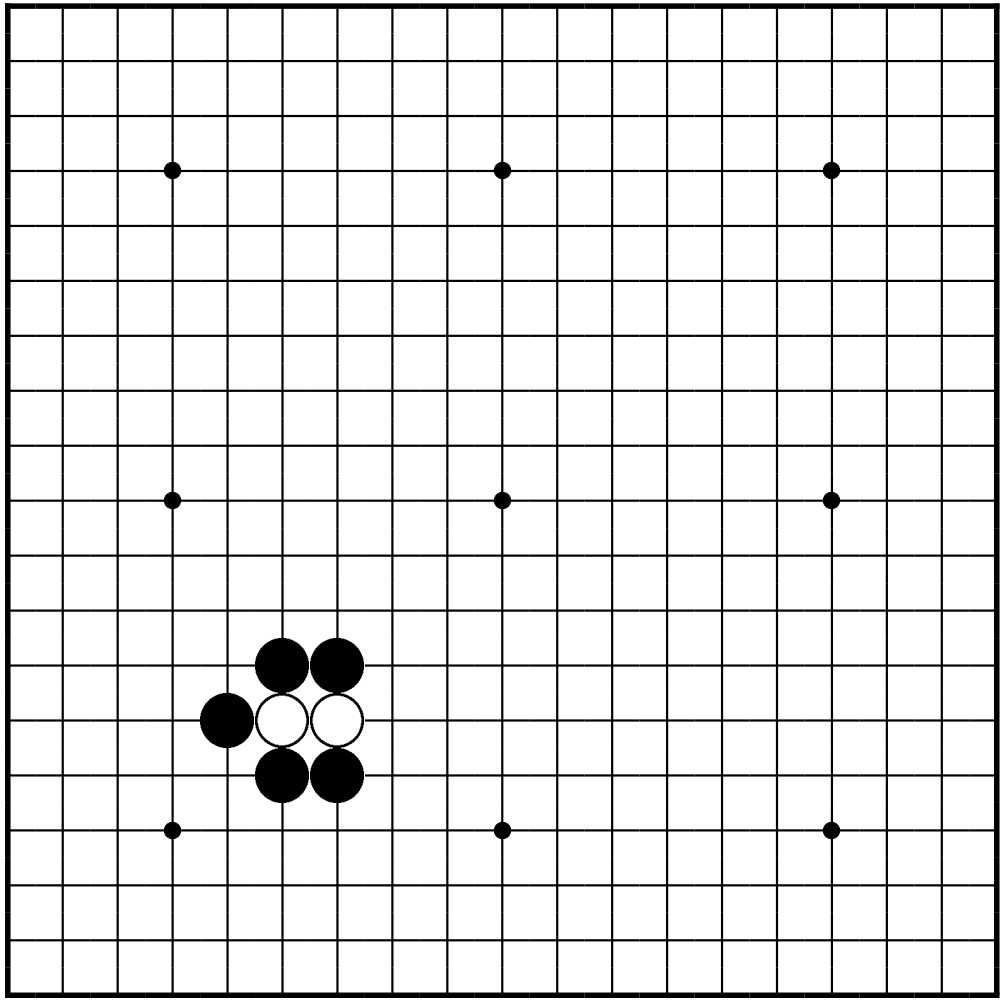
Adjacent stones of one colour form what we call a *chain* (also called *block*). This diagram shows two black and one white chain.

Chains and Liberties



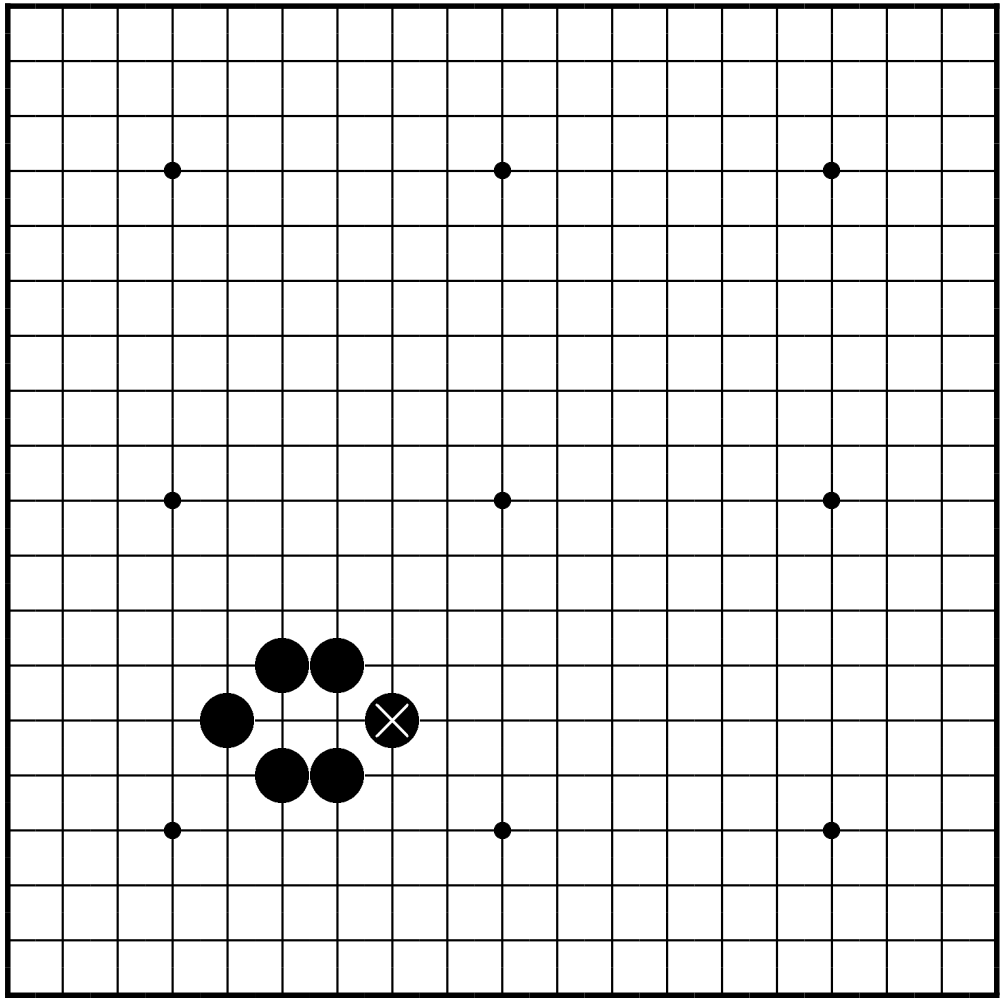
The extra black stone links the two black chains into a single chain. Points adjacent to a chain are called *liberties* of that chain. The black chain has 7 liberties: e3, d3, c4, d5, e6, f6, g5.

The Capture Rule



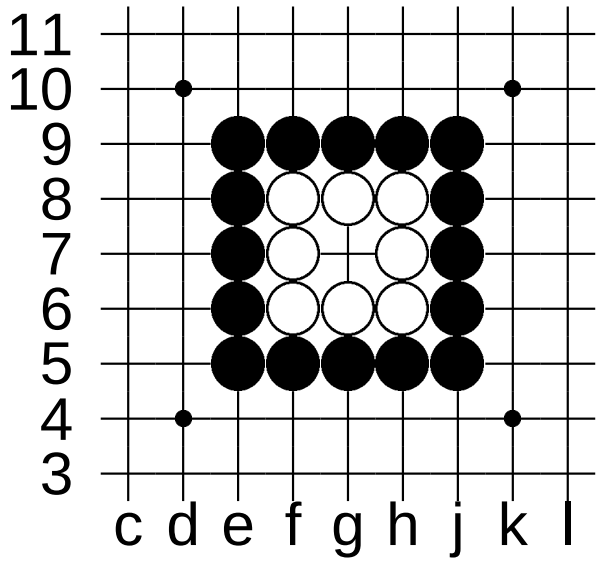
1. Rule: Each chain needs at least one liberty, for example, the white chain has one liberty. If ● is going to occupy that too then ..

The Capture Rule



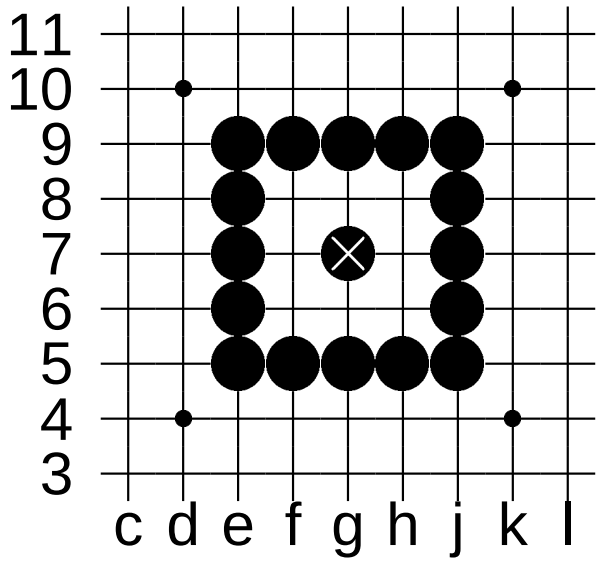
.. the white stones are captured, i.e. they are taken off the board and kept as prisoners till the end of the game.
The resulting free space surrounded by stones of one colour is call *eye*.

The Suicide Rule



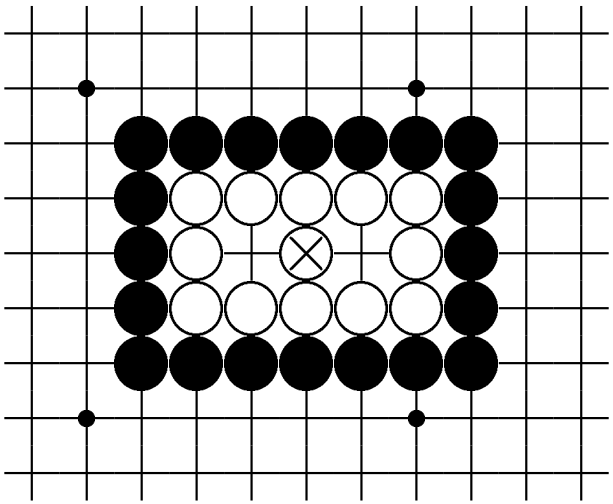
2. Rule: A move that takes the last liberty of a chain of same colour without capturing an opposite chain is forbidden. That means that ◯ on g7 is forbidden ..

The Suicide Rule



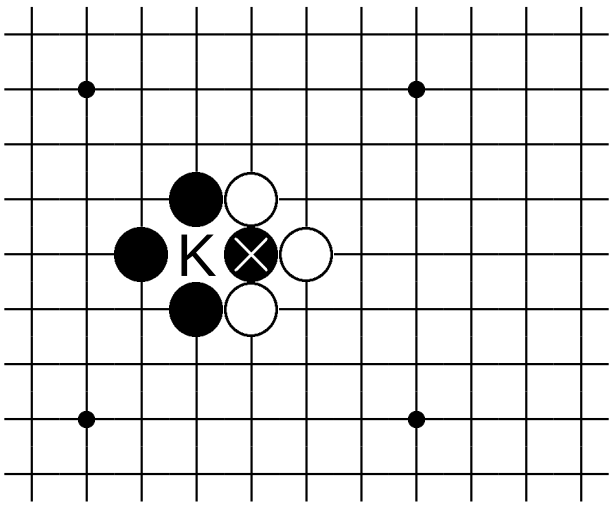
.. whereas ⊗ is allowed because it has liberties as a result of capture.

A new Quality: Life



⊗ creates two eyes. A ● move inside any one eye would violate the suicide rule because no capture would take place because ○ would still have one liberty in the other eye. Therefore, ○ can not be captured, i.e. it is *alive*.

The Ko Rule



3. Rule: A move may not restore the exact position before the previous move.

The point marked K is a forbidden point for ○ to play right now. It may be played in a later move.

Outline

Typical Computer Approaches

- Knowledge based programs

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- Monte-Carlo programs

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- Monte-Carlo programs
- Learning from professional games or selfplay

Outline

What is missing in Computer Go?

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- An understanding of the nature of Go? $(\surd)$
  Hardly used so far: Go has continuous aspects and much of the interaction on the board is local.

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Ultralocal Data: The exact shape shape of chains is taken into account in addition to neighbourhood relations,...

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Global Relations: When a Ko-fight starts then already settled regions start to depend on each other and large scale global trading starts.

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This system couples all units on the board (i.e. all (empty) points and chains) and thus a fixpoint of the dynamical system is a ‘global consequence’ of the whole board. A change of strength of one chain would influence the strength of weak neighbouring chains and so on but would stop at strong chains.

State Variables

To each *point i* (i.e. each empty intersection) are attached 2 real floating point type numbers:

w_i . . . probability to be occupied by ○ at end of game
b_i . . . probability to be occupied by ● at end of game

and to each *chain j* is attached one number:

s_j . . . probability for this chain to survive.

All values are in the interval 0 . . . 1.
For explanation purposes we also introduce

$\bar{w}_i$, $\bar{b}_i$ . . . probability that at least one neighbouring point is occupied by resp. ○ or ● at the end of game.

Relations

Apart from

$$b_i + w_i = 1 \tag{1}$$

we make the assumption $w_i/b_i = \bar{w}_i/\bar{b}_i$, i.e.

$$w_i \bar{b}_i = b_i \bar{w}_i \tag{2}$$

which gives correct results at least in the extreme cases $(\bar{w}_i, \bar{b}_i) = (1,1),\ (1,0),\ (0,1)$. From (1), (2) we get

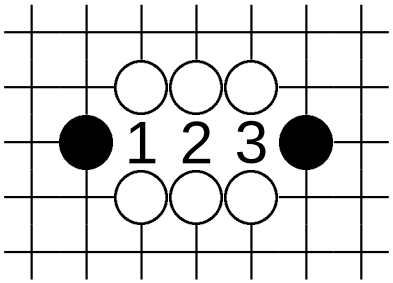
$$w_i = \frac{\bar{w}_i}{\bar{b}_i}(1 - w_i) = \frac{\bar{w}_i}{\bar{b}_i} - \frac{\bar{w}_i}{\bar{b}_i} w_i$$

$$w_i \left(1 + \frac{\bar{w}_i}{\bar{b}_i}\right) = \frac{\bar{w}_i}{\bar{b}_i}$$

$$w_i = \frac{\bar{w}_i}{\bar{b}_i} \left(1 + \frac{\bar{w}_i}{\bar{b}_i}\right)^{-1} = \frac{\bar{w}_i}{\bar{w}_i + \bar{b}_i}$$

where $\bar{w}_i, \bar{b}_i$ have to be expressed in terms of b_j, w_j, s_j from the neighbouring points and chains.

Example



$$w_i = \frac{\bar{w}_i}{\bar{w}_i + \bar{b}_i}$$

All chains are initially fully alive: $s_j = 1$.

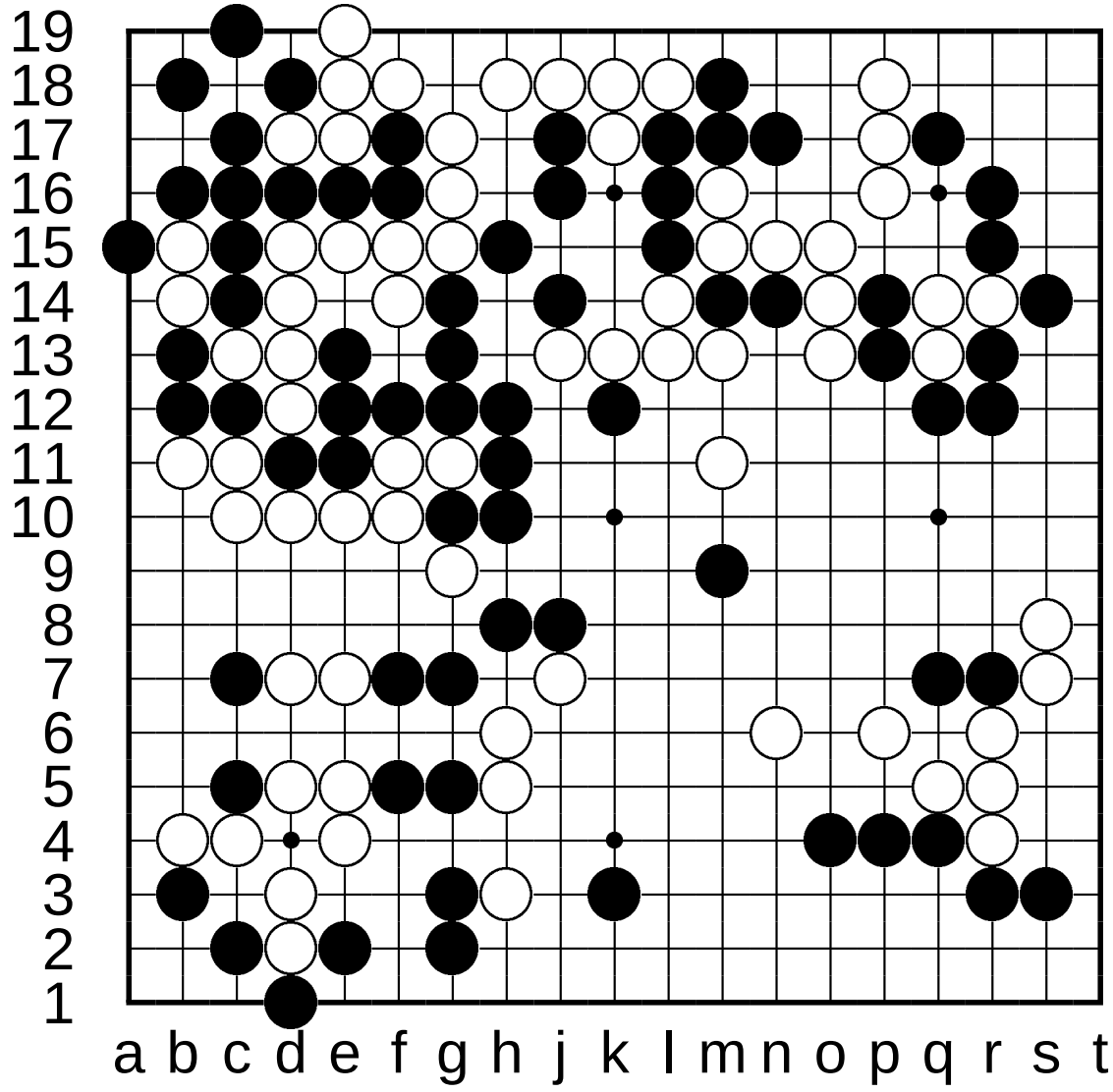
$$\rightarrow w_1 = \frac{1}{1+1} = \frac{1}{2} = b_1 = w_3 = b_3 \quad \text{(by symmetry)}$$

$$\begin{aligned} w_2 &= \frac{1}{1+\bar{b}_2} \\ &= \frac{1}{1+3/4} \\ &= \frac{4}{7} \\ b_2 &= \frac{3}{7} \end{aligned}$$

$$\begin{aligned} \rightarrow \bar{b}_2 &= \text{probability of } \bullet \text{ on 1 or } \bullet \text{ on 3} \\ &= 1 - \text{probability of } \bigcirc \text{ on 1 and } \bigcirc \text{ on 3} \\ &= 1 - w_1 w_3 \\ &= 1 - \frac{1}{4} \\ &= \frac{3}{4} \end{aligned}$$

A similar computation is done for all chains.

Full Board Example



The corresponding dynamical System

All b_, w_ variables are initializied to 0.5 and all s_ variales to 1. In total there are 489 variables and as many equations:

$$b_{t19} = -w_{t19} + 1,$$

$$w_{t19} = \frac{b_{s19} b_{t18} - 1}{b_{s19} b_{t18} + w_{s19} w_{t18} - 2},$$

$$b_{t18} = -w_{t18} + 1,$$

$$w_{t18} = \frac{b_{s18} b_{t17} b_{t19} - 1}{b_{s18} b_{t17} b_{t19} + w_{s18} w_{t17} w_{t19} - 2},$$

$$b_{t17} = -w_{t17} + 1,$$

$$w_{t17} = \frac{b_{s17} b_{t16} b_{t18} - 1}{b_{s17} b_{t16} b_{t18} + w_{s17} w_{t16} w_{t18} - 2},$$

$$b_{t16} = -w_{t16} + 1,$$

$$w_{t16} = \frac{b_{s16} b_{t15} b_{t17} - 1}{b_{s16} b_{t15} b_{t17} + w_{s16} w_{t15} w_{t17} - 2},$$

The corresponding dynamical System

$$b\_t15 = -w\_t15 + 1,$$

$$w\_t15 = \frac{b\_s15*b\_t14*b\_t16 - 1}{b\_s15*b\_t14*b\_t16 + w\_s15*w\_t14*w\_t16 - 2},$$

$$b\_t14 = -w\_t14 + 1,$$

$$w\_t14 = \frac{b\_t13*b\_t15*s\_s14 - 1}{b\_t13*b\_t15*s\_s14 - s\_s14*w\_t13*w\_t15 + w\_t13*w\_t15 - 2},$$

$$b\_t13 = -w\_t13 + 1,$$

$$w\_t13 = \frac{b\_s13*b\_t12*b\_t14 - 1}{b\_s13*b\_t12*b\_t14 + w\_s13*w\_t12*w\_t14 - 2},$$

$$b\_t12 = -w\_t12 + 1,$$

$$w\_t12 = \frac{b\_s12*b\_t11*b\_t13 - 1}{b\_s12*b\_t11*b\_t13 + w\_s12*w\_t11*w\_t13 - 2},$$

$$b\_t11 = -w\_t11 + 1,$$

$$w\_t11 = \frac{b\_s11*b\_t10*b\_t12 - 1}{b\_s11*b\_t10*b\_t12 + w\_s11*w\_t10*w\_t12 - 2},$$

The corresponding dynamical System

$$b_{t10} = -w_{t10} + 1,$$

$$w_{t10} = \frac{b_{s10} b_{t11} b_{t9} - 1}{b_{s10} b_{t11} b_{t9} + w_{s10} w_{t11} w_{t9} - 2},$$

$$b_{t9} = -w_{t9} + 1,$$

$$w_{t9} = \frac{b_{s9} b_{t10} b_{t8} - 1}{b_{s9} b_{t10} b_{t8} + w_{s9} w_{t10} w_{t8} - 2},$$

$$b_{t8} = -w_{t8} + 1,$$

$$w_{t8} = \frac{b_{t7} b_{t9} s_{s7} - b_{t7} b_{t9} + 1}{b_{t7} b_{t9} s_{s7} - b_{t7} b_{t9} - s_{s7} w_{t7} w_{t9} + 2},$$

$$b_{t7} = -w_{t7} + 1,$$

$$w_{t7} = \frac{b_{t6} b_{t8} s_{s7} - b_{t6} b_{t8} + 1}{b_{t6} b_{t8} s_{s7} - b_{t6} b_{t8} - s_{s7} w_{t6} w_{t8} + 2},$$

$$b_{t6} = -w_{t6} + 1,$$

$$w_{t6} = \frac{b_{s6} b_{t5} b_{t7} - 1}{b_{s6} b_{t5} b_{t7} + w_{s6} w_{t5} w_{t7} - 2}$$

The corresponding dynamical System

$$b_{t5} = -w_{t5} + 1,$$

$$w_{t5} = \frac{b_{s5} b_{t4} b_{t6} - 1}{b_{s5} b_{t4} b_{t6} + w_{s5} w_{t4} w_{t6} - 2}$$

$$b_{t4} = -w_{t4} + 1,$$

$$w_{t4} = \frac{b_{s4} b_{t3} b_{t5} - 1}{b_{s4} b_{t3} b_{t5} + w_{s4} w_{t3} w_{t5} - 2}$$

$$b_{t3} = -w_{t3} + 1,$$

$$w_{t3} = \frac{b_{t2} b_{t4} s_{r3} - 1}{b_{t2} b_{t4} s_{r3} - s_{r3} w_{t2} w_{t4} + w_{t2} w_{t4} - 2}$$

$$b_{t2} = -w_{t2} + 1,$$

$$w_{t2} = \frac{b_{s2} b_{t1} b_{t3} - 1}{b_{s2} b_{t1} b_{t3} + w_{s2} w_{t1} w_{t3} - 2}$$

$$b_{t1} = -w_{t1} + 1,$$

$$w_{t1} = \frac{b_{s1} b_{t2} - 1}{b_{s1} b_{t2} + w_{s1} w_{t2} - 2}$$

The corresponding dynamical System

$$b\_s19 = -w\_s19 + 1,$$

$$w\_s19 = \frac{b\_r19*b\_s18*b\_t19 - 1}{b\_r19*b\_s18*b\_t19 + w\_r19*w\_s18*w\_t19 - 2},$$

$$b\_s18 = -w\_s18 + 1,$$

$$w\_s18 = \frac{b\_r18*b\_s17*b\_s19*b\_t18 - 1}{b\_r18*b\_s17*b\_s19*b\_t18 + w\_r18*w\_s17*w\_s19*w\_t18 - 2},$$

$$b\_s17 = -w\_s17 + 1,$$

$$w\_s17 = \frac{b\_r17*b\_s16*b\_s18*b\_t17 - 1}{b\_r17*b\_s16*b\_s18*b\_t17 + w\_r17*w\_s16*w\_s18*w\_t17 - 2},$$

$$b\_s16 = -w\_s16 + 1,$$

$$w\_s16 = \frac{b\_s15*b\_s17*b\_t16*s\_r16 - 1}{b\_s15*b\_s17*b\_t16*s\_r16 - s\_r16*w\_s15*w\_s17*w\_t16 + w\_s15*w\_s17*w\_t16 - 2},$$

$$b\_s15 = -w\_s15 + 1,$$

The corresponding dynamical System

```
w_s15=(b_s16*b_t15*s_r16*s_s14 - 1)/(b_s16*b_t15*s_r16*s_s14
    + s_r16*s_s14*w_s16*w_t15 - s_r16*w_s16*w_t15 - s_s14*w_s16*w_t15
    + w_s16*w_t15 - 2),
s_s14= - s_q13*w_s13*w_s15*w_t14 + 1,
b_s13= - w_s13 + 1,
w_s13=(b_s12*b_t13*s_r13*s_s14 - 1)/(b_s12*b_t13*s_r13*s_s14
    + s_r13*s_s14*w_s12*w_t13 - s_r13*w_s12*w_t13 - s_s14*w_s12*w_t13
    + w_s12*w_t13 - 2),
b_s12= - w_s12 + 1,
w_s12
                          b_s11*b_s13*b_t12*s_r13 - 1
=-------------------------------------------------------------------------,
  b_s11*b_s13*b_t12*s_r13 - s_r13*w_s11*w_s13*w_t12 + w_s11*w_s13*w_t12 - 2
b_s11= - w_s11 + 1,
                     b_r11*b_s10*b_s12*b_t11 - 1
w_s11=-------------------------------------------------------,
       b_r11*b_s10*b_s12*b_t11 + w_r11*w_s10*w_s12*w_t11 - 2
```

The corresponding dynamical System

```
b_s10= - w_s10 + 1,

                    b_r10*b_s11*b_s9*b_t10 - 1
w_s10=-----------------------------------------------------,
       b_r10*b_s11*b_s9*b_t10 + w_r10*w_s11*w_s9*w_t10 - 2

b_s9= - w_s9 + 1,

                  b_r9*b_s10*b_t9*s_s7 - b_r9*b_s10*b_t9 + 1
w_s9=-------------------------------------------------------------------,
      b_r9*b_s10*b_t9*s_s7 - b_r9*b_s10*b_t9 - s_s7*w_r9*w_s10*w_t9 + 2

s_s7= - b_r8*b_s6*b_s9*b_t7*b_t8*s_r7 + 1,

b_s6= - w_s6 + 1,

w_s6=(b_s5*b_t6*s_r4*s_s7 - b_s5*b_t6*s_r4 - b_s5*b_t6*s_s7 + b_s5*b_t6 - 1)/

(b_s5*b_t6*s_r4*s_s7 - b_s5*b_t6*s_r4 - b_s5*b_t6*s_s7 + b_s5*b_t6

  + s_r4*s_s7*w_s5*w_t6 - 2),

b_s5= - w_s5 + 1,

                  b_s4*b_s6*b_t5*s_r4 - b_s4*b_s6*b_t5 + 1
w_s5=-------------------------------------------------------------------,
      b_s4*b_s6*b_t5*s_r4 - b_s4*b_s6*b_t5 - s_r4*w_s4*w_s6*w_t5 + 2

b_s4= - w_s4 + 1,
```

The corresponding dynamical System

```
w_s4=(b_s5*b_t4*s_r3*s_r4 - b_s5*b_t4*s_r3 + 1)/(b_s5*b_t4*s_r3*s_r4
    - b_s5*b_t4*s_r3 + s_r3*s_r4*w_s5*w_t4 - s_r4*w_s5*w_t4 + 2),

b_s2= - w_s2 + 1,

                          b_r2*b_s1*b_t2*s_r3 - 1
w_s2=-------------------------------------------------------------------,
      b_r2*b_s1*b_t2*s_r3 - s_r3*w_r2*w_s1*w_t2 + w_r2*w_s1*w_t2 - 2

b_s1= - w_s1 + 1,

              b_r1*b_s2*b_t1 - 1
w_s1=-------------------------------------,
      b_r1*b_s2*b_t1 + w_r1*w_s2*w_t1 - 2

b_r19= - w_r19 + 1,

                 b_q19*b_r18*b_s19 - 1
w_r19=-----------------------------------------,
       b_q19*b_r18*b_s19 + w_q19*w_r18*w_s19 - 2

b_r18= - w_r18 + 1,

                    b_q18*b_r17*b_r19*b_s18 - 1
w_r18=-----------------------------------------------------,
       b_q18*b_r17*b_r19*b_s18 + w_q18*w_r17*w_r19*w_s18 - 2

b_r17= - w_r17 + 1,
```

The corresponding dynamical System

```
w_r17=(b_r18*b_s17*s_q17*s_r16 - 1)/(b_r18*b_s17*s_q17*s_r16
    + s_q17*s_r16*w_r18*w_s17 - s_q17*w_r18*w_s17 - s_r16*w_r18*w_s17
    + w_r18*w_s17 - 2),
s_r16= - s_q13*w_q15*w_q16*w_r17*w_s15*w_s16 + 1,
s_r13= - s_q13*w_p12*w_q11*w_r11*w_s12*w_s13 + 1,
b_r11= - w_r11 + 1,
w_r11
                          b_q11*b_r10*b_s11*s_r13 - 1
=-------------------------------------------------------------------------,
  b_q11*b_r10*b_s11*s_r13 - s_r13*w_q11*w_r10*w_s11 + w_q11*w_r10*w_s11 - 2
b_r10= - w_r10 + 1,
                   b_q10*b_r11*b_r9*b_s10 - 1
w_r10=-----------------------------------------------------,
       b_q10*b_r11*b_r9*b_s10 + w_q10*w_r11*w_r9*w_s10 - 2
b_r9= - w_r9 + 1,
                  b_q9*b_r10*b_r8*b_s9 - 1
w_r9=-------------------------------------------------,
      b_q9*b_r10*b_r8*b_s9 + w_q9*w_r10*w_r8*w_s9 - 2
```

The corresponding dynamical System

```
b_r8= - w_r8 + 1,

w_r8=(b_q8*b_r9*s_r7*s_s7 - b_q8*b_r9*s_r7 + 1)/(b_q8*b_r9*s_r7*s_s7

    - b_q8*b_r9*s_r7 + s_r7*s_s7*w_q8*w_r9 - s_s7*w_q8*w_r9 + 2),

s_r7= - s_r4*s_s7*w_p7*w_q6*w_q8*w_r8 + 1,

s_r4= - b_p5*b_q6*b_s4*b_s5*b_s6*s_q4*s_r3*s_r7 + 1,

s_r3= - s_r4*w_q3*w_r2*w_s2*w_s4*w_t3 + 1,

b_r2= - w_r2 + 1,

                          b_q2*b_r1*b_s2*s_r3 - 1
w_r2=-------------------------------------------------------------------,
      b_q2*b_r1*b_s2*s_r3 - s_r3*w_q2*w_r1*w_s2 + w_q2*w_r1*w_s2 - 2

b_r1= - w_r1 + 1,

              b_q1*b_r2*b_s1 - 1
w_r1=-------------------------------------,
      b_q1*b_r2*b_s1 + w_q1*w_r2*w_s1 - 2

b_q19= - w_q19 + 1,

                 b_p19*b_q18*b_r19 - 1
w_q19=-----------------------------------------,
       b_p19*b_q18*b_r19 + w_p19*w_q18*w_r19 - 2
```

The corresponding dynamical System

```
b_q18= - w_q18 + 1,
w_q18=(b_q19*b_r18*s_p18*s_q17 - b_q19*b_r18*s_q17 + 1)/(
   b_q19*b_r18*s_p18*s_q17 - b_q19*b_r18*s_q17 + s_p18*s_q17*w_q19*w_r18
    - s_p18*w_q19*w_r18 + 2),
s_q17= - s_p18*w_q16*w_q18*w_r17 + 1,
b_q16= - w_q16 + 1,
w_q16=(b_q15*s_p18*s_q17*s_r16 - b_q15*s_q17*s_r16 + 1)/(
   b_q15*s_p18*s_q17*s_r16 - b_q15*s_q17*s_r16 - s_p18*s_q17*s_r16*w_q15
    + s_p18*s_q17*w_q15 + s_p18*s_r16*w_q15 - s_p18*w_q15 + 2),
b_q15= - w_q15 + 1,
w_q15=(b_p15*b_q16*s_q13*s_r16 - b_p15*b_q16*s_r16 + 1)/(
   b_p15*b_q16*s_q13*s_r16 - b_p15*b_q16*s_r16 + s_q13*s_r16*w_p15*w_q16
    - s_q13*w_p15*w_q16 + 2),
s_q13= - b_q15*s_p13*s_r13*s_r16*s_s14 + 1,
b_q11= - w_q11 + 1,
```

The corresponding dynamical System

```
w_q11

                          b_p11*b_q10*b_r11*s_r13 - 1
=-------------------------------------------------------------------------,
  b_p11*b_q10*b_r11*s_r13 - s_r13*w_p11*w_q10*w_r11 + w_p11*w_q10*w_r11 - 2

b_q10= - w_q10 + 1,

                    b_p10*b_q11*b_q9*b_r10 - 1
w_q10=-----------------------------------------------------,
       b_p10*b_q11*b_q9*b_r10 + w_p10*w_q11*w_q9*w_r10 - 2

b_q9= - w_q9 + 1,

                  b_p9*b_q10*b_q8*b_r9 - 1
w_q9=-------------------------------------------------,
      b_p9*b_q10*b_q8*b_r9 + w_p9*w_q10*w_q8*w_r9 - 2

b_q8= - w_q8 + 1,

                          b_p8*b_q9*b_r8*s_r7 - 1
w_q8=-------------------------------------------------------------,
      b_p8*b_q9*b_r8*s_r7 - s_r7*w_p8*w_q9*w_r8 + w_p8*w_q9*w_r8 - 2

b_q6= - w_q6 + 1,

      s_p6*s_r4*s_r7 - s_p6*s_r7 - s_r4*s_r7 + s_r7 - 1
w_q6=-------------------------------------------------,
        s_p6*s_r4 - s_p6*s_r7 - s_r4*s_r7 + s_r7 - 2
```

The corresponding dynamical System

```
s_q4= - s_r4*w_n4*w_o3*w_o5*w_p3*w_p5*w_q3 + 1,

b_q3= - w_q3 + 1,

w_q3=(b_p3*b_q2*s_q4*s_r3 - 1)/(b_p3*b_q2*s_q4*s_r3 + s_q4*s_r3*w_p3*w_q2

    - s_q4*w_p3*w_q2 - s_r3*w_p3*w_q2 + w_p3*w_q2 - 2),

b_q2= - w_q2 + 1,

                 b_p2*b_q1*b_q3*b_r2 - 1
w_q2=-----------------------------------------------,
      b_p2*b_q1*b_q3*b_r2 + w_p2*w_q1*w_q3*w_r2 - 2

b_q1= - w_q1 + 1,

              b_p1*b_q2*b_r1 - 1
w_q1=-------------------------------------,
      b_p1*b_q2*b_r1 + w_p1*w_q2*w_r1 - 2

b_p19= - w_p19 + 1,

                  b_o19*b_q19*s_p18 - b_o19*b_q19 + 1
w_p19=-------------------------------------------------------------,
       b_o19*b_q19*s_p18 - b_o19*b_q19 - s_p18*w_o19*w_q19 + 2

s_p18= - b_o16*b_o17*b_o18*b_p15*b_p19*b_q16*b_q18*s_q17 + 1,
```

The corresponding dynamical System

```
b_p15= - w_p15 + 1,

w_p15=(b_q15*s_m15*s_p13*s_p18 - b_q15*s_m15*s_p13 - b_q15*s_p13*s_p18
        + b_q15*s_p13 - 1)/(b_q15*s_m15*s_p13*s_p18 - b_q15*s_m15*s_p13
    - b_q15*s_p13*s_p18 + b_q15*s_p13 - s_m15*s_p13*s_p18*w_q15
    + s_m15*s_p18*w_q15 - 2),

s_p13= - s_m15*s_q13*w_p12*w_p15 + 1,

b_p12= - w_p12 + 1,

w_p12=(b_o12*b_p11*s_p13*s_r13 - 1)/(b_o12*b_p11*s_p13*s_r13
    + s_p13*s_r13*w_o12*w_p11 - s_p13*w_o12*w_p11 - s_r13*w_o12*w_p11
    + w_o12*w_p11 - 2),

b_p11= - w_p11 + 1,

                    b_o11*b_p10*b_p12*b_q11 - 1
w_p11=-----------------------------------------------------,
       b_o11*b_p10*b_p12*b_q11 + w_o11*w_p10*w_p12*w_q11 - 2

b_p10= - w_p10 + 1,
```

The corresponding dynamical System

$$w\_p10=\frac{b\_o10*b\_p11*b\_p9*b\_q10 - 1}{b\_o10*b\_p11*b\_p9*b\_q10 + w\_o10*w\_p11*w\_p9*w\_q10 - 2},$$

$$b\_p9= - w\_p9 + 1,$$

$$w\_p9=\frac{b\_o9*b\_p10*b\_p8*b\_q9 - 1}{b\_o9*b\_p10*b\_p8*b\_q9 + w\_o9*w\_p10*w\_p8*w\_q9 - 2},$$

$$b\_p8= - w\_p8 + 1,$$

$$w\_p8=\frac{b\_o8*b\_p7*b\_p9*b\_q8 - 1}{b\_o8*b\_p7*b\_p9*b\_q8 + w\_o8*w\_p7*w\_p9*w\_q8 - 2},$$

$$b\_p7= - w\_p7 + 1,$$

$$w\_p7=(b\_o7*b\_p8*s\_p6*s\_r7 - b\_o7*b\_p8*s\_r7 + 1)/(b\_o7*b\_p8*s\_p6*s\_r7 - b\_o7*b\_p8*s\_r7 + s\_p6*s\_r7*w\_o7*w\_p8 - s\_p6*w\_o7*w\_p8 + 2),$$

$$s\_p6= - b\_o6*b\_p5*b\_p7*b\_q6 + 1,$$

$$b\_p5= - w\_p5 + 1,$$

The corresponding dynamical System

```
w_p5=(b_o5*s_p6*s_q4*s_r4 - b_o5*s_p6*s_q4 - b_o5*s_q4*s_r4 + b_o5*s_q4 - 1)/

(b_o5*s_p6*s_q4*s_r4 - b_o5*s_p6*s_q4 - b_o5*s_q4*s_r4 + b_o5*s_q4

  - s_p6*s_q4*s_r4*w_o5 + s_p6*s_r4*w_o5 - 2),

b_p3= - w_p3 + 1,

                          b_o3*b_p2*b_q3*s_q4 - 1
w_p3=-------------------------------------------------------------,
      b_o3*b_p2*b_q3*s_q4 - s_q4*w_o3*w_p2*w_q3 + w_o3*w_p2*w_q3 - 2

b_p2= - w_p2 + 1,

                  b_o2*b_p1*b_p3*b_q2 - 1
w_p2=-----------------------------------------------,
      b_o2*b_p1*b_p3*b_q2 + w_o2*w_p1*w_p3*w_q2 - 2

b_p1= - w_p1 + 1,

               b_o1*b_p2*b_q1 - 1
w_p1=-------------------------------------,
      b_o1*b_p2*b_q1 + w_o1*w_p2*w_q1 - 2

b_o19= - w_o19 + 1,

                  b_n19*b_o18*b_p19 - 1
w_o19=-----------------------------------------,
       b_n19*b_o18*b_p19 + w_n19*w_o18*w_p19 - 2
```

The corresponding dynamical System

```
b_o18= - w_o18 + 1,

w_o18

                b_n18*b_o17*b_o19*s_p18 - b_n18*b_o17*b_o19 + 1
=-------------------------------------------------------------------------,
  b_n18*b_o17*b_o19*s_p18 - b_n18*b_o17*b_o19 - s_p18*w_n18*w_o17*w_o19 + 2

b_o17= - w_o17 + 1,

w_o17=(b_o16*b_o18*s_l16*s_p18 - b_o16*b_o18*s_l16 + 1)/(

   b_o16*b_o18*s_l16*s_p18 - b_o16*b_o18*s_l16 + s_l16*s_p18*w_o16*w_o18

    - s_p18*w_o16*w_o18 + 2),

b_o16= - w_o16 + 1,

w_o16=(b_n16*b_o17*s_m15*s_p18 - b_n16*b_o17*s_m15 - b_n16*b_o17*s_p18

        + b_n16*b_o17 - 1)/(b_n16*b_o17*s_m15*s_p18 - b_n16*b_o17*s_m15

    - b_n16*b_o17*s_p18 + b_n16*b_o17 + s_m15*s_p18*w_n16*w_o17 - 2),

b_o12= - w_o12 + 1,
```

The corresponding dynamical System

```
w_o12

                b_n12*b_o11*b_p12*s_m15 - b_n12*b_o11*b_p12 + 1
=-------------------------------------------------------------------------,
  b_n12*b_o11*b_p12*s_m15 - b_n12*b_o11*b_p12 - s_m15*w_n12*w_o11*w_p12 + 2

b_o11= - w_o11 + 1,

                    b_n11*b_o10*b_o12*b_p11 - 1
w_o11=-----------------------------------------------------,
       b_n11*b_o10*b_o12*b_p11 + w_n11*w_o10*w_o12*w_p11 - 2

b_o10= - w_o10 + 1,

                   b_n10*b_o11*b_o9*b_p10 - 1
w_o10=-----------------------------------------------------,
       b_n10*b_o11*b_o9*b_p10 + w_n10*w_o11*w_o9*w_p10 - 2

b_o9= - w_o9 + 1,

                  b_n9*b_o10*b_o8*b_p9 - 1
w_o9=-------------------------------------------------,
      b_n9*b_o10*b_o8*b_p9 + w_n9*w_o10*w_o8*w_p9 - 2

b_o8= - w_o8 + 1,

                  b_n8*b_o7*b_o9*b_p8 - 1
w_o8=-------------------------------------------------,
      b_n8*b_o7*b_o9*b_p8 + w_n8*w_o7*w_o9*w_p8 - 2
```

The corresponding dynamical System

```
b_o7= - w_o7 + 1,

                  b_n7*b_o6*b_o8*b_p7 - 1
w_o7=-----------------------------------------------,
      b_n7*b_o6*b_o8*b_p7 + w_n7*w_o6*w_o8*w_p7 - 2

b_o6= - w_o6 + 1,

w_o6=(b_o5*b_o7*s_n6*s_p6 - b_o5*b_o7*s_n6 - b_o5*b_o7*s_p6 + b_o5*b_o7 - 1)/

(b_o5*b_o7*s_n6*s_p6 - b_o5*b_o7*s_n6 - b_o5*b_o7*s_p6 + b_o5*b_o7

  + s_n6*s_p6*w_o5*w_o7 - 2),

b_o5= - w_o5 + 1,

                         b_n5*b_o6*b_p5*s_q4 - 1
w_o5=-------------------------------------------------------------,
      b_n5*b_o6*b_p5*s_q4 - s_q4*w_n5*w_o6*w_p5 + w_n5*w_o6*w_p5 - 2

b_o3= - w_o3 + 1,

                         b_n3*b_o2*b_p3*s_q4 - 1
w_o3=-------------------------------------------------------------,
      b_n3*b_o2*b_p3*s_q4 - s_q4*w_n3*w_o2*w_p3 + w_n3*w_o2*w_p3 - 2

b_o2= - w_o2 + 1,
```

The corresponding dynamical System

$$w_{o2}=\frac{b_{n2}b_{o1}b_{o3}b_{p2}-1}{b_{n2}b_{o1}b_{o3}b_{p2}+w_{n2}w_{o1}w_{o3}w_{p2}-2}$$

$$b_{o1}=-w_{o1}+1$$

$$w_{o1}=\frac{b_{n1}b_{o2}b_{p1}-1}{b_{n1}b_{o2}b_{p1}+w_{n1}w_{o2}w_{p1}-2}$$

$$b_{n19}=-w_{n19}+1$$

$$w_{n19}=\frac{b_{m19}b_{n18}b_{o19}-1}{b_{m19}b_{n18}b_{o19}+w_{m19}w_{n18}w_{o19}-2}$$

$$b_{n18}=-w_{n18}+1$$

$$w_{n18}=\frac{b_{n19}b_{o18}s_{l16}-1}{b_{n19}b_{o18}s_{l16}-s_{l16}w_{n19}w_{o18}+w_{n19}w_{o18}-2}$$

$$b_{n16}=-w_{n16}+1$$

$$w_{n16}=\frac{b_{o16}s_{l16}s_{m15}-b_{o16}s_{l16}+1}{b_{o16}s_{l16}s_{m15}-b_{o16}s_{l16}+s_{l16}s_{m15}w_{o16}-s_{m15}w_{o16}+2}$$

The corresponding dynamical System

```
b_n13= - w_n13 + 1,

w_n13=(b_n12*s_l14*s_m14*s_m15 - b_n12*s_l14*s_m14 - b_n12*s_m14*s_m15

        + b_n12*s_m14 - 1)/(b_n12*s_l14*s_m14*s_m15 - b_n12*s_l14*s_m14

    - b_n12*s_m14*s_m15 + b_n12*s_m14 - s_l14*s_m14*s_m15*w_n12

    + s_l14*s_m15*w_n12 - 2),

b_n12= - w_n12 + 1,

                    b_m12*b_n11*b_n13*b_o12 - 1
w_n12=-----------------------------------------------------,
       b_m12*b_n11*b_n13*b_o12 + w_m12*w_n11*w_n13*w_o12 - 2

b_n11= - w_n11 + 1,

w_n11

                b_n10*b_n12*b_o11*s_m11 - b_n10*b_n12*b_o11 + 1
=-------------------------------------------------------------------------,
  b_n10*b_n12*b_o11*s_m11 - b_n10*b_n12*b_o11 - s_m11*w_n10*w_n12*w_o11 + 2

b_n10= - w_n10 + 1,

                   b_m10*b_n11*b_n9*b_o10 - 1
w_n10=-----------------------------------------------------,
       b_m10*b_n11*b_n9*b_o10 + w_m10*w_n11*w_n9*w_o10 - 2
```

The corresponding dynamical System

```
b_n9= - w_n9 + 1,

                          b_n10*b_n8*b_o9*s_m9 - 1
w_n9=-------------------------------------------------------------------,
     b_n10*b_n8*b_o9*s_m9 - s_m9*w_n10*w_n8*w_o9 + w_n10*w_n8*w_o9 - 2

b_n8= - w_n8 + 1,

                 b_m8*b_n7*b_n9*b_o8 - 1
w_n8=-----------------------------------------,
     b_m8*b_n7*b_n9*b_o8 + w_m8*w_n7*w_n9*w_o8 - 2

b_n7= - w_n7 + 1,

                 b_m7*b_n8*b_o7*s_n6 - b_m7*b_n8*b_o7 + 1
w_n7=-----------------------------------------------------------------,
     b_m7*b_n8*b_o7*s_n6 - b_m7*b_n8*b_o7 - s_n6*w_m7*w_n8*w_o7 + 2

s_n6= - b_m6*b_n5*b_n7*b_o6 + 1,

b_n5= - w_n5 + 1,

                 b_m5*b_n4*b_o5*s_n6 - b_m5*b_n4*b_o5 + 1
w_n5=-----------------------------------------------------------------,
     b_m5*b_n4*b_o5*s_n6 - b_m5*b_n4*b_o5 - s_n6*w_m5*w_n4*w_o5 + 2

b_n4= - w_n4 + 1,
```

The corresponding dynamical System

$$w_{n4}=\frac{b_{m4}b_{n3}b_{n5}s_{q4}-1}{b_{m4}b_{n3}b_{n5}s_{q4}-s_{q4}w_{m4}w_{n3}w_{n5}+w_{m4}w_{n3}w_{n5}-2}$$

$$b_{n3}=-w_{n3}+1,$$

$$w_{n3}=\frac{b_{m3}b_{n2}b_{n4}b_{o3}-1}{b_{m3}b_{n2}b_{n4}b_{o3}+w_{m3}w_{n2}w_{n4}w_{o3}-2}$$

$$b_{n2}=-w_{n2}+1,$$

$$w_{n2}=\frac{b_{m2}b_{n1}b_{n3}b_{o2}-1}{b_{m2}b_{n1}b_{n3}b_{o2}+w_{m2}w_{n1}w_{n3}w_{o2}-2}$$

$$b_{n1}=-w_{n1}+1,$$

$$w_{n1}=\frac{b_{m1}b_{n2}b_{o1}-1}{b_{m1}b_{n2}b_{o1}+w_{m1}w_{n2}w_{o1}-2}$$

$$b_{m19}=-w_{m19}+1,$$

$$w_{m19}=\frac{b_{l19}b_{n19}s_{l16}-1}{b_{l19}b_{n19}s_{l16}-s_{l16}w_{l19}w_{n19}+w_{l19}w_{n19}-2}$$

The corresponding dynamical System

```
s_m15= - b_n13*b_n16*b_o12*b_o16*b_p15*s_l16*s_m14*s_p13 + 1,

s_m14= - s_l14*s_m15*w_n13 + 1,

b_m12= - w_m12 + 1,

w_m12=(b_l12*b_n12*s_l14*s_m11 - b_l12*b_n12*s_l14 - b_l12*b_n12*s_m11
        + b_l12*b_n12 - 1)/(b_l12*b_n12*s_l14*s_m11 - b_l12*b_n12*s_l14
    - b_l12*b_n12*s_m11 + b_l12*b_n12 + s_l14*s_m11*w_l12*w_n12 - 2),

s_m11= - b_l11*b_m10*b_m12*b_n11 + 1,

b_m10= - w_m10 + 1,

w_m10=(b_l10*b_n10*s_m11*s_m9 - b_l10*b_n10*s_m9 + 1)/(b_l10*b_n10*s_m11*s_m9
    - b_l10*b_n10*s_m9 + s_m11*s_m9*w_l10*w_n10 - s_m11*w_l10*w_n10 + 2),

s_m9= - w_l9*w_m10*w_m8*w_n9 + 1,

b_m8= - w_m8 + 1,

                          b_l8*b_m7*b_n8*s_m9 - 1
w_m8=-------------------------------------------------------------,
      b_l8*b_m7*b_n8*s_m9 - s_m9*w_l8*w_m7*w_n8 + w_l8*w_m7*w_n8 - 2
```

The corresponding dynamical System

$$b\_m7 = -w\_m7 + 1,$$

$$w\_m7 = \frac{b\_l7*b\_m6*b\_m8*b\_n7 - 1}{b\_l7*b\_m6*b\_m8*b\_n7 + w\_l7*w\_m6*w\_m8*w\_n7 - 2},$$

$$b\_m6 = -w\_m6 + 1,$$

$$w\_m6 = \frac{b\_l6*b\_m5*b\_m7*s\_n6 - b\_l6*b\_m5*b\_m7 + 1}{b\_l6*b\_m5*b\_m7*s\_n6 - b\_l6*b\_m5*b\_m7 - s\_n6*w\_l6*w\_m5*w\_m7 + 2},$$

$$b\_m5 = -w\_m5 + 1,$$

$$w\_m5 = \frac{b\_l5*b\_m4*b\_m6*b\_n5 - 1}{b\_l5*b\_m4*b\_m6*b\_n5 + w\_l5*w\_m4*w\_m6*w\_n5 - 2},$$

$$b\_m4 = -w\_m4 + 1,$$

$$w\_m4 = \frac{b\_l4*b\_m3*b\_m5*b\_n4 - 1}{b\_l4*b\_m3*b\_m5*b\_n4 + w\_l4*w\_m3*w\_m5*w\_n4 - 2},$$

$$b\_m3 = -w\_m3 + 1,$$

$$w\_m3 = \frac{b\_l3*b\_m2*b\_m4*b\_n3 - 1}{b\_l3*b\_m2*b\_m4*b\_n3 + w\_l3*w\_m2*w\_m4*w\_n3 - 2},$$

The corresponding dynamical System

```
b_m2= - w_m2 + 1,

                 b_l2*b_m1*b_m3*b_n2 - 1
w_m2=-----------------------------------------------,
      b_l2*b_m1*b_m3*b_n2 + w_l2*w_m1*w_m3*w_n2 - 2

b_m1= - w_m1 + 1,

              b_l1*b_m2*b_n1 - 1
w_m1=-------------------------------------,
      b_l1*b_m2*b_n1 + w_l1*w_m2*w_n1 - 2

b_l19= - w_l19 + 1,

                  b_k19*b_m19*s_h18 - b_k19*b_m19 + 1
w_l19=-------------------------------------------------------------,
       b_k19*b_m19*s_h18 - b_k19*b_m19 - s_h18*w_k19*w_m19 + 2

s_l16= - s_h18*s_l14*s_m15*w_k15*w_k16*w_m19*w_n16*w_n18*w_o17 + 1,

s_l14= - b_h13*b_j12*b_k14*b_l12*b_m12*b_n13*s_j14*s_k12*s_l16*s_m14 + 1,

b_l12= - w_l12 + 1,

w_l12=(b_l11*b_m12*s_k12*s_l14 - b_l11*b_m12*s_k12 + 1)/(

   b_l11*b_m12*s_k12*s_l14 - b_l11*b_m12*s_k12 + s_k12*s_l14*w_l11*w_m12

    - s_l14*w_l11*w_m12 + 2),

b_l11= - w_l11 + 1,
```

The corresponding dynamical System

```
w_l11

                b_k11*b_l10*b_l12*s_m11 - b_k11*b_l10*b_l12 + 1
=-------------------------------------------------------------------------------,
  b_k11*b_l10*b_l12*s_m11 - b_k11*b_l10*b_l12 - s_m11*w_k11*w_l10*w_l12 + 2

b_l10= - w_l10 + 1,

                   b_k10*b_l11*b_l9*b_m10 - 1
w_l10=-----------------------------------------------------,
       b_k10*b_l11*b_l9*b_m10 + w_k10*w_l11*w_l9*w_m10 - 2

b_l9= - w_l9 + 1,

                           b_k9*b_l10*b_l8*s_m9 - 1
w_l9=-------------------------------------------------------------------,
      b_k9*b_l10*b_l8*s_m9 - s_m9*w_k9*w_l10*w_l8 + w_k9*w_l10*w_l8 - 2

b_l8= - w_l8 + 1,

                  b_k8*b_l7*b_l9*b_m8 - 1
w_l8=-----------------------------------------------,
      b_k8*b_l7*b_l9*b_m8 + w_k8*w_l7*w_l9*w_m8 - 2

b_l7= - w_l7 + 1,

                  b_k7*b_l6*b_l8*b_m7 - 1
w_l7=-----------------------------------------------,
      b_k7*b_l6*b_l8*b_m7 + w_k7*w_l6*w_l8*w_m7 - 2

b_l6= - w_l6 + 1,
```

The corresponding dynamical System

$$w_{l6} = \frac{b_{k6} b_{l5} b_{l7} b_{m6} - 1}{b_{k6} b_{l5} b_{l7} b_{m6} + w_{k6} w_{l5} w_{l7} w_{m6} - 2}$$

$$b_{l5} = -w_{l5} + 1,$$

$$w_{l5} = \frac{b_{k5} b_{l4} b_{l6} b_{m5} - 1}{b_{k5} b_{l4} b_{l6} b_{m5} + w_{k5} w_{l4} w_{l6} w_{m5} - 2}$$

$$b_{l4} = -w_{l4} + 1,$$

$$w_{l4} = \frac{b_{k4} b_{l3} b_{l5} b_{m4} - 1}{b_{k4} b_{l3} b_{l5} b_{m4} + w_{k4} w_{l3} w_{l5} w_{m4} - 2}$$

$$b_{l3} = -w_{l3} + 1,$$

$$w_{l3} = \frac{b_{l2} b_{l4} b_{m3} s_{k3} - 1}{b_{l2} b_{l4} b_{m3} s_{k3} - s_{k3} w_{l2} w_{l4} w_{m3} + w_{l2} w_{l4} w_{m3} - 2}$$

$$b_{l2} = -w_{l2} + 1,$$

$$w_{l2} = \frac{b_{k2} b_{l1} b_{l3} b_{m2} - 1}{b_{k2} b_{l1} b_{l3} b_{m2} + w_{k2} w_{l1} w_{l3} w_{m2} - 2}$$

$$b_{l1} = -w_{l1} + 1,$$

The corresponding dynamical System

```
              b_k1*b_l2*b_m1 - 1
w_l1=-------------------------------------,
      b_k1*b_l2*b_m1 + w_k1*w_l2*w_m1 - 2

b_k19= - w_k19 + 1,

                  b_j19*b_l19*s_h18 - b_j19*b_l19 + 1
w_k19=-------------------------------------------------------------,
       b_j19*b_l19*s_h18 - b_j19*b_l19 - s_h18*w_j19*w_l19 + 2

b_k16= - w_k16 + 1,

w_k16=(b_k15*s_h18*s_j16*s_l16 - b_k15*s_j16*s_l16 + 1)/(

   b_k15*s_h18*s_j16*s_l16 - b_k15*s_j16*s_l16 - s_h18*s_j16*s_l16*w_k15

    + s_h18*s_j16*w_k15 + s_h18*s_l16*w_k15 - s_h18*w_k15 + 2),

b_k15= - w_k15 + 1,

w_k15

                          b_j15*b_k14*b_k16*s_l16 - 1
=-------------------------------------------------------------------------,
  b_j15*b_k14*b_k16*s_l16 - s_l16*w_j15*w_k14*w_k16 + w_j15*w_k14*w_k16 - 2

b_k14= - w_k14 + 1,

                         b_k15*s_j14*s_l14 - b_k15*s_j14 + 1
w_k14=-----------------------------------------------------------------,
       b_k15*s_j14*s_l14 - b_k15*s_j14 + s_j14*s_l14*w_k15 - s_l14*w_k15 + 2
```

The corresponding dynamical System

```
s_k12= - s_l14*w_j12*w_k11*w_l12 + 1,

b_k11= - w_k11 + 1,

w_k11

                          b_j11*b_k10*b_l11*s_k12 - 1
=-------------------------------------------------------------------------,
  b_j11*b_k10*b_l11*s_k12 - s_k12*w_j11*w_k10*w_l11 + w_j11*w_k10*w_l11 - 2

b_k10= - w_k10 + 1,

                   b_j10*b_k11*b_k9*b_l10 - 1
w_k10=-----------------------------------------------------,
       b_j10*b_k11*b_k9*b_l10 + w_j10*w_k11*w_k9*w_l10 - 2

b_k9= - w_k9 + 1,

                  b_j9*b_k10*b_k8*b_l9 - 1
w_k9=-------------------------------------------------,
      b_j9*b_k10*b_k8*b_l9 + w_j9*w_k10*w_k8*w_l9 - 2

b_k8= - w_k8 + 1,

                          b_k7*b_k9*b_l8*s_j8 - 1
w_k8=-------------------------------------------------------------------,
      b_k7*b_k9*b_l8*s_j8 - s_j8*w_k7*w_k9*w_l8 + w_k7*w_k9*w_l8 - 2

b_k7= - w_k7 + 1,
```

The corresponding dynamical System

$$w_{k7}=\frac{b_{k6}b_{k8}b_{l7}s_{j7}-b_{k6}b_{k8}b_{l7}+1}{b_{k6}b_{k8}b_{l7}s_{j7}-b_{k6}b_{k8}b_{l7}-s_{j7}w_{k6}w_{k8}w_{l7}+2}$$

$$b_{k6}=-w_{k6}+1,$$

$$w_{k6}=\frac{b_{j6}b_{k5}b_{k7}b_{l6}-1}{b_{j6}b_{k5}b_{k7}b_{l6}+w_{j6}w_{k5}w_{k7}w_{l6}-2}$$

$$b_{k5}=-w_{k5}+1,$$

$$w_{k5}=\frac{b_{j5}b_{k4}b_{k6}b_{l5}-1}{b_{j5}b_{k4}b_{k6}b_{l5}+w_{j5}w_{k4}w_{k6}w_{l5}-2}$$

$$b_{k4}=-w_{k4}+1,$$

$$w_{k4}=\frac{b_{j4}b_{k5}b_{l4}s_{k3}-1}{b_{j4}b_{k5}b_{l4}s_{k3}-s_{k3}w_{j4}w_{k5}w_{l4}+w_{j4}w_{k5}w_{l4}-2}$$

$$s_{k3}=-w_{j3}w_{k2}w_{k4}w_{l3}+1,$$

$$b_{k2}=-w_{k2}+1,$$

$$w_{k2}=\frac{b_{j2}b_{k1}b_{l2}s_{k3}-1}{b_{j2}b_{k1}b_{l2}s_{k3}-s_{k3}w_{j2}w_{k1}w_{l2}+w_{j2}w_{k1}w_{l2}-2}$$

$$b_{k1}=-w_{k1}+1,$$

The corresponding dynamical System

```
                b_j1*b_k2*b_l1 - 1
w_k1=-------------------------------------,
       b_j1*b_k2*b_l1 + w_j1*w_k2*w_l1 - 2

b_j19= - w_j19 + 1,

                   b_h19*b_k19*s_h18 - b_h19*b_k19 + 1
w_j19=-------------------------------------------------------------,
        b_h19*b_k19*s_h18 - b_h19*b_k19 - s_h18*w_h19*w_k19 + 2

s_j16= - s_h18*w_h16*w_h17*w_j15*w_k16 + 1,

b_j15= - w_j15 + 1,

w_j15=(b_k15*s_h15*s_j14*s_j16 - 1)/(b_k15*s_h15*s_j14*s_j16

    - s_h15*s_j14*s_j16*w_k15 + s_h15*s_j14*w_k15 + s_h15*s_j16*w_k15

    - s_h15*w_k15 + s_j14*s_j16*w_k15 - s_j14*w_k15 - s_j16*w_k15 + w_k15 - 2

   ),

s_j14= - s_l14*w_h14*w_j15*w_k14 + 1,

b_j12= - w_j12 + 1,

w_j12=(b_j11*s_h12*s_k12*s_l14 - b_j11*s_h12*s_k12 + 1)/(

   b_j11*s_h12*s_k12*s_l14 - b_j11*s_h12*s_k12 - s_h12*s_k12*s_l14*w_j11

    + s_h12*s_l14*w_j11 + s_k12*s_l14*w_j11 - s_l14*w_j11 + 2),
```

The corresponding dynamical System

```
b_j11= - w_j11 + 1,

w_j11

                          b_j10*b_j12*b_k11*s_h12 - 1
=-------------------------------------------------------------------------,
  b_j10*b_j12*b_k11*s_h12 - s_h12*w_j10*w_j12*w_k11 + w_j10*w_j12*w_k11 - 2

b_j10= - w_j10 + 1,

w_j10

                         b_j11*b_j9*b_k10*s_h12 - 1
=-----------------------------------------------------------------------,
  b_j11*b_j9*b_k10*s_h12 - s_h12*w_j11*w_j9*w_k10 + w_j11*w_j9*w_k10 - 2

b_j9= - w_j9 + 1,

                           b_h9*b_j10*b_k9*s_j8 - 1
w_j9=-----------------------------------------------------------------,
      b_h9*b_j10*b_k9*s_j8 - s_j8*w_h9*w_j10*w_k9 + w_h9*w_j10*w_k9 - 2

s_j8= - s_j7*w_g8*w_h7*w_h9*w_j9*w_k8 + 1,

s_j7= - b_h7*b_j6*b_k7*s_j8 + 1,

b_j6= - w_j6 + 1,
```

The corresponding dynamical System

```
w_j6=(b_j5*b_k6*s_h5*s_j7 - b_j5*b_k6*s_h5 - b_j5*b_k6*s_j7 + b_j5*b_k6 - 1)/

(b_j5*b_k6*s_h5*s_j7 - b_j5*b_k6*s_h5 - b_j5*b_k6*s_j7 + b_j5*b_k6

  + s_h5*s_j7*w_j5*w_k6 - 2),

b_j5= - w_j5 + 1,

                 b_j4*b_j6*b_k5*s_h5 - b_j4*b_j6*b_k5 + 1
w_j5=-------------------------------------------------------------,
      b_j4*b_j6*b_k5*s_h5 - b_j4*b_j6*b_k5 - s_h5*w_j4*w_j6*w_k5 + 2

b_j4= - w_j4 + 1,

                 b_h4*b_j3*b_j5*b_k4 - 1
w_j4=-----------------------------------------------,
      b_h4*b_j3*b_j5*b_k4 + w_h4*w_j3*w_j5*w_k4 - 2

b_j3= - w_j3 + 1,

w_j3=(b_j2*b_j4*s_h3*s_k3 - b_j2*b_j4*s_k3 + 1)/(b_j2*b_j4*s_h3*s_k3

    - b_j2*b_j4*s_k3 + s_h3*s_k3*w_j2*w_j4 - s_h3*w_j2*w_j4 + 2),

b_j2= - w_j2 + 1,

                 b_h2*b_j1*b_j3*b_k2 - 1
w_j2=-----------------------------------------------,
      b_h2*b_j1*b_j3*b_k2 + w_h2*w_j1*w_j3*w_k2 - 2
```

The corresponding dynamical System

```
b_j1= - w_j1 + 1,

              b_h1*b_j2*b_k1 - 1
w_j1=-------------------------------------,
      b_h1*b_j2*b_k1 + w_h1*w_j2*w_k1 - 2

b_h19= - w_h19 + 1,

                 b_g19*b_j19*s_h18 - b_g19*b_j19 + 1
w_h19=-------------------------------------------------------------,
       b_g19*b_j19*s_h18 - b_g19*b_j19 - s_h18*w_g19*w_j19 + 2

s_h18= - b_g18*b_h17*b_h19*b_j19*b_k16*b_k19*b_l19*s_j16*s_l16 + 1,

b_h17= - w_h17 + 1,

w_h17=(b_h16*s_g17*s_h18*s_j16 - b_h16*s_g17*s_j16 - b_h16*s_h18*s_j16

        + b_h16*s_j16 - 1)/(b_h16*s_g17*s_h18*s_j16 - b_h16*s_g17*s_j16

    - b_h16*s_h18*s_j16 + b_h16*s_j16 - s_g17*s_h18*s_j16*w_h16

    + s_g17*s_h18*w_h16 - 2),

b_h16= - w_h16 + 1,

w_h16=(b_h17*s_g17*s_h15*s_j16 - b_h17*s_h15*s_j16 + 1)/(

   b_h17*s_g17*s_h15*s_j16 - b_h17*s_h15*s_j16 - s_g17*s_h15*s_j16*w_h17

    + s_g17*s_h15*w_h17 + s_g17*s_j16*w_h17 - s_g17*w_h17 + 2),
```

The corresponding dynamical System

```
s_h15= - s_g17*w_h14*w_h16*w_j15 + 1,

b_h14= - w_h14 + 1,

w_h14=(b_h13*s_h12*s_h15*s_j14 - 1)/(b_h13*s_h12*s_h15*s_j14

    - s_h12*s_h15*s_j14*w_h13 + s_h12*s_h15*w_h13 + s_h12*s_j14*w_h13

    - s_h12*w_h13 + s_h15*s_j14*w_h13 - s_h15*w_h13 - s_j14*w_h13 + w_h13 - 2

   ),

b_h13= - w_h13 + 1,

                        b_h14*s_h12*s_l14 - b_h14*s_h12 + 1
w_h13=-----------------------------------------------------------------------
       b_h14*s_h12*s_l14 - b_h14*s_h12 + s_h12*s_l14*w_h14 - s_l14*w_h14 + 2

,

s_h12= - s_f10*s_g17*s_g9*w_e14*w_f13*w_h13*w_h14*w_h9*w_j10*w_j11*w_j12 + 1,

b_h9= - w_h9 + 1,

w_h9=(b_j9*s_g9*s_h12*s_j8 - b_j9*s_h12*s_j8 + 1)/(b_j9*s_g9*s_h12*s_j8

    - b_j9*s_h12*s_j8 - s_g9*s_h12*s_j8*w_j9 + s_g9*s_h12*w_j9

    + s_g9*s_j8*w_j9 - s_g9*w_j9 + 2),
```

The corresponding dynamical System

```
b_h7= - w_h7 + 1,
w_h7=(s_g7*s_h5*s_j7*s_j8 - s_g7*s_h5*s_j8 - s_g7*s_j7*s_j8 + s_g7*s_j8 - 1)/
(2*s_g7*s_h5*s_j7*s_j8 - s_g7*s_h5*s_j7 - s_g7*s_h5*s_j8 - s_g7*s_j7*s_j8
  + s_g7*s_j8 - s_h5*s_j7*s_j8 + s_h5*s_j7 - 2),
s_h5= - b_g6*b_h4*b_h7*b_j5*b_j6*s_f5 + 1,
b_h4= - w_h4 + 1,
w_h4=(b_g4*b_j4*s_h3*s_h5 - b_g4*b_j4*s_h3 - b_g4*b_j4*s_h5 + b_g4*b_j4 - 1)/
(b_g4*b_j4*s_h3*s_h5 - b_g4*b_j4*s_h3 - b_g4*b_j4*s_h5 + b_g4*b_j4
  + s_h3*s_h5*w_g4*w_j4 - 2),
s_h3= - b_h2*b_h4*b_j3*s_g3 + 1,
b_h2= - w_h2 + 1,
w_h2=(b_h1*b_j2*s_g3*s_h3 - b_h1*b_j2*s_g3 + 1)/(b_h1*b_j2*s_g3*s_h3
    - b_h1*b_j2*s_g3 + s_g3*s_h3*w_h1*w_j2 - s_h3*w_h1*w_j2 + 2),
b_h1= - w_h1 + 1,
```

The corresponding dynamical System

```
              b_g1*b_h2*b_j1 - 1
w_h1=-----------------------------------,
      b_g1*b_h2*b_j1 + w_g1*w_h2*w_j1 - 2

b_g19= - w_g19 + 1,

                  b_f19*b_g18*b_h19 - 1
w_g19=-----------------------------------------,
       b_f19*b_g18*b_h19 + w_f19*w_g18*w_h19 - 2

b_g18= - w_g18 + 1,

w_g18=(b_g19*s_d17*s_g17*s_h18 - b_g19*s_d17*s_g17 - b_g19*s_d17*s_h18

        + b_g19*s_d17 - b_g19*s_g17*s_h18 + b_g19*s_g17 + b_g19*s_h18 - b_g19

        + 1)/(b_g19*s_d17*s_g17*s_h18 - b_g19*s_d17*s_g17 - b_g19*s_d17*s_h18

               + b_g19*s_d17 - b_g19*s_g17*s_h18 + b_g19*s_g17 + b_g19*s_h18

               - b_g19 - s_d17*s_g17*s_h18*w_g19 + 2),

s_g17= - b_e14*b_f13*b_g18*b_h16*b_h17*s_b13*s_e16*s_h12*s_h15 + 1,

s_g9= - b_f9*b_g8*b_h9*s_h12 + 1,

b_g8= - w_g8 + 1,
```

The corresponding dynamical System

```
w_g8=(b_f8*s_g7*s_g9*s_j8 - b_f8*s_g7*s_j8 + 1)/(b_f8*s_g7*s_g9*s_j8
    - b_f8*s_g7*s_j8 - s_g7*s_g9*s_j8*w_f8 + s_g7*s_g9*w_f8 + s_g9*s_j8*w_f8
    - s_g9*w_f8 + 2),
s_g7= - s_e7*w_f6*w_f8*w_g6*w_g8*w_h7 + 1,
b_g6= - w_g6 + 1,
w_g6=(b_f6*s_f5*s_g7*s_h5 - b_f6*s_f5*s_g7 + 1)/(b_f6*s_f5*s_g7*s_h5
    - b_f6*s_f5*s_g7 - s_f5*s_g7*s_h5*w_f6 + s_f5*s_h5*w_f6 + s_g7*s_h5*w_f6
    - s_h5*w_f6 + 2),
b_g4= - w_g4 + 1,
w_g4=(b_f4*b_h4*s_f5*s_g3 - 1)/(b_f4*b_h4*s_f5*s_g3 + s_f5*s_g3*w_f4*w_h4
    - s_f5*w_f4*w_h4 - s_g3*w_f4*w_h4 + w_f4*w_h4 - 2),
s_g3= - s_h3*w_f2*w_f3*w_g1*w_g4*w_h2 + 1,
b_g1= - w_g1 + 1,
                    b_f1*b_h1*s_g3 - 1
w_g1=-----------------------------------------------,
     b_f1*b_h1*s_g3 - s_g3*w_f1*w_h1 + w_f1*w_h1 - 2
```

The corresponding dynamical System

```
b_f19= - w_f19 + 1,

                b_g19*s_d17 - b_g19 + 1
w_f19=----------------------------------------,
       b_g19*s_d17 - b_g19 - s_d17*w_g19 + 2

b_f13= - w_f13 + 1,

              s_g17*s_h12 - s_h12 + 1
w_f13=-----------------------------------,
       2*s_g17*s_h12 - s_g17 - s_h12 + 2

s_f10= - b_a11*b_b10*b_c9*b_d9*b_e9*b_f9*s_b13*s_h12 + 1,

b_f9= - w_f9 + 1,

w_f9=(b_e9*b_f8*s_f10*s_g9 - b_e9*b_f8*s_f10 - b_e9*b_f8*s_g9 + b_e9*b_f8 - 1

      )/(b_e9*b_f8*s_f10*s_g9 - b_e9*b_f8*s_f10 - b_e9*b_f8*s_g9 + b_e9*b_f8

          + s_f10*s_g9*w_e9*w_f8 - 2),

b_f8= - w_f8 + 1,

                          b_e8*b_f9*b_g8*s_g7 - 1
w_f8=-------------------------------------------------------------------,
      b_e8*b_f9*b_g8*s_g7 - s_g7*w_e8*w_f9*w_g8 + w_e8*w_f9*w_g8 - 2

b_f6= - w_f6 + 1,
```

The corresponding dynamical System

```
w_f6=(b_e6*b_g6*s_f5*s_g7 - 1)/(b_e6*b_g6*s_f5*s_g7 + s_f5*s_g7*w_e6*w_g6
    - s_f5*w_e6*w_g6 - s_g7*w_e6*w_g6 + w_e6*w_g6 - 2),
s_f5= - s_e4*s_h5*w_f4*w_f6*w_g4*w_g6 + 1,
b_f4= - w_f4 + 1,
w_f4=(b_f3*b_g4*s_e4*s_f5 - b_f3*b_g4*s_f5 + 1)/(b_f3*b_g4*s_e4*s_f5
    - b_f3*b_g4*s_f5 + s_e4*s_f5*w_f3*w_g4 - s_e4*w_f3*w_g4 + 2),
b_f3= - w_f3 + 1,
                          b_e3*b_f2*b_f4*s_g3 - 1
w_f3=-------------------------------------------------------------,
      b_e3*b_f2*b_f4*s_g3 - s_g3*w_e3*w_f2*w_f4 + w_e3*w_f2*w_f4 - 2
b_f2= - w_f2 + 1,
w_f2=(b_f1*b_f3*s_e2*s_g3 - 1)/(b_f1*b_f3*s_e2*s_g3 + s_e2*s_g3*w_f1*w_f3
    - s_e2*w_f1*w_f3 - s_g3*w_f1*w_f3 + w_f1*w_f3 - 2),
b_f1= - w_f1 + 1,
               b_e1*b_f2*b_g1 - 1
w_f1=-----------------------------------,
      b_e1*b_f2*b_g1 + w_e1*w_f2*w_g1 - 2
```

The corresponding dynamical System

```
s_e16= - s_b14*s_d17*s_g17*w_a16*w_b17*w_c18 + 1,

b_e14= - w_e14 + 1,

            s_g17*s_h12 - s_h12 + 1
w_e14=-----------------------------------,
       2*s_g17*s_h12 - s_g17 - s_h12 + 2

b_e9= - w_e9 + 1,

                 b_d9*b_e8*b_f9*s_f10 - b_d9*b_e8*b_f9 + 1
w_e9=-------------------------------------------------------------------,
      b_d9*b_e8*b_f9*s_f10 - b_d9*b_e8*b_f9 - s_f10*w_d9*w_e8*w_f9 + 2

b_e8= - w_e8 + 1,

                 b_d8*b_e9*b_f8*s_e7 - b_d8*b_e9*b_f8 + 1
w_e8=-------------------------------------------------------------------,
      b_d8*b_e9*b_f8*s_e7 - b_d8*b_e9*b_f8 - s_e7*w_d8*w_e9*w_f8 + 2

s_e7= - b_d6*b_d8*b_e6*b_e8*s_c7*s_g7 + 1,

b_e6= - w_e6 + 1,

w_e6=(b_d6*b_f6*s_e4*s_e7 - b_d6*b_f6*s_e4 - b_d6*b_f6*s_e7 + b_d6*b_f6 - 1)/

(b_d6*b_f6*s_e4*s_e7 - b_d6*b_f6*s_e4 - b_d6*b_f6*s_e7 + b_d6*b_f6

  + s_e4*s_e7*w_d6*w_f6 - 2),
```

The corresponding dynamical System

```
s_e4= - b_d4*b_d6*b_e3*b_e6*b_f4*s_c5*s_f5 + 1,

b_e3= - w_e3 + 1,

w_e3=(b_f3*s_d3*s_e2*s_e4 - b_f3*s_d3*s_e2 - b_f3*s_e2*s_e4 + b_f3*s_e2 - 1)/

(b_f3*s_d3*s_e2*s_e4 - b_f3*s_d3*s_e2 - b_f3*s_e2*s_e4 + b_f3*s_e2

  - s_d3*s_e2*s_e4*w_f3 + s_d3*s_e4*w_f3 - 2),

s_e2= - s_d3*w_e1*w_e3*w_f2 + 1,

b_e1= - w_e1 + 1,

                              b_f1*s_d1*s_e2 - 1
w_e1=-------------------------------------------------------------------,
      b_f1*s_d1*s_e2 + s_d1*s_e2*w_f1 - s_d1*w_f1 - s_e2*w_f1 + w_f1 - 2

b_d19= - w_d19 + 1,

               s_c19*s_d17*s_d18 - s_c19*s_d18 + 1
w_d19=-----------------------------------------------------,
       s_c19*s_d17 - s_c19*s_d18 + s_d17*s_d18 - s_d17 + 2

s_d18= - s_d17*w_c18*w_d19 + 1,

s_d17= - b_d19*b_f19*b_g18*s_d18*s_e16 + 1,

b_d9= - w_d9 + 1,
```

The corresponding dynamical System

$$w_{d9}=\frac{b_{c9}b_{d8}b_{e9}s_{f10} - b_{c9}b_{d8}b_{e9} + 1}{b_{c9}b_{d8}b_{e9}s_{f10} - b_{c9}b_{d8}b_{e9} - s_{f10}w_{c9}w_{d8}w_{e9} + 2},$$

$$b_{d8}= - w_{d8} + 1,$$

$$w_{d8}=\frac{b_{c8}b_{d9}b_{e8}s_{e7} - b_{c8}b_{d9}b_{e8} + 1}{b_{c8}b_{d9}b_{e8}s_{e7} - b_{c8}b_{d9}b_{e8} - s_{e7}w_{c8}w_{d9}w_{e8} + 2},$$

$$b_{d6}= - w_{d6} + 1,$$

$$w_{d6}=(b_{c6}b_{e6}s_{e4}s_{e7} - b_{c6}b_{e6}s_{e4} - b_{c6}b_{e6}s_{e7} + b_{c6}b_{e6} - 1)/(b_{c6}b_{e6}s_{e4}s_{e7} - b_{c6}b_{e6}s_{e4} - b_{c6}b_{e6}s_{e7} + b_{c6}b_{e6} + s_{e4}s_{e7}w_{c6}w_{e6} - 2),$$

$$b_{d4}= - w_{d4} + 1,$$

$$w_{d4}=\frac{- s_{b4}s_{d3}s_{e4} + s_{b4}s_{d3} + s_{b4}s_{e4} - s_{b4} + s_{d3}s_{e4} - s_{d3} - s_{e4}}{s_{b4}s_{d3} + s_{b4}s_{e4} - s_{b4} + s_{d3}s_{e4} - s_{d3} - s_{e4} - 1},$$

$$s_{d3}= - b_{c3}b_{d4}b_{e3}s_{c2}s_{d1}s_{e2} + 1,$$

The corresponding dynamical System

```
s_d1= - s_d3*w_c1*w_e1 + 1,

s_c19= - w_b19*w_c18*w_d19 + 1,

b_c18= - w_c18 + 1,

w_c18=(s_b18*s_c19*s_d18*s_e16 - 1)/(2*s_b18*s_c19*s_d18*s_e16

    - s_b18*s_c19*s_d18 - s_b18*s_c19*s_e16 + s_b18*s_c19 - s_b18*s_d18*s_e16

    + s_b18*s_d18 + s_b18*s_e16 - s_b18 - s_c19*s_d18*s_e16 + s_c19*s_d18

    + s_c19*s_e16 - s_c19 + s_d18*s_e16 - s_d18 - s_e16 - 1),

b_c9= - w_c9 + 1,

                 b_b9*b_c8*b_d9*s_f10 - b_b9*b_c8*b_d9 + 1
w_c9=-------------------------------------------------------------------,
      b_b9*b_c8*b_d9*s_f10 - b_b9*b_c8*b_d9 - s_f10*w_b9*w_c8*w_d9 + 2

b_c8= - w_c8 + 1,

                         b_b8*b_c9*b_d8*s_c7 - 1
w_c8=-----------------------------------------------------------------,
      b_b8*b_c9*b_d8*s_c7 - s_c7*w_b8*w_c9*w_d8 + w_b8*w_c9*w_d8 - 2

s_c7= - s_e7*w_b7*w_c6*w_c8 + 1,

b_c6= - w_c6 + 1,
```

The corresponding dynamical System

```
w_c6=(b_b6*b_d6*s_c5*s_c7 - 1)/(b_b6*b_d6*s_c5*s_c7 + s_c5*s_c7*w_b6*w_d6
    - s_c5*w_b6*w_d6 - s_c7*w_b6*w_d6 + w_b6*w_d6 - 2),
s_c5= - s_b4*s_e4*w_b5*w_c6 + 1,
b_c3= - w_c3 + 1,
w_c3=(s_b3*s_b4*s_c2*s_d3 - s_b3*s_b4*s_c2 - s_b3*s_c2*s_d3 + s_b3*s_c2 - 1)/
(2*s_b3*s_b4*s_c2*s_d3 - s_b3*s_b4*s_c2 - s_b3*s_b4*s_d3 - s_b3*s_c2*s_d3
  + s_b3*s_c2 - s_b4*s_c2*s_d3 + s_b4*s_d3 - 2),
s_c2= - s_d3*w_b2*w_c1*w_c3 + 1,
b_c1= - w_c1 + 1,
                              b_b1*s_c2*s_d1 - 1
w_c1=-------------------------------------------------------------------,
      b_b1*s_c2*s_d1 + s_c2*s_d1*w_b1 - s_c2*w_b1 - s_d1*w_b1 + w_b1 - 2
b_b19= - w_b19 + 1,
w_b19=(b_a19*s_b18*s_c19 - 1)/(b_a19*s_b18*s_c19 + s_b18*s_c19*w_a19
    - s_b18*w_a19 - s_c19*w_a19 + w_a19 - 2),
s_b18= - w_a18*w_b17*w_b19*w_c18 + 1,
```

The corresponding dynamical System

```
b_b17= - w_b17 + 1,

w_b17=(b_a17*s_b18*s_e16 - 1)/(b_a17*s_b18*s_e16 + s_b18*s_e16*w_a17

    - s_b18*w_a17 - s_e16*w_a17 + w_a17 - 2),

s_b14= - b_a14*s_a15*s_b13*s_e16 + 1,

s_b13= - s_b14*s_f10*s_g17*w_a12*w_a13 + 1,

b_b10= - w_b10 + 1,

                b_a10*b_b9*s_f10 - b_a10*b_b9 + 1
w_b10=-----------------------------------------------------,
       b_a10*b_b9*s_f10 - b_a10*b_b9 - s_f10*w_a10*w_b9 + 2

b_b9= - w_b9 + 1,

                 b_a9*b_b10*b_b8*b_c9 - 1
w_b9=-------------------------------------------------,
      b_a9*b_b10*b_b8*b_c9 + w_a9*w_b10*w_b8*w_c9 - 2

b_b8= - w_b8 + 1,

                 b_a8*b_b7*b_b9*b_c8 - 1
w_b8=-------------------------------------------------,
      b_a8*b_b7*b_b9*b_c8 + w_a8*w_b7*w_b9*w_c8 - 2

b_b7= - w_b7 + 1,
```

The corresponding dynamical System

```
                          b_a7*b_b6*b_b8*s_c7 - 1
w_b7=-------------------------------------------------------------------,
      b_a7*b_b6*b_b8*s_c7 - s_c7*w_a7*w_b6*w_b8 + w_a7*w_b6*w_b8 - 2

b_b6= - w_b6 + 1,

                   b_a6*b_b5*b_b7*b_c6 - 1
w_b6=-----------------------------------------------,
      b_a6*b_b5*b_b7*b_c6 + w_a6*w_b5*w_b7*w_c6 - 2

b_b5= - w_b5 + 1,

w_b5=(b_a5*b_b6*s_b4*s_c5 - b_a5*b_b6*s_c5 + 1)/(b_a5*b_b6*s_b4*s_c5

    - b_a5*b_b6*s_c5 + s_b4*s_c5*w_a5*w_b6 - s_b4*w_a5*w_b6 + 2),

s_b4= - b_a4*b_b5*b_c3*b_d4*s_b3*s_c5 + 1,

s_b3= - s_b4*w_a3*w_b2*w_c3 + 1,

b_b2= - w_b2 + 1,

w_b2=(b_a2*b_b1*s_b3*s_c2 - 1)/(b_a2*b_b1*s_b3*s_c2 + s_b3*s_c2*w_a2*w_b1

    - s_b3*w_a2*w_b1 - s_c2*w_a2*w_b1 + w_a2*w_b1 - 2),

b_b1= - w_b1 + 1,

                b_a1*b_b2*b_c1 - 1
w_b1=-------------------------------------,
      b_a1*b_b2*b_c1 + w_a1*w_b2*w_c1 - 2
```

The corresponding dynamical System

```
b_a19= - w_a19 + 1,

              b_a18*b_b19 - 1
w_a19=------------------------------,
       b_a18*b_b19 + w_a18*w_b19 - 2

b_a18= - w_a18 + 1,

                        b_a17*b_a19*s_b18 - 1
w_a18=-------------------------------------------------------,
       b_a17*b_a19*s_b18 - s_b18*w_a17*w_a19 + w_a17*w_a19 - 2

b_a17= - w_a17 + 1,

                  b_a16*b_a18*b_b17 - 1
w_a17=-----------------------------------------,
       b_a16*b_a18*b_b17 + w_a16*w_a18*w_b17 - 2

b_a16= - w_a16 + 1,

w_a16=(b_a17*s_a15*s_e16 - 1)/(b_a17*s_a15*s_e16 + s_a15*s_e16*w_a17

    - s_a15*w_a17 - s_e16*w_a17 + w_a17 - 2),

s_a15= - s_b14*w_a14*w_a16 + 1,

b_a14= - w_a14 + 1,

                        b_a13*s_a15*s_b14 - b_a13*s_a15 + 1
w_a14=-------------------------------------------------------------------,
       b_a13*s_a15*s_b14 - b_a13*s_a15 + s_a15*s_b14*w_a13 - s_b14*w_a13 + 2
```

The corresponding dynamical System

$$b_{a13} = -w_{a13} + 1,$$

$$w_{a13} = \frac{b_{a12} b_{a14} s_{b13} - 1}{b_{a12} b_{a14} s_{b13} - s_{b13} w_{a12} w_{a14} + w_{a12} w_{a14} - 2}$$

$$b_{a12} = -w_{a12} + 1,$$

$$w_{a12} = \frac{b_{a11} b_{a13} s_{b13} - 1}{b_{a11} b_{a13} s_{b13} - s_{b13} w_{a11} w_{a13} + w_{a11} w_{a13} - 2}$$

$$b_{a11} = -w_{a11} + 1,$$

$$w_{a11} = \frac{b_{a10} b_{a12} s_{f10} - b_{a10} b_{a12} + 1}{b_{a10} b_{a12} s_{f10} - b_{a10} b_{a12} - s_{f10} w_{a10} w_{a12} + 2}$$

$$b_{a10} = -w_{a10} + 1,$$

$$w_{a10} = \frac{b_{a11} b_{a9} b_{b10} - 1}{b_{a11} b_{a9} b_{b10} + w_{a11} w_{a9} w_{b10} - 2}$$

$$b_{a9} = -w_{a9} + 1,$$

$$w_{a9} = \frac{b_{a10} b_{a8} b_{b9} - 1}{b_{a10} b_{a8} b_{b9} + w_{a10} w_{a8} w_{b9} - 2}$$

The corresponding dynamical System

$$
\begin{aligned}
b\_a8 &= -w\_a8 + 1,\\
w\_a8 &= \frac{b\_a7*b\_a9*b\_b8 - 1}{b\_a7*b\_a9*b\_b8 + w\_a7*w\_a9*w\_b8 - 2},\\
b\_a7 &= -w\_a7 + 1,\\
w\_a7 &= \frac{b\_a6*b\_a8*b\_b7 - 1}{b\_a6*b\_a8*b\_b7 + w\_a6*w\_a8*w\_b7 - 2},\\
b\_a6 &= -w\_a6 + 1,\\
w\_a6 &= \frac{b\_a5*b\_a7*b\_b6 - 1}{b\_a5*b\_a7*b\_b6 + w\_a5*w\_a7*w\_b6 - 2},\\
b\_a5 &= -w\_a5 + 1,\\
w\_a5 &= \frac{b\_a4*b\_a6*b\_b5 - 1}{b\_a4*b\_a6*b\_b5 + w\_a4*w\_a6*w\_b5 - 2},\\
b\_a4 &= -w\_a4 + 1,\\
w\_a4 &= \frac{b\_a3*b\_a5*s\_b4 - b\_a3*b\_a5 + 1}{b\_a3*b\_a5*s\_b4 - b\_a3*b\_a5 - s\_b4*w\_a3*w\_a5 + 2},
\end{aligned}
$$

The corresponding dynamical System

$$b\_a3 = -w\_a3 + 1,$$

$$w\_a3 = \frac{b\_a2*b\_a4*s\_b3 - 1}{b\_a2*b\_a4*s\_b3 - s\_b3*w\_a2*w\_a4 + w\_a2*w\_a4 - 2},$$

$$b\_a2 = -w\_a2 + 1,$$

$$w\_a2 = \frac{b\_a1*b\_a3*b\_b2 - 1}{b\_a1*b\_a3*b\_b2 + w\_a1*w\_a3*w\_b2 - 2},$$

$$b\_a1 = -w\_a1 + 1,$$

$$w\_a1 = \frac{b\_a2*b\_b1 - 1}{b\_a2*b\_b1 + w\_a2*w\_b1 - 2}$$

Outline

Statistics on Professional Games

A batch mode testing environment has been written to compare the 10.4 million moves of 50,000 professional GoGoD games each with best moves as proposed by the influence module (IM). This produces the following statistics:

- for each move number a diagram showing the rank distribution of the professional move according to the influence module
- a listing of board positions where the professional move ranked lowest (most urgent improvements for IM)
- time measurements

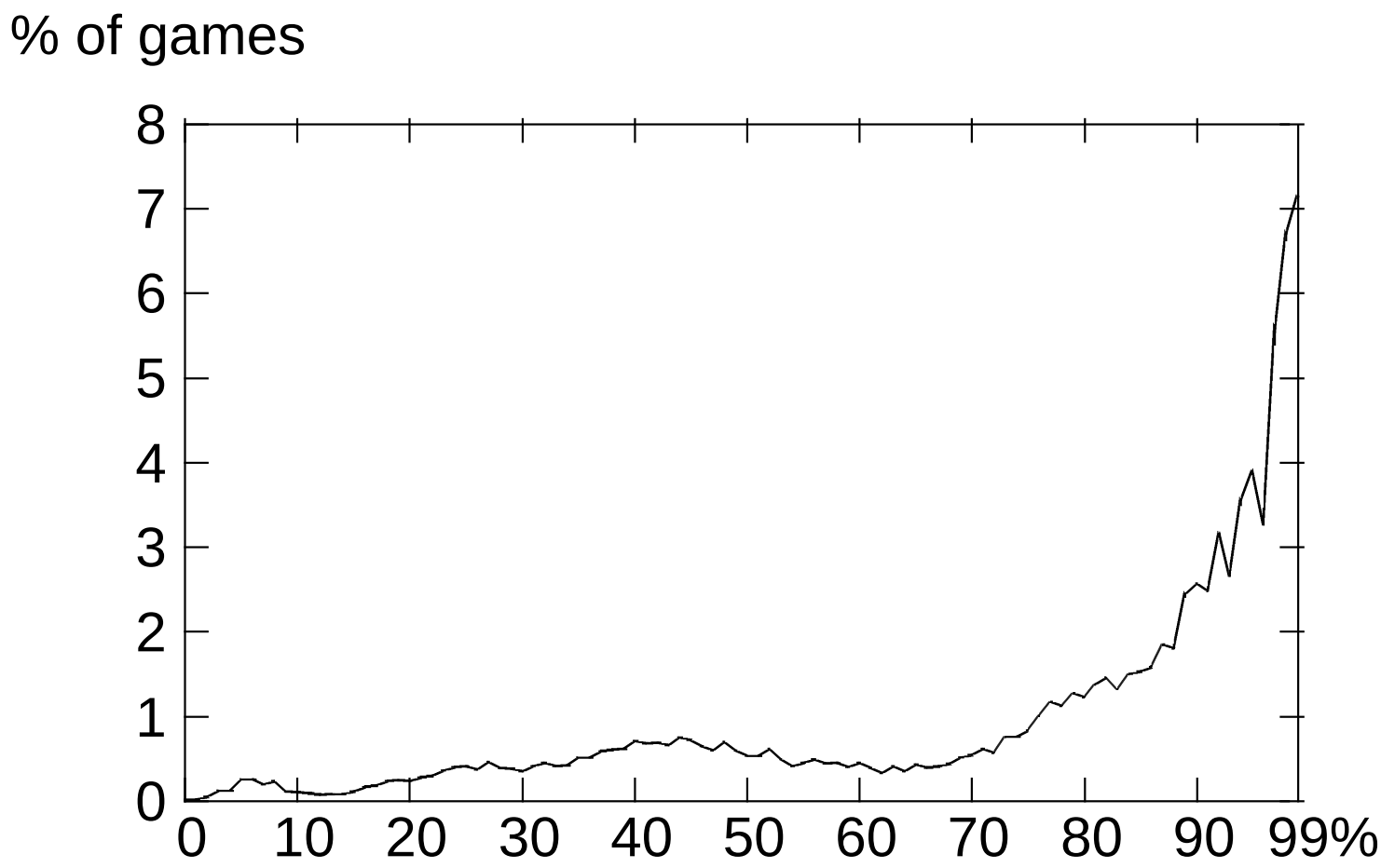


Figure: A statistics of the ranking of the next professional move after move 100 in 50000 professional games

Playing Games

To help improving the influence field to become useful as an evaluation function in a game playing program the following interfaces have been written:

- a runtime library libgotoolse.so for linux (and DLL for Windows if needed) including a program in Pascal and in C (by Sam Owre) to test the runtime library,
- a possibility to run autoplay games on Fuego between the influence module move selection and move selection by other Fuego players (by SO),
- a possibility to run Fuego with IM move selection on CGOS (Computer GO Server) against other programs (by SO).

Outline

Lessons Learned

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- How quickly is the system converging?
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- How many (real) solutions with values in the interval $0 \ldots 1$ does the dynamical system have?
  $\rightarrow$ in general only one with the only exception of two opposite colour chains attached to each other, both with only one liberty which is a very unstable situation in Go

More Learned

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  $\rightarrow$ both computations must be combined in one
- Should the influence/evaluation function depend on and make use of who moves next?
  $\rightarrow$ not in its first approximation, only in higher order approximations when interlinking it with treesearch for its improvement (see next project).

Outline

How Computer Algebra was used

For smaller positions the polynomial system was formulated and solved exactly using CA and it was confirmed that there was only one real solution with values in the interval 0..1. Also it was checked that the numerical solution coincided with the symbolic solution.

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CA was also used to formulate and print the system shown in the full board example.

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CA was also used to formulate and print the system shown in the full board example.
The numerical model is hybrid discrete-polynomial by using functions like `maximum` (e.g. to account for the possibility of choosing the order of moves when catching a chain and doing the least probably move last...).

Outline

Acknowledgment

This work was sponsored by a DARPA seedlings grant.

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The End