

The Mathematical Association of America

# Placement Test Program User's Guide



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# College Placement Testing in Mathematics

Educational accomplishments in mathematics often exert a strong influence on career accomplishments. College-level mathematics study must build on and extend prior experiences. Students entering higher education have diverse preparations for college mathematics due to many factors including academic background, time since high school graduation, age, and work experience. As a result, mathematics departments in colleges and universities have difficulty in placing students in their first college mathematics courses by using only data such as high school rank-in-class, grade point average, or record of high school mathematics courses.

Placement tests can be an effective component of a comprehensive placement process. However, it is important to recognize that the development of testing instruments is a nontrivial process. The Mathematical Association of America recommends that college placement tests in mathematics should:

**MEASURE DEVELOPED MATHEMATICAL REASONING SKILLS.** College admission tests such as the SAT or ACT measure students' general readiness for college, whereas placement tests seek to measure students' knowledge and skills that are prerequisite for specific entry-level college mathematics courses. Nationally administered tests such as SAT and ACT measure a broad range of quantitative skills, and this measure is often too general to distinguish between readiness for entry-level mathematics courses such as college algebra, trigonometry, pre-calculus and calculus. Therefore, very often, high school record and admission test scores need to be supplemented to make decisions about placing entering students into their initial mathematics courses.

**EMPHASIZE REALISTIC AND CURRENT EXPECTATIONS.** Placement tests should not reflect obsolescent expectations in mathematics preparation in the secondary schools. Placement tests must be carefully reviewed as more is learned about what contributes to success in post secondary education and in light of changes in content and effectiveness of pre-collegiate mathematics programs.

**AVOID SINGULAR FOCUS ON COMPUTATIONAL SKILLS.** Good placement tests assess computational skills in unexpected contexts and a balance of procedural fluency, conceptual understanding, and strategic reasoning.

**INCORPORATE APPROPRIATE TECHNOLOGY.** Calculators and computers are an integral part of most pre-collegiate mathematics instruction. Even though prerequisite skills for a college mathematics course can be assessed without computers or calculators, students may be more comfortable working on a placement test in the familiar environment that includes use of technology. Therefore, calculators and computers should be considered for use in placement testing programs.

**USE APPROPRIATE TESTING METHODS.** Great care should be used in the design and administration of placement test programs. Informed consultants and helpful literature should be utilized in the design of placement test programs.

Further information on design of effective college placement programs for mathematics can be obtained from the Mathematical Association of America, 1529 Eighteenth Street, NW, Washington, DC 20036.

— MAA Board of Governors  
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# Trends in Collegiate Mathematics Placement

Students enter college with diverse goals, academic records, and dispositions. A mechanism of some kind is therefore needed to place these incoming students into the appropriate first mathematics course. While such educational decisions should not be based solely on a single test score, tests typically play a dominant role in how an institution views each student's placement profile.

In recent years there have been a number of developments that impinge on college mathematics placement testing, including significant changes in high school and college curricula. At the high school level, more students are taking more years of mathematics, with algebra beginning in earlier grades. Some schools offer traditional courses, while others feature integrated curricula. Over the decades 1990–2010 states developed standards for K–12 mathematics, often following the path of the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics*, emphasizing both process and content. Beginning in 2010 most states adopted the *Common Core State Standards for Mathematics*, pointing to more uniformity and stronger mathematical practice. Colleges have responded to the variety of student interests and backgrounds by creating more courses, including courses for students who have studied calculus in high school, quantitative reasoning courses, statistics courses, and discrete mathematics courses.

Advances in technology have also led to new options in placement testing. Partly because hand-held calculators are heavily used in high school, placement testing in the presence of calculators is more common. Online delivery of placement tests is more common, as is computer-adaptive testing and testing accompanied by tutorial materials to shore up weaknesses in preparation for additional testing.

Other promising developments in testing have stemmed less from technological breakthroughs than from innovative ideas:

- Programs of prognostic testing, in which feeder high schools administer to their college-bound students an early college placement test so as to provide them with the most appropriate mathematics instruction in their senior year;

- Customized placement testing, in which a test maker offers colleges the option of composing tailor-made placement examinations from a catalog of well-crafted items, thereby reaping the advantages of both locally and externally developed placement examinations;

- Item types other than the traditional multiple-choice items, including student-generated responses that are machine-gradable; and

- Research-based items that probe understanding of concepts critical for success in college mathematics courses.

While all these choices complicate the life of the individual responsible for instituting a college's program of mathematics placement testing, they offer the hope of improving the service that the institution delivers to its students.

# The Placement Test Suite of the MAA and Maplesoft

Widespread and increasing concern with mathematics placement problems in colleges and universities prompted the MAA, in 1977, to establish its Placement Test Program (PTP) to assist with the development of local placement test programs. Development and administration of PTP was led by the MAA's Committee on Placement Examinations (COPE). COPE was replaced several years ago by the Committee on Testing (COT). Both COPE and COT were composed of faculty members affiliated with a variety of types of collegiate institutions who are experienced in placement testing, and COPE and COT appointed ad hoc task forces of similar faculty members to attend to specific developments, such as new tests or significant modifications to existing tests. In 1999, PTP was discontinued by the MAA, and in 2002, MAA entered into a contract with Maplesoft to market and deliver MAA placement tests, now designated as the Placement Testing Suite (PTS). Since 2002, the MAA committee that oversees PTS has been the Committee on Articulation and Placement (CAP) in conjunction with an MAA director of PTS, who is advised by a task force.

The PTS currently provides subscribers with a battery of placement tests in mathematics for entering college students and five tests designed for "prognostic" testing of college-bound secondary school students. Placement tests are available in:

- Arithmetic and Basic Skills (A-S)\*
- Basic Algebra (BA)\*
- Algebra (A)\*
- Advanced Algebra (AA)
- Trigonometry and Elementary Functions (T)
- Calculus Readiness (CR)\*
- Calculus Concept Readiness (CCR)
- High School Elementary Test (HS-E)\*
- High School Intermediate Test (HS-I)

\*Both calculator and non-calculator versions of this test are currently available.

Four static parallel forms of the first six tests above are available.

Two forms of the high school tests are available.

Algorithmic forms of the first four tests are available, thereby having the capability of generating thousands of forms that are highly similar, essentially parallel.

Each test consists of 25 to 32 multiple-choice questions that can be administered in 45 minutes or less. Several of the tests contain sub-tests that are designed to meet more specific needs. These sub-tests can be administered in 25 minutes or less. Most tests come in multiple parallel forms. The content areas of the parallel forms are the same, but the questions may differ slightly.

Specific information on each of the tests, including topics covered and recommended time limits, is given in Appendix A of this guide.

## How PTS Tests Are Developed

The PTS tests are written by panels of college mathematics teachers who are directly involved in teaching students the courses served by the PTS tests. Each panel writes one or more preliminary versions of a new test and administers these at selected institutions. After consideration of detailed analyses, the panel may



prepare another preliminary version or submit the new test to the MAA for final approval by the appropriate committee.

## PTS Test Security

Since the PTS tests are not designed to assign course credit, many of the security problems associated with credit-by-examination are avoided. Nevertheless, subscribers should take precautions to safeguard their tests and to prevent cheating during testing sessions. Parallel forms may be used for this purpose. Probably the most effective means of handling the problem of student dishonesty on placement tests is to make sure that students are fully informed of the very real advantages to them of correct placement.

PTS users are post-secondary institutions, and PTS is not marketed to secondary institutions. Secondary school teachers and administrators who are interested in learning more about the PTS program should contact MAA at 1529 Eighteenth Street, NW, Washington, DC 20036 or Maplesoft, a division of Waterloo Maple Inc.

# Assessing the Need for Local Mathematics Placement

The first step that should be taken by any college considering a placement test program is to assess its mathematics placement needs. Consideration should be given to:

- The number and variety of entry-level courses
- The extent to which these courses presuppose previous mathematical learning
- The variability of the mathematical preparation of students
- Evidence of placement problems, such as substantial numbers of early course withdrawals, expression of student dissatisfaction (courses too hard or too easy), or faculty concern about the disparity in mathematical preparation of students in their courses

The basis on which students are currently placed into mathematics courses should be carefully scrutinized. In the absence of a formal placement program, it is likely that the primary basis for placement is high school background.

While it is true that the total record of high school mathematics performance contains more data than are obtainable from a placement test, there are factors that may make high school background an unreliable basis for placement:

- The variety of high school programs presented by students
- The large number of entering students making processing of high school data impractical
- Changes in high school course content or standards
- The existence of a substantial group of students for whom high school mathematics background is not available or is of little use (e.g., students who have been out of school for several years)

Another commonly used criterion for placement is a student's score on a general mathematical college admissions test. While a score on such a test may be useful in conjunction with other information, it alone is inadequate to determine whether the student has the prerequisite skills for one entry-level course versus another.

Consequently, many colleges prefer to avoid making inferences about a student's preparation that are based only on high school background or admission test scores, but rely in addition on a direct measurement of the student's skills taken under conditions that are uniform for all students, i.e., a placement test.

# Selecting/Adapting PTS Tests and Setting Cutoff Scores

## Prior to Selecting Tests

Before a test is selected, it is necessary to identify the topics that are taught in each mathematics course in the sequence. Selections of appropriate tests are based on the match between the topics taught in each course and the topics included on the tests. A placement test is not the same as a final examination. In general, it is not necessary to test all the topics taught in the previous course in the sequence to determine whether a student is ready to be placed into the next course. Placement tests tend to be more modest in scope. More often, only a sample of the topics from the previous course and the next course needs to be tested.

## Selecting PTS Tests

Guided by the considerations described, subscribers need to determine whether any of the existing PTS tests are a satisfactory match to the list of topics in each of their courses. The content of each PTS test is given in the appendix at the end of this user's guide.

## Adapting PTS Tests to Local Needs

If none of the PTS tests adequately meets local needs, construction of a custom-tailored test may be more appropriate. If subscribers wish, they may add locally developed items to an existing PTS test. If new tests are used, the following points should be kept in mind:

1. Even minor changes (e.g., moving questions from one location in the test to another, or adding or deleting choices from an existing question) may produce a test that differs significantly from the original with regard to topics covered and difficulty.
2. When a test has been altered, the new test should be regarded as just that: new. An experimental administration should be conducted to check for typographical errors and to determine whether the test difficulty has shifted.
3. If the test has been given at the appropriate time during the academic year, it may even be possible to use the results from the experimental administration for setting preliminary placement cut-scores (see below).

## Item Analyses

Any altered test should be given careful analysis before it is used for placement purposes. Detailed scrutiny of a test is usually based on an examination of data resulting from an appropriate administration of the test. Statistical procedures for test and item analysis are described in most introductory educational measurement texts. Some references are given below.

Introductory References on Educational and Psychological Measurement:

Anastasi, Anne (1976). *Psychological Testing*, 4th edition. New York: MacMillan Publishing Co., Inc.

Hopkins, Kenneth D., Stanley, Julian C. & Hopkins, B.R. (1990). *Educational and Psychological Measurement and Evaluation*, 7th ed. Englewood Cliffs, NJ: Prentice-Hall.

Linn, Robert L. & Gronlund, Norman E. (1995). *Measurement and Assessment in Teaching*, 7th edition. Englewood Cliffs, NJ: Prentice-Hall, Inc.

Additional References on Educational and Psychological Measurement:

Allen, Mary J. & Yen, Wendy M. (1979). *Introduction to Measurement Theory*. Monterey, CA: Brooks/Cole Publishing Co.

Crocker, Linda M. & Algina, James (1986). *Introduction to Classical and Modern Test Theory*. New York: Harcourt Brace Jovanovich College Publishers.

Oftentimes, it is possible to obtain assistance with these analyses from the testing center on campus.

## Norms, Reliability, Validity

Panels of experienced college mathematics teachers construct PTS tests for standard placement situations. Careful construction of syllabi and extensive pre-testing of items are employed to produce tests that should exhibit high content validity. However, the validity of a PTS test for a specific placement use will necessarily depend on the content and prerequisites of the courses into which students are being placed, those best measured by user mathematics departments. The test construction procedures and extensive statistical analyses of items are designed to give PTS tests a high level of reliability, and PTS users are encouraged to conduct local validity and reliability studies. Because of the intended use of PTS, national norms are inappropriate.

Validity studies are of several types: Comparative, criterion, judgment, and predictive are types often used for placement tests. Comparative studies compare performance on the placement instrument with other measures, such as SAT or ACT scores. Criterion studies compare performance on the placement test to other criteria for entering the target course, for example, success in the prerequisite course. Judgment studies use opinions of experts, usually faculty members, who identify performance levels on the placement test believed necessary for success in the target course. Judgment studies produce evidence that a placement test has face validity; that is, the test, in the view of experts, has valid content to give valid placement information. Predictive studies compare performance on a placement test to success in the target course measured by passing the first test or achieving a passing grade in the course. For example, students in a beginning calculus course take a CR test, and student performance in the calculus course is compared to scores on the test. No placement test is likely to be a strong predictor of course grades, but correlations and comparisons still give valuable information.

Reliability is the extent to which a test consistently measures whatever it measures. One way to measure reliability is to split a test into two equal pieces and compute the correlation between scores on the two pieces. In placement testing, different items may measure different concepts or skills, so performance on different items may not be highly correlated. Therefore, the reliability estimates for placement tests are often less than those expected on achievement tests. The latter are often above 0.85. Placement test reliability estimates are usually in the 0.60 to 0.80 range.

## PTS Tests for 15 Standard Entry-Level Courses

### Remedial

Arithmetic	A-S, CB-A-S
Elementary Algebra	A-S, CB-A-S, BA, CB-BA
Arithmetic and Elementary Algebra	A-S, CB-A-S, BA, CB-BA, A, CB-A
Intermediate Algebra	BA, CB-BA, A, CB-A

**Precalculus**

Precalculus and Elementary Functions	A, CB-A, AA, T
College Algebra/Trigonometry	A, CB-A, AA, T
College Algebra	A, CB-A, AA
Trigonometry	A, CB-A, AA, T

**Service**

Calculus (Intuitive)	CCR, CR, CB-CR, BA, CB-BA, A, CB-A
Statistics (non-Calculus based)	BA, CB-BA, A, CB-A
Finite Mathematics	BA, CB-BA, A, CB-A
Survey/Liberal Arts Mathematics	BA, CB-BA, A-S, CB-A-S
Algebra (Terminal)	BA, CB-BA, A, CB-A, AA
Mathematics for Elementary Teachers	BA, CB-BA, A-S, CB-A-S

**Calculus (Engineering-Physical Science)**

Calculus and Analytical Geometry	CCR, CR, CB-CR, GCB-CR, T, A, CB-A, AA
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**Setting Institutional Policies for Placement Test Use**

Placement in mathematics courses is done to measure students' mathematics preparation so that they can be placed in the appropriate courses in the sequence of mathematics courses. The intent of placement is to maximize either or both students' learning and institutional use of resources. The testing done is part of a carefully structured process designed to yield information that can be used to make specific decisions. In this section, we briefly describe some issues that should be considered in determining how the placement tests are to be used.

**Use of Multiple Measures.** First, it is important to note that a single test should rarely be used to determine course placement. It is preferable in most cases to use multiple measures whenever making placement decisions. This is done to “triangulate” the information needed to place a student: that is, to determine whether the student has the minimal level of mathematics preparation needed to begin study at the next course in the sequence. Multiple sources of information could include the following:

- A. High school courses taken in the subject and grades in those courses;
- B. Advanced Placement or college-level courses completed successfully;
- C. performance on a placement test in the subject; and
- D. observation of performance in a specially structured problem situation.

In general, use of multiple sources should yield a more accurate picture of each student's mathematics preparation than would be available from a single test. The information from all sources should be integrated before making placement decisions. If scores from several tests are available, it may be possible to combine them in some way to obtain a final score.

**Setting Standards for Performance**

The points at which placement decisions based on test results change are called cut-scores. These scores reflect the minimum level of mathematics preparation needed to begin study in a course.

**Nature of Cut-Scores.** Cut-scores should be set so that students have a reasonable chance for successful learning in courses. After careful consideration of all available information about student preparation, a

specific cut-score may not be clearly identified. Therefore, final decisions have to be based on other considerations. In the following section, some simple methods for determining cut-scores are described.

**Adjusting Cut-Scores.** Cut-scores need to be adjusted periodically. Typically, adjustment does not occur each year; rather, adjustment occurs every several years. The goal, of course, is that those individuals who are placed into a particular course are truly ready to begin learning in that course. If either students themselves or their professors judge that students are not ready for instruction, then cut-scores are not effective and need to be adjusted.

## Some Simple Methods for Setting Cut-Scores

Setting cut-scores is a complicated procedure, and experience is valuable. It is sometimes useful to consult someone with experience prior to proceeding to set cut-scores. Some simple approaches are described below to suggest ways of setting cut-scores. In the following, adjacent courses are courses that are sequenced, for example, precalculus and calculus 1, one following the other.

**Faculty Judgment of Expected Student Performance.** This is similar to the judgment validity study described briefly above. In this method, faculty members teaching the mathematics courses in the sequence make a judgment about how well students in each course in the sequence would do on the test. The point of reference for these judgments is usually either the end of the previous course in the sequence or the beginning of the next. There are several ways to arrive at these judgments. One simple method is to have faculty rate each question on the test in response to a stimulus, such as "What percent of students beginning this course would be expected to get this question correct?" Each test item is rated for each course in the sequence. If there were three courses in the sequence, then there would need to be three sets of ratings.

The judgments are then summarized as follows to arrive at a set of cutoff scores:

1. Compute the sum for all items for each faculty rater for a course. This sum, as a fraction (i.e., 10.5 as opposed to 1050%) is the faculty member's estimate of the number of correct answers for a student beginning the course.
2. Compute the average of the sum of ratings for all faculty members for a given course. Do this for each course separately.
3. Calculate the midpoint between the averages for adjacent courses (obtained in 2, above). This value is the cutoff score between the two courses. Students who score less than this number can be placed in the lower course, and those who score higher than this number will be placed in the higher course.

It is not critical that all faculty members give their ratings for each course in the sequence. If someone does not teach courses in the sequence, asking him or her to give ratings would probably not make much sense.

**Use of Enrolled Students' Performance.** This is similar to the predictive validity study described above, and the use of scores in adjacent courses is sometimes called contrasting groups studies. This method uses the actual test performance of students who are enrolled in the course or adjacent courses to determine what the cutoff scores should be. The following is a simple example of this approach:

1. Place all students into the appropriate course in the sequence using existing placement tests or decision criteria.
2. Give all students the placement test as soon after the beginning of the course as possible. Typically, this would be during the first week of the course.
3. Calculate the average placement score for students in each course.
4. Calculate the midpoint between the averages for adjacent courses and use this as the cut-score between those two courses. This technique is useful if only a single score is used for placement. It is

also predicated on the assumption that students are already placed in the course in some way that is acceptable to the institution.

The methods needed to develop a cut-score for placement become more complicated with an increase in the number of tests that need to be combined in arriving at a placement decision.

**Sampling within Courses.** Rather than giving the test to the entire group of students in each class, it may be easier to use only a small sample of students. The test could be given to groups of students who are very close to some specific level of performance. For example, the test could be given only to students who had a grade of C or better in the course. To do this would require waiting until the end of the course to obtain their course grade. A cut-score between adjacent courses can be obtained using the procedure described above (for enrolled students). It is important to note that if the test is administered at the beginning of the semester, the cutoff score is likely to be different than if the test is given at the end of the course. In general, the cut-scores will be higher if students are tested at the end of a course than if they are tested at the beginning, before any instruction has occurred.

**Use of Institutional Quotas.** Still another approach is to set the cutoff scores based on the numbers of students that can be accommodated in each course. To do this, all students would be given the placement test and the distribution of test scores estimated. Next, the percentage (or number) of students who could be taught in each course would be used to allocate students into courses in the sequence. As many students as can be accommodated in the first course would be taken from the bottom of the test score distribution. The next higher-scoring X percent would be assigned to next higher course, etc. The score halfway between adjacent courses in the sequence would be the cutoff score between the two courses.

**Making Decisions Near Cut-Scores.** It is important to recognize that test scores, like all measures of human ability, contain errors. Two important indices of measurement error are the test reliability and the standard error of measurement. Such errors may be both positive and negative. That is, errors can inflate a student's test score or deflate it. Although we hope to minimize these errors, it is usually not possible to remove them completely. This can oftentimes become crucial when placement decisions are made near the cut-scores. For this reason, it is important for advisors to treat these scores as useful but not as absolute indicators of students' mathematics preparation. When making placement decisions based on test scores, therefore, it is best to use multiple indices of students' mathematics preparation and to recognize that tests are fallible and may need to be tempered by human judgments.

**Concluding Comments on Setting Cutoff Scores.** It is important to note that there is no single best way to set cutoff scores and that the cutoff scores that are obtained will depend in part upon the method(s) used to set them. Further, reconciling results from the different methods is not likely to be an easy task. Assistance in implementing these procedures is generally advisable. Finally, adjustments in whatever initial cutoff scores are used will invariably need to be made based on subsequent experience.

# Administering a Mathematics Placement Test Program

## Use of the Placement Test

Users should carefully consider when and where the placement test will be administered and how the results will be used. Some questions to consider:

Will the results be advisory or enforced (mandatory placement according to score)?

Will the test be proctored or un-proctored?

Will calculators be allowed?

If results are enforced, will students be allowed a second attempt after a recommended review, is there a waiver process that recognizes other supporting information, and is there a well-planned appeal process? And in either case, will the test score be used to waive degree requirements or grant credit? Finally, there needs to be a plan for administering the test to students who have special needs (more time, large print, quiet environment, etc.).

## Notifying Campus Personnel

Users should consider notifying those colleagues on campus who advise or administer students. Depending on the campus, this may include other advisers, new student enrollment personnel, deans, and registration supervisors. These are the stakeholders involved with recruitment, retention, and registration of students. They will get asked about the placement test program by students and parents. These stakeholders may also identify others who should be informed. In many cases, it is a good idea to explain the purpose and use of the placement test at several of the “all adviser” meetings.

## Notifying High Schools about the Program

Users should consider notifying high school mathematics departments about their program. The existence of a placement test program is a recognition of a shared educational responsibility between high school and college. Most high school mathematics teachers will welcome a placement test program that they understand, since it demonstrates to their students that learning mathematics in high school does have long-term benefits. However, a placement test program, the content and goals of which are unknown, may have a somewhat ominous appearance.

The sample tests that are supplied to PTS subscribers may help communicate appropriate information to high schools. For more information, see the section below entitled “Sample Tests.”

## Notifying Students about the Program

All students should be informed in advance about the testing procedure: when they will be tested, the nature of the test questions, and how the results will be used. Some students have to be convinced that proper placement is to their advantage, and this requires careful thought and communication.

Users may wish to suggest that students who have not recently taken a mathematics course review briefly before taking the test. It is certainly appropriate that students receive credit on the test for material that they can relearn quickly. Material that they can relearn only slowly is best redone in a course. Many students are retrospectively optimistic about the amount of material they could have relearned if asked. The most satisfactory procedure is to ask in advance and see what they actually do.



## Sample Tests

Sample tests are available for the principal PTS placement tests: A-S [Arithmetic and Skills], BA [Basic Algebra], AA [Advanced Algebra], T [Trigonometry], CR [Calculus Readiness], and CCR [Calculus Concept Readiness]. Sample Tests BA and AA together cover Test A, which is a combined test. The sample tests reflect the general nature and level of difficulty of the placement tests. Users should note that tests and sample tests have no test items in common, thus no security is breached in making the sample tests public. Users are encouraged to use the sample tests in communicating with local high schools, high school students, and their own students.

Since each sample test displays only a few test items, they do not reflect the full range of topics covered on any test. The lists of topics covered on PTS tests are included on the sample tests and at the end of this guide (Appendix).

## Scheduling Testing Sessions

Users should plan to administer the tests as early as possible and to get interpretation of the test results to the students as quickly as possible. Some colleges arrange for students to take placement tests during the summer preceding their freshman year. This permits the students to know the test results when they start to plan their fall schedules. Additional information, like ACT or SAT scores and mathematics courses taken in high school, can also be used to place students. This arrangement produces far more satisfactory results than one that calls for changes after schedules are set.

Information obtained from the early testing also permits the department to make final adjustments that match its course offerings to student needs.

If placement testing during the summer is not feasible, users should plan to complete testing and placement before the first class day. Students who have registered for a course but are subsequently moved to a lower-level course as a result of their score on a placement test often perceive this as a kind of failure, not a good way to start a semester. Furthermore, the logistics of moving students after classes have begun can be awkward and time consuming, usually resulting in lost class time. To ensure that all students have taken the placement test, it is preferable to require them to take the test before registration.

## Setting Time Limits

Recommended time limits are given for each PTS test (see Appendix A). If tests have been modified by adding or deleting items, time limits should be adjusted accordingly. A rule of thumb for any placement test is about one minute of actual test time for each question from the PTS collection.

The time limit should remain fixed for all administrations of any test so that scores at one administration can be compared with those at another. Time limits should only be changed when experience shows a clear need to do so.

## Mandatory versus Advisory Placement

The decision as to whether placement should be mandatory or advisory may depend on:

- Number of students to be placed
- Number of faculty advisors available to counsel students
- Familiarity of the faculty advisors with high school and college mathematics curricula
- Time available to faculty advisors to peruse high school records, SAT scores, etc.
- Experience with the quality of advice that faculty advisors have given mathematics students in the past
- Legal or policy constraints

## Testing Students with Special Needs

While the uniform administration of a short, standardized test may be sufficient to assess the preparation of most students, it will not be sufficient for all. The ability of placement tests to provide quick and routine assessments must not become the occasion for a wholly mechanical processing of students.

Users may wish to make some plans for identifying and assessing, on an individual basis, the background of students whose previous training has been different from that which is customary or for whom English is a second language.

Users should also make plans for administering the tests to students with special needs under appropriate conditions.

## Exceptionally Low Scores

Users will need to consider what they will do about students whose scores are so low as to make them appear unprepared for any available course. No student should be denied access to mathematics solely on the basis of a single administration of a PTS test, however low the resulting score. If initial placement scores indicate that a student is not prepared for any available course, a more thorough evaluation should be made of the student's preparation through further testing or other means before any final decision is made.

PTS tests are designed with the understanding that students taking them will normally be going into mathematics courses where their work will receive continuing evaluation, and that in the instances where further evaluation shows that their test scores do not accurately measure their preparation, corrective action will be taken.

Every effort should be made to provide opportunities for students to learn mathematics at appropriate levels.

# Collecting and Analyzing Placement Data

A placement program should include a carefully organized plan for collecting and analyzing data related to that program. The data to be collected should be designed to determine whether the program is as effective as it could be or, if changes are needed, what those changes should be. In examining program effectiveness, it is important to provide evidence about the utility of the specific cutoff score(s) used on the placement tests. This support must be obtained from data external to the test. In this regard, one should seek to answer the following questions:

1. Are the students in a course prepared to learn in that course, or would they learn better in another course?
  - a. Do students feel they are prepared for the course in which they are enrolled?
  - b. Do instructors feel students are prepared for the course?
  - c. Do students learn better in the course in which they are enrolled than they would in some other course in the sequence?
  - d. Do students remain in the course, or do they drop enrollment before completion?
2. Are students enrolled in courses so as to make the best use of institutional resources?

A placement testing program should ensure that these kinds of questions are answered in the affirmative. If evidence can be gathered to support such answers, then the program is successful, and the use of the cut-off scores can probably be supported. If such evidence is not available, then the tests used or the cut-scores need to be reviewed.

## Record Systems

Annual improvements in placement effectiveness are based upon precise knowledge of the outcome of previous results. Record keeping is an important component of developing and maintaining an effective placement program. Information should be maintained to support program effectiveness. Whenever possible, use of existing campus record systems should be considered. These systems may allow for greater efficiency and convenience than an ad hoc system of data maintained by the mathematics department.

Permanent records should be maintained for the placement program. The following are some of the more important kinds of information:

- Copy of the form of the test given
- Cutoff scores used
- Description of the students tested. Some basic descriptive statistics include the following: the number tested, the mean score, the range of scores, the standard deviation of the scores, and a frequency distribution of test scores
- Comparison of the recommended placements versus the actual enrollments (if placement is not mandatory)

Individual records should also be maintained for all students tested. These data can be used for several purposes.

1. If students do not take a mathematics course immediately following the placement test, then it may be necessary to consult records to determine the proper placement at some point in the future.
2. If students take the test(s) more than one time, a record of each attempt should be maintained. The date(s) the test(s) were taken and the form of the test should also be recorded.

3. These records can also be used for examining the effectiveness of the placement program. Efforts to compare test scores with student dropout rates, with instructor satisfaction with students' readiness for the course, or with students' own feelings of readiness for the course can be made with these data.

At most institutions, individual student scores are subject to the Family Educational Rights and Privacy Act, which restricts the right of access to a student's test or grade information. This law is not intended to prevent use of the data for legitimate academic purposes by instructors, but it does prevent inappropriate circulation of private student records. If you are in doubt, you should consult institutional administrators on appropriate policies to ensure compliance.

## Measuring Program Effectiveness

**Types of Data.** The evidence for use of the placement test comes from information external to the test. This information includes such evidence as the following:

1. Final course grades
2. Mid-semester grades
3. First examination grades
4. Instructor judgments of students who are over- or underprepared for the course
5. Measures of student satisfaction with their readiness for the course
6. Withdrawals from the course

The specific data to be used will depend on the concerns of the institution.

**Course Grades.** The first measure is one that generally comes to mind when one considers validating the use of a placement test. Various analyses, however, have indicated that even in excellent placement programs, weak correlations will be observed between placement test scores and course final grades. The reasons for this are listed below in the section "Using PTS Test Scores as Grade Predictors." Consequently, use of final course grades should not be the sole basis for determining program effectiveness.

**First Examination Grades or Midterm Grades.** These indices often provide better measures than do final grades of student performance in the course. This is particularly true for first examination grades. Even so, such indices are still subject to many of the same weaknesses noted for final course grades. Therefore, use of either first examination grade or midterm grade should be part of a larger set of information and not the sole indicator of program effectiveness. Students who are ready to learn in the course should do well enough on the first examination or midterm examination to have a passing grade. Placement in a course too advanced for a student will probably be indicated on the first examination or on midterm grades.

**Instructor Ratings of Student Readiness.** Comparison of test scores with instructor ratings of student preparedness provides another potentially useful means of judging program effectiveness. Often, instructors can provide excellent ratings of students who are over-, under-, or properly prepared for the course. Such ratings are not to be confused with course grades, but provide a means of rating students outside the normal testing carried out in a course. If the program is effective at placing students, then instructors should indicate that their students are generally ready to learn in the course.

**Student Self-Rating of Preparedness.** Sometimes, measures of student satisfaction with course placement can point to areas in which students feel under-, over-, or properly prepared. If the program is effective, then students would indicate satisfaction with their readiness to learn in the course.

**Course Withdrawals.** The numbers of withdrawals will be useful for measuring another aspect of program effectiveness. If students are properly placed, then they are less likely to drop enrollment. Depending

on the procedure for dropping a course, you may need to solicit cooperation of your Registrar's Office, faculty advisors, instructors, or a combination of these.

## **Identification of Trends in Student Preparation**

One possible use of PTS test scores is to track year-to-year trends in student preparation. This information may be useful for explaining enrollment shifts or in supporting requests for increases in staff. Approaching such tasks rationally is facilitated by an objective measure of student preparation.

## **Records for Program Justification**

A department can best defend expenditures on its placement program with good summary information that can be accessed at any time to support and supplement its general rationale for involvement in placement activities. People who are not involved in testing are frequently confused by the many different legitimate goals of different testing programs: admissions, placement, skill diagnosis, certification, achievement, etc. All reports should clearly reflect the policies, goals, and results of your testing program.

# Limitations of PTS Tests

The PTS tests are designed to help place students into entry-level mathematics courses. To use these tests for other purposes may produce results that are unsatisfactory, misleading, and incorrect. Even when PTS tests are used for placement purposes, the most satisfactory results are obtained through the careful development of a testing program.

## Using PTS Test Scores as Grade Predictors

PTS test scores are not good predictors of course grades for the following reasons:

Course grades are within course indicators of student achievement. They have little real meaning outside the course context. A grade of “A” in a lower course does not mean the same amount of achievement as an “A” in a higher course. Correlations between test scores and such measures, therefore, have little clear meaning.

Placement tests attempt to determine whether or not a student has certain skills at the time the test is administered. They do not incorporate features to account for the vast array of other factors (physical, emotional, academic, psychological, etc.) that affect students between the time they take the placement test and when they receive a final grade.

It is difficult to second-guess the academic forces within our own departments. Different instructors may contribute differentially to the amount students learn in a given course. They may follow different grading policies. Shifts in the student population taking a course may produce semester-to-semester fluctuations in grades and in the amount of learning a grade represents. Getting meaningful grade predictions or a meaningful analysis of grade variability is a formidable task.

## Using PTS Tests as Credit, Competency, or Certification Examinations

Although appropriate PTS tests or items may reasonably serve as components of credit, competency, or certification examinations, they should not serve as the entire examination. The following limitations that the design of the PTS tests impose on these uses should be considered:

Topics are selected for PTS tests on the basis of their relevance to course placement. The construction of a test for other purposes should begin with a selection of topics appropriate for those purposes. It seems unlikely that these will fit exactly with the topics of the PTS tests.

Questions on a PTS test are selected to cover a broad area in a short time. They achieve this goal at the price of being relatively less than comprehensive. For example, they consciously avoid requiring the combination of several different ideas to produce an answer. The assumption is that such combinations will take place in the course that the placement test serves. Since the individual ideas of mathematics are of little use unless one is able to select and combine them to solve a problem, the lack of such questions is a serious deficiency for most credit or proficiency examinations.

Although the PTS tests are distributed only to collegiate institutions and it is assumed that these institutions maintain control of the test copies, no attempt has been made to impose the type of tight security system associated with examinations for credit.

# Appendix

## The Arithmetic and Skills Test (A-S)

The Arithmetic and Skills Test (A-S) contains 32 questions. The recommended time limit for this test is 40 minutes.

Test A-S covers the following topics:

<u>Topic</u>	<u>Number of Test Questions</u>
Integers and Fractions . . . . .	5
Decimals . . . . .	4
Order of Operations . . . . .	2
Linear Equations . . . . .	1
Formula Evaluation . . . . .	3
Exponents and Radicals . . . . .	3
Geometry . . . . .	5
Order Relations . . . . .	2
Word Problems . . . . .	12
Proportion . . . . .	1
Probability . . . . .	1
Percent . . . . .	3
Averaging . . . . .	1
Graph and Table Interpretation . . . . .	4
Approximation and Estimation . . . . .	6

Note: Some questions are counted more than once.

## The Calculator-based Arithmetic and Skills Test (CB-A-S)

The Calculator-Based Arithmetic and Skills Test (CB-A-S) contains 32 items. A scientific calculator will be needed for some questions on the test. The recommended time limit for this test is 40 minutes.

Test CB-A-S covers the following topics:

<u>Topic</u>	<u>Number of Test Questions</u>
Integers . . . . .	2
Fractions . . . . .	4
Decimals . . . . .	3
Percents . . . . .	2
Exponents . . . . .	5
Radicals . . . . .	3
Proportions . . . . .	1
Order Relations on Numbers . . . . .	2
Word Problems . . . . .	8
Geometry . . . . .	3
Linear Equations . . . . .	2
Formula Evaluation . . . . .	2
Approximation . . . . .	9
Graph and Table Interpretation . . . . .	4
Estimation . . . . .	1
Calculator Active . . . . .	18
Scientific Notation . . . . .	1
Order of Operations . . . . .	2

Note: Some questions are counted more than once.



## The Algebra Tests: BA, AA, A

The algebra tests cover topics in elementary, intermediate, and college algebra.

The Basic Algebra Test (BA) consists of 25 questions from elementary and intermediate algebra. The recommended time limit for this test is 30 minutes.

The Advanced Algebra Test (AA) consists of 25 questions from intermediate and college algebra. The recommended time limit for this test is 30 minutes.

The Basic and Advanced Algebra Test (A) is a combined test consisting of 32 questions. It includes items from elementary, intermediate, and college algebra. The recommended time limit for this test is 45 minutes.

The topics covered by each algebra test are listed in the tables below.

### The Basic Algebra Test (BA)

<u>Topic</u>	<u>Number of Test Questions</u>
Arithmetic of Rational Numbers . . . . .	2
Order of Operations . . . . .	2
Operations with Algebraic Expressions . . . . .	8
Algebraic Fractions . . . . .	3
Exponents and Radicals . . . . .	4
Linear Equations and Inequalities . . . . .	7
Systems of Linear Equations . . . . .	1
Fractional and Quadratic Equations . . . . .	2
Word Problems . . . . .	4
Graphing . . . . .	3

### The Algebra Test (A)

<u>Topic</u>	<u>Number of Test Questions</u>
Arithmetic of Rational Numbers . . . . .	3
Operations with Algebraic Expressions . . . . .	8
Linear Equations and Inequalities . . . . .	8
Factoring and Algebraic Fractions . . . . .	7
Exponents and Radicals . . . . .	6
Graphing . . . . .	3
Fractional and Quadratic Equations and Quadratic Inequalities . . . . .	4
Logarithms . . . . .	2
Functions . . . . .	2
Complex Numbers . . . . .	1
Absolute Value . . . . .	2
Systems of Equations . . . . .	2

## The Advanced Algebra Test (AA)

<u>Topic</u>	<u>Number of Test Questions</u>
Arithmetic of Rational Numbers . . . . .	1
Operations with Algebraic Expressions . . . . .	4
Linear Equations and Inequalities . . . . .	5
Factoring and Algebraic Fractions . . . . .	4
Exponents and Radicals . . . . .	5
Graphing . . . . .	4
Fractional and Quadratic Equations and Quadratic Inequalities . . . . .	4
Logarithms . . . . .	3
Functions . . . . .	2
Complex Numbers . . . . .	1
Absolute Value . . . . .	1
Systems of Equations . . . . .	1

Note: Some questions are counted more than once.

## The Calculator-based Basic Algebra Test (CB-BA)

The Calculator-Based Basic Algebra Test and the Basic Algebra Test have two questions in common. A scientific calculator is needed for some of the questions on the CB-BA test. The suggested time for the CB-BA test is 35 minutes.

Test CB-BA covers the following topics:

<u>Topic</u>	<u>Number of Test Questions</u>
Arithmetic of Rational Numbers . . . . .	1
Expand, Factor, and Simplify . . . . .	3
Algebraic Fractions . . . . .	5
Exponents and Radicals Including Equations . . . . .	5
Linear Equations and Inequalities . . . . .	8
Quadratic Equations . . . . .	2
Graphing . . . . .	3
Absolute Value . . . . .	2
Systems of Equations . . . . .	2
Problem Solving . . . . .	4
Geometry . . . . .	3
Estimation and Approximation . . . . .	7
Calculator Active . . . . .	8

Note: Some questions are counted more than once.

## The Calculator-based Algebra Test (CB-A)

The Calculator-Based Algebra Test and the Algebra Test have 16 questions in common. A scientific calculator is needed for some of the questions on the CB-A test. The suggested time for the CB-A test is 45 minutes. There is no overlap between the CB-BA and CB-A tests.

Test CB-A covers the following topics:

<u>Topic</u>	<u>Number of Test Questions</u>
Arithmetic of Rational Numbers . . . . .	2
Operations with Algebraic Expressions . . . . .	10
Factoring and Algebraic Fractions . . . . .	7
Exponents and Radicals . . . . .	8
Graphing . . . . .	5
Linear Equations and Inequalities . . . . .	8
Quadratic Equations and Inequalities . . . . .	2
Functions . . . . .	3
Logarithms . . . . .	2
Absolute Value . . . . .	2
Systems of Equations . . . . .	2
Problem Solving . . . . .	4
Estimation and Approximation . . . . .	5
Calculator Active . . . . .	8

Note: Some questions are counted more than once.

## The Trigonometry/Elementary Functions Test (T)

The Trigonometry/Elementary Functions Test (T) covers basic skills from trigonometry and elementary functions. The test consists of two parts. Part I contains 15 questions from plane trigonometry. Part II consists of 15 questions from topics in elementary functions. Parts I and II are completely independent and may be administered separately.

The suggested time limits for these tests are:

Part I	25 minutes
Part II	25 minutes
Parts I and II	45 minutes

<b>Topic</b>	<b>Number of Test Questions</b>
<b>Part I - Trigonometry</b>	
Definition of Trigonometric Functions . . . . .	2
Right Triangles . . . . .	2
Cofunctions . . . . .	1
Evaluation of Special Angles . . . . .	6
Related Angles . . . . .	1
Radian Measure . . . . .	6
Graphing . . . . .	1
Identities . . . . .	3
Laws of Sines and Cosines . . . . .	1
Trigonometric Equations . . . . .	2
Inverse Trigonometric Functions . . . . .	1
<b>Part II - Elementary Functions</b>	
Distance . . . . .	1
Straight Line . . . . .	3
Conics . . . . .	2
Functions: Notation, Composition . . . . .	5
Graphs and Their Properties . . . . .	6
Logarithmic and Exponential Functions . . . . .	4
Higher Degree Polynomials . . . . .	1
Absolute Value . . . . .	1
Inequalities . . . . .	1

Note: Some questions are counted more than once.

## The Calculus Readiness Test (CR)

The Calculus Readiness Test (CR) is a 25-question test designed to show a student's potential for handling calculus, either business calculus or the engineering-physical science type of calculus. It does not cover the content of a typical precalculus course but instead emphasizes word problems, graphing and graphical interpretation, numerical awareness, the formation of new concepts, and the application of formulas to new situations.

The test consists of two parts. Part I contains 20 questions involving no trigonometry. Part II contains five elementary trigonometry questions designed only to indicate whether a student has a minimal grasp of trigonometry. These questions can easily be omitted.

The suggested time limit for the entire test is 30 minutes. If the five trigonometry questions are omitted, the recommended time limit is reduced to 25 minutes.

Test CR covers the following topics:

<b>Topic</b>	<b>Number of Test Questions</b>
Geometry and Measurement . . . . .	5
Graphs of Functions . . . . .	8
Word Problems, Modeling . . . . .	5
Concept Formulation . . . . .	3
Numerical Awareness . . . . .	2
Exponential Functions . . . . .	2
Exponents and Logarithms . . . . .	2
Equations and Factoring . . . . .	2
Functional Notation . . . . .	2
Inequalities, Absolute Value . . . . .	2
Trigonometry . . . . .	5

Note: Some questions are counted more than once.

## The Calculator-based Calculus Readiness Test (CB-CR)

The Calculator-Based Calculus Readiness Test is designed in the same way as the Calculus Readiness Test. There are 15 questions that are the same on the two tests. A scientific calculator is needed for some of the questions on the CB-CR test. The suggested time for the entire test is 40 minutes. The suggested time for Part I of the test is 35 minutes.

Test CB-CR covers the following topics:

<u>Topic</u>	<u>Number of Test Questions</u>
Geometry and Measurement . . . . .	7
Graphs of Functions . . . . .	8
Word Problems, Modeling . . . . .	5
Concept Formulation . . . . .	3
Numerical Awareness . . . . .	3
Exponential Functions . . . . .	2
Exponents and Logarithms . . . . .	3
Equations and Factoring . . . . .	3
Functional Notation . . . . .	2
Inequalities, Absolute Value . . . . .	2
Trigonometry . . . . .	5
Calculator Active . . . . .	9

Note: Some questions are counted more than once.

## The Graphing Calculator-based Calculus Readiness Test (GCB-CR)

The Graphing Calculator-Based Calculus Readiness Test is designed in the same way as the Calculus Readiness Test. A graphing calculator is needed for some of the questions on the GCB-CR test. The suggested time for the test is 45 minutes.

Test GCB-CR covers the following topics:

<u>Topic</u>	<u>Number of Test Questions</u>
Geometry and Measurement . . . . .	5
Graphs of Functions . . . . .	4
Word Problems, Modeling . . . . .	6
Concept Formulation . . . . .	3
Numerical Awareness . . . . .	2
Exponential Functions . . . . .	4
Exponents and Logarithms . . . . .	4
Equations and Factoring . . . . .	1
Functional Notation . . . . .	2
Inequalities, Absolute Value . . . . .	1
Trigonometry . . . . .	6
Operations and Algebraic Expressions . . . . .	2
Polynomial Equations and Inequalities . . . . .	4
Translation of Axes . . . . .	1
Graphical Problem Solving . . . . .	2
Scientific Calculator Active . . . . .	9
Graphing Calculator Active . . . . .	4

Note: Some questions are counted more than once.

## The Calculus Concept Readiness Test (CCR)

The Calculus Concept Readiness Test (CCR) is a 25-question test designed to measure reasoning abilities and understandings central to precalculus and foundational for beginning calculus. These include a strong understanding of rate of change and a process view of function and the ability to use covariational reasoning to understand how two variables change together.

The test consists of 20 questions involving no trigonometry and five trigonometry questions designed to indicate whether a student has a functional grasp of trigonometry.

The suggested time limit for the entire test is 30 minutes. If the five trigonometry questions are omitted, the recommended time limit is reduced to 25 minutes.

Test CCR covers the following topics:

<b>Topic</b>	<b>Number of Test Questions</b>
<b>Reasoning Strands</b>	
Quantitative Reasoning involves identifying and relating measurable attributes of an object or situation in a problem context. . . . .	9
Proportional Reasoning involves thinking about how two quantities change such that their ratio remains constant; attending to how one variable changes so that it is always a constant multiple of another variable. . . . .	3
Covariational Reasoning involves thinking about how two quantities in a functional relationship are changing together; attending to how one variable changes while imagining successive amounts of equal changes in another variable. It involves coordinating two varying quantities that change in tandem while attending to how the quantities change in relation to each other. . . . .	14
Process View of Function conceives of a function as an entity that accepts a continuum of input values to produce a continuum of output values; it views a function as a generalized process that accepts input and produces output; and it appropriately coordinates multiple function processes. . . . .	11
Notational Reasoning involves making sense of symbols used in mathematical expressions and giving meaning to the mathematical ideas communicated by conventional notation. . . . .	9
Graphical Reasoning involves making sense of graphs that represent functions and giving meaning to attributes of the graph that convey aspects of a function's behavior. . . . .	9
Computational Abilities refers to facility with manipulations and procedures needed to evaluate functions, solve equations, compose functions, and invert linear and exponential functions — within the context of algebraic representations. . . . .	6



**Content Areas**

Proportions: Ratios of quantities in constant proportion ..... 2  
Algebra: Algebraic expressions, equations, inequalities ..... 9  
Functions: Concept, properties, operations ..... 13  
Representations of Functions: Symbolic, graphical, tabular, contextual (verbal) ..... 9  
Analytic Geometry: Circle, parabola, line ..... 7  
Trigonometry: Functions and applications ..... 5  
Models: Functions as models ..... 5

Note: Some questions are counted more than once.