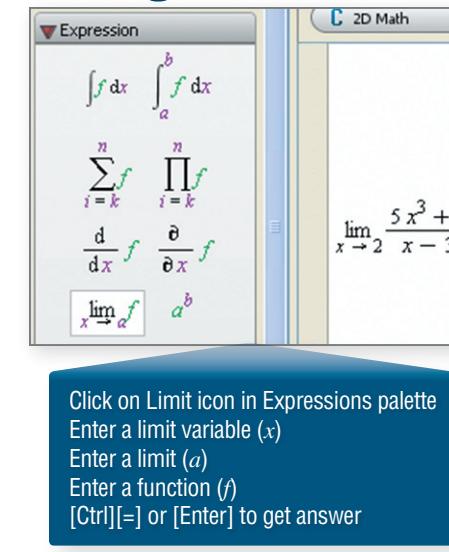


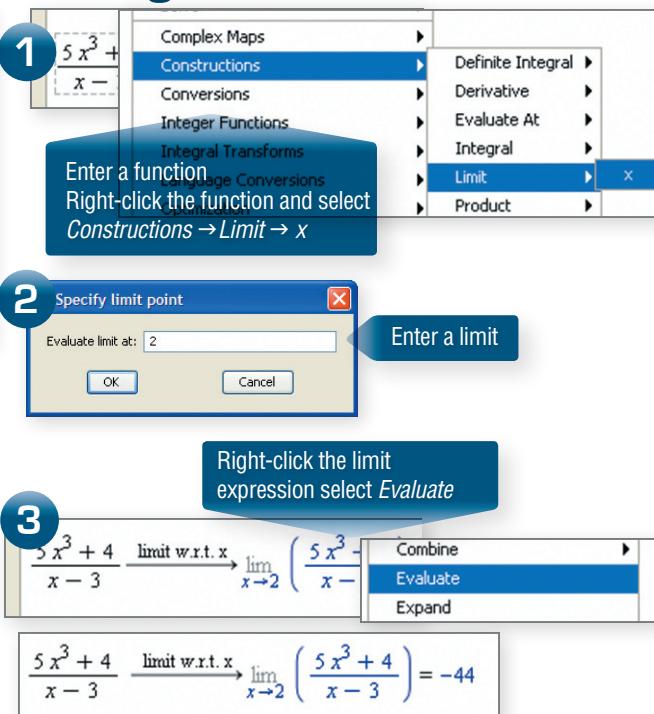
# Clickable Calculus™ with Maple™

## Limits

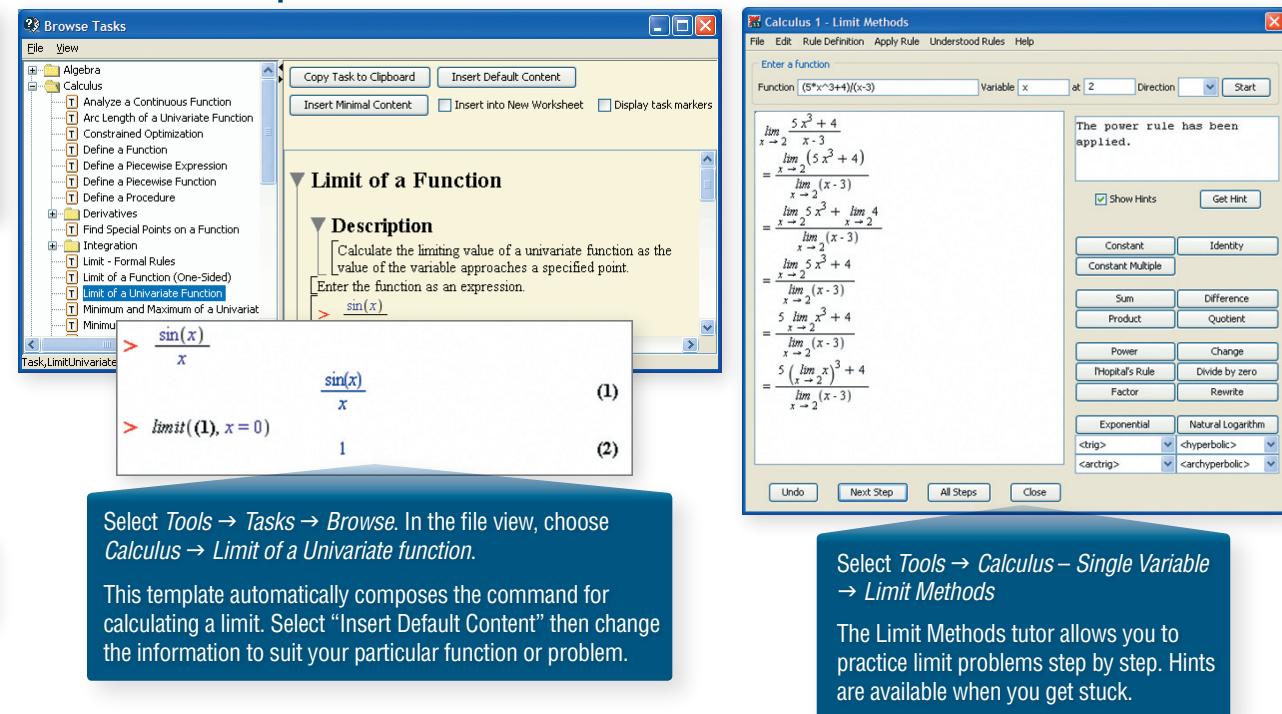
### Using Palettes



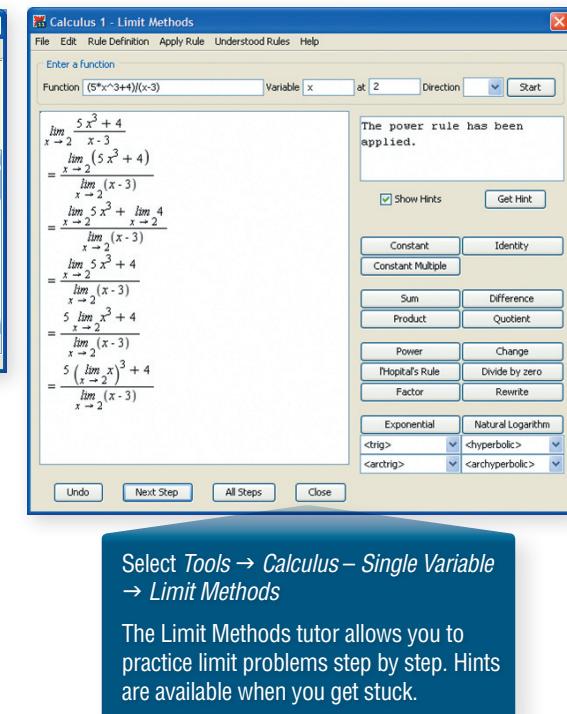
### Using Context Menus



### Task Template

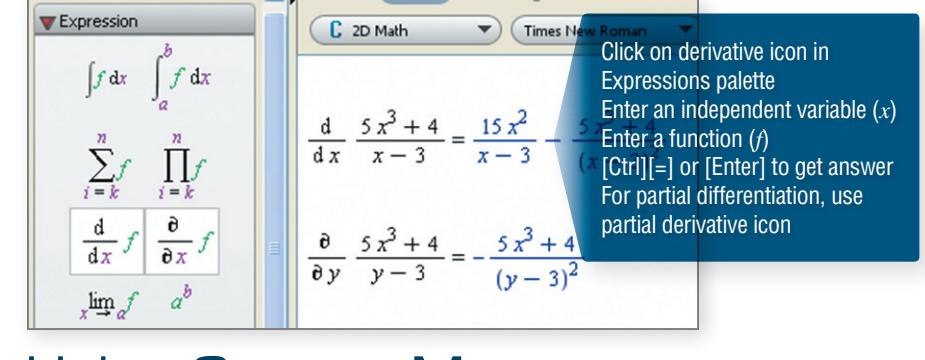


### Interactive Tutor

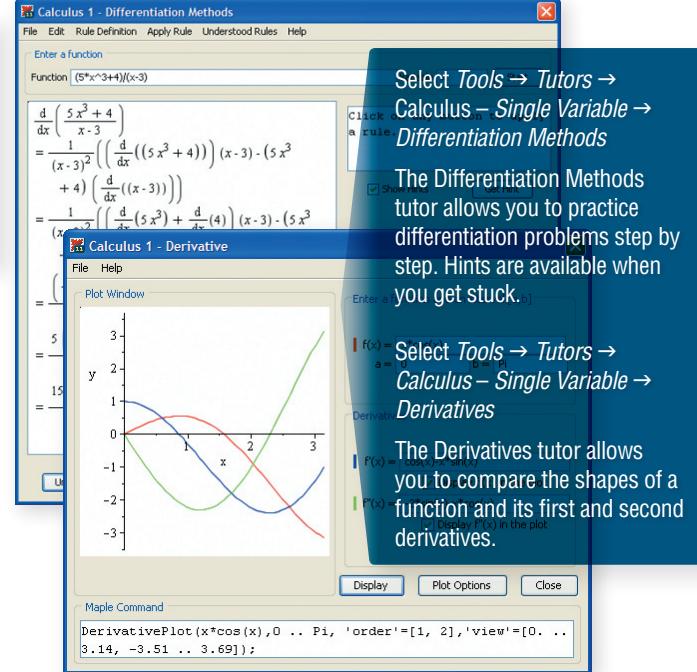


## Differentiation

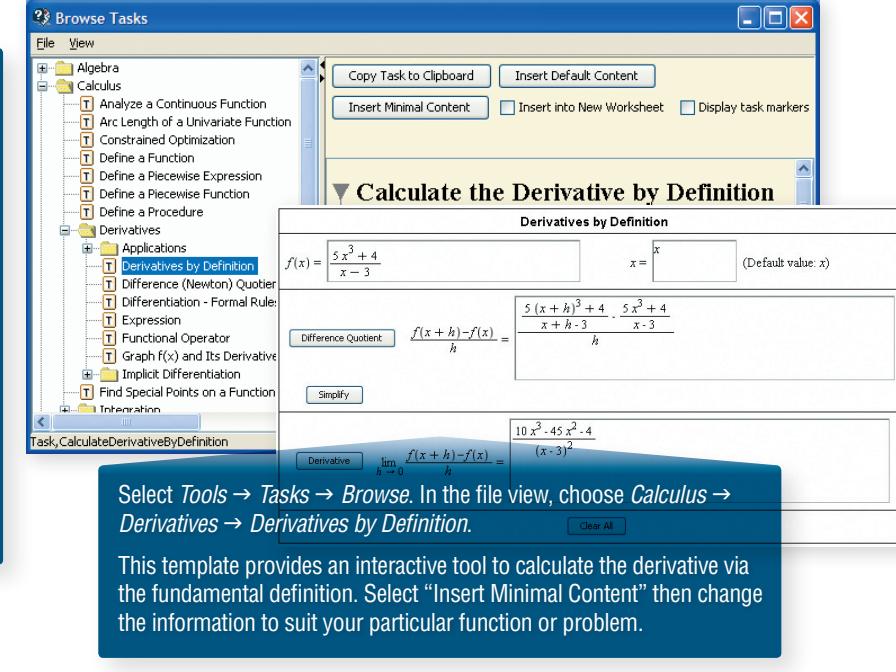
### Using Palettes



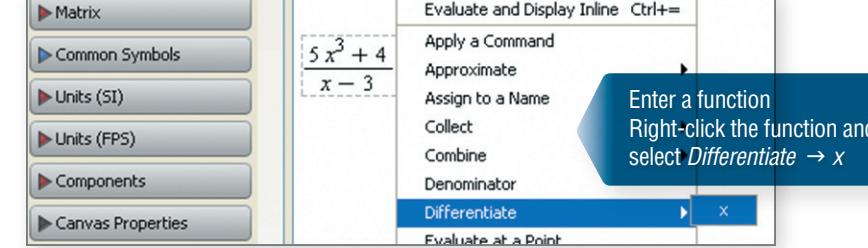
### Interactive Tutor



### Task Template

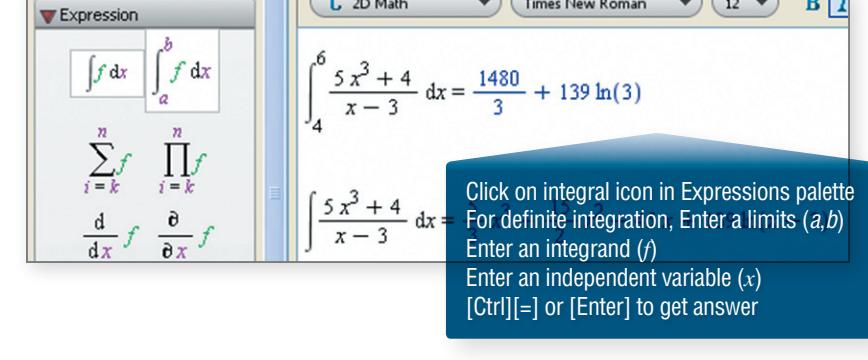


### Using Context Menus

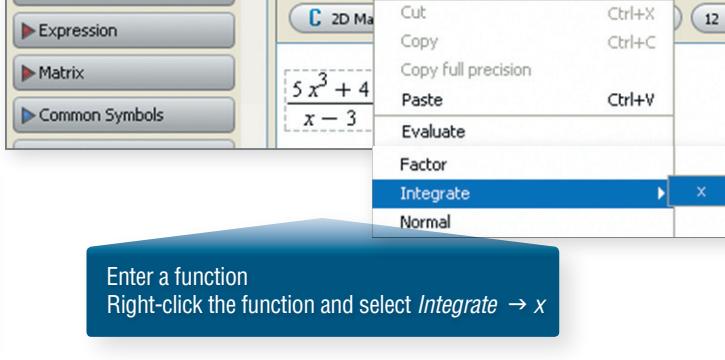


## Integration

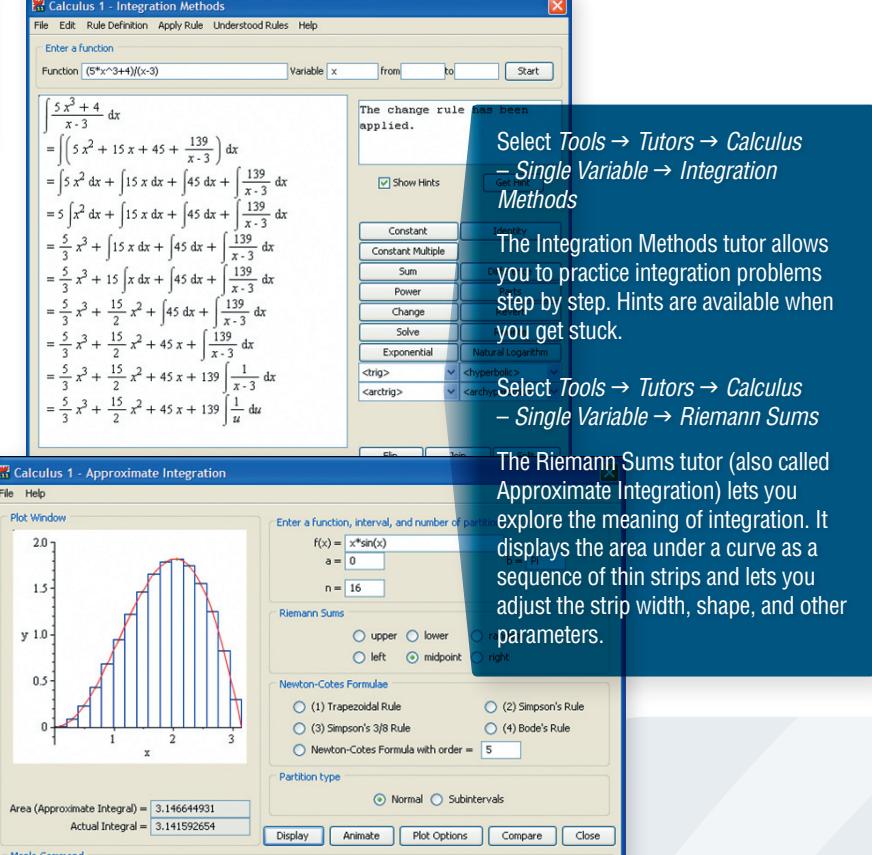
### Using Palettes



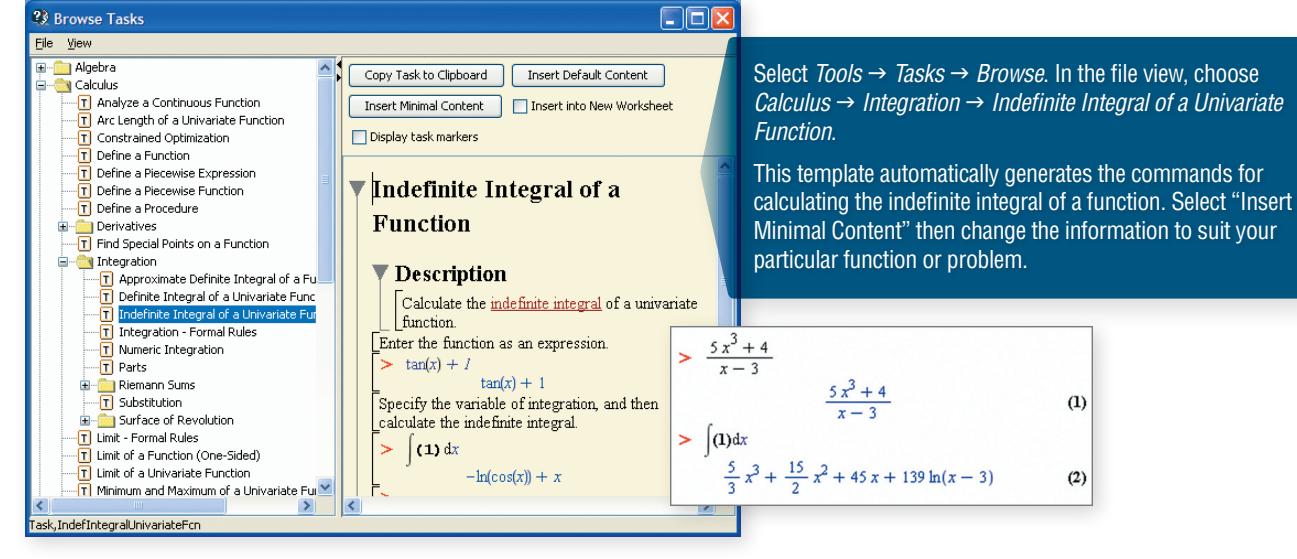
### Using Context Menus



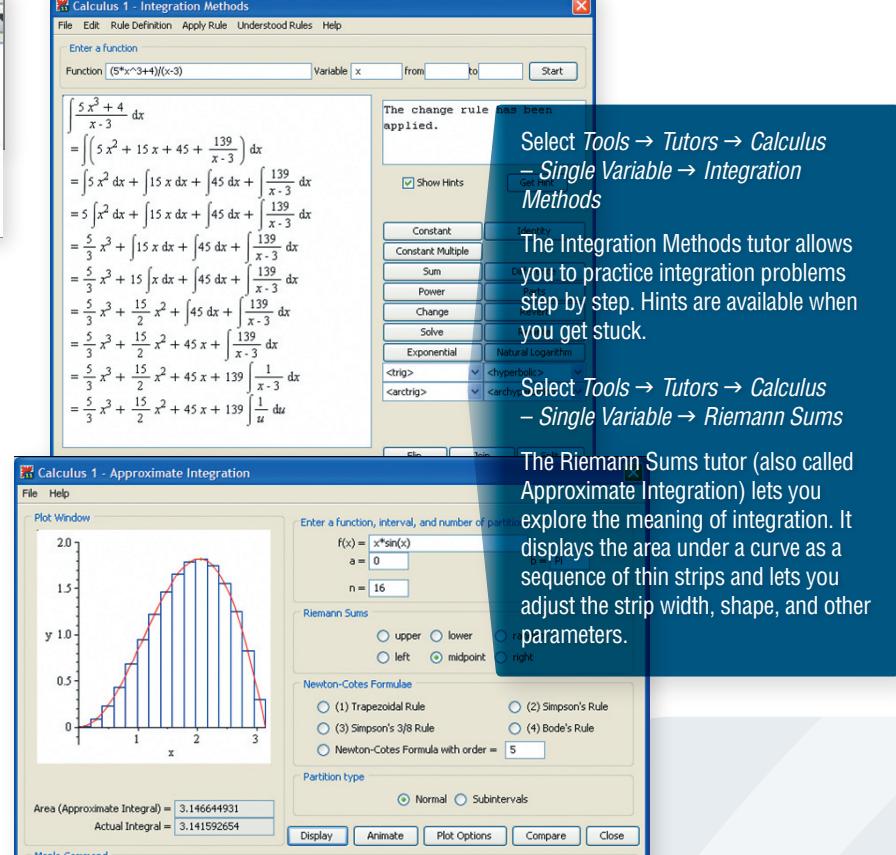
### Interactive Tutor



### Task Template

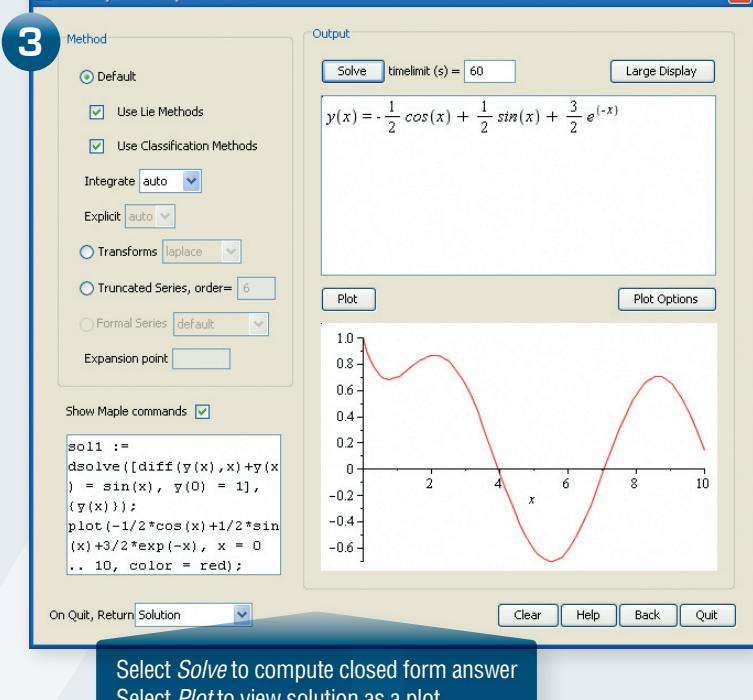
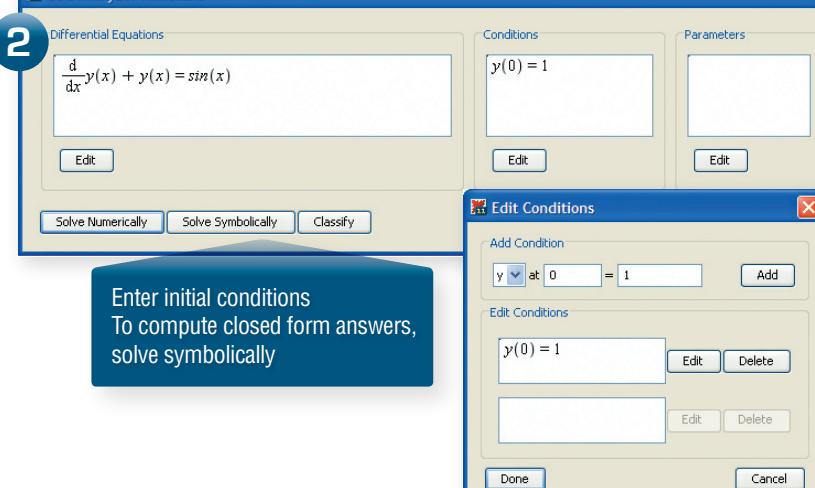
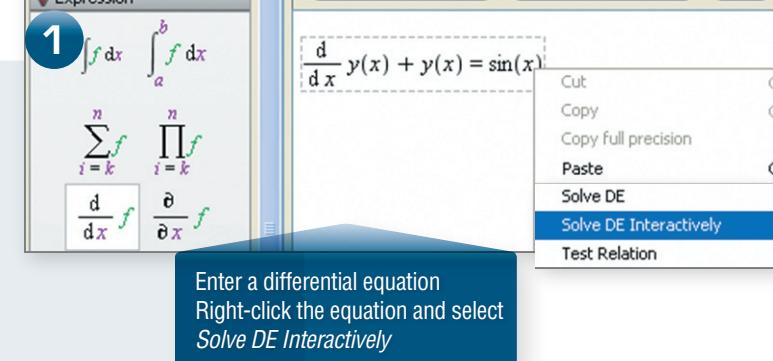


### Task Template

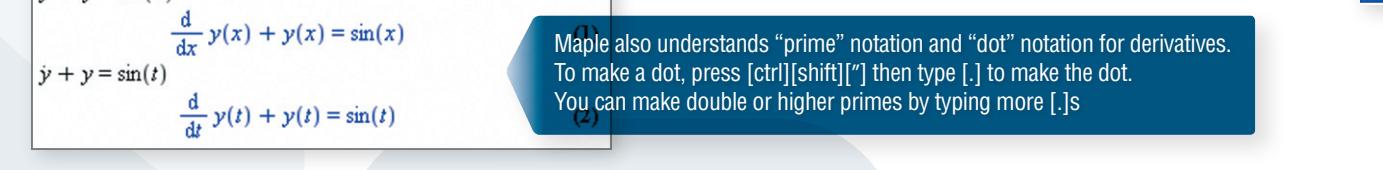


## Differential Equations

### Interactive Assistant

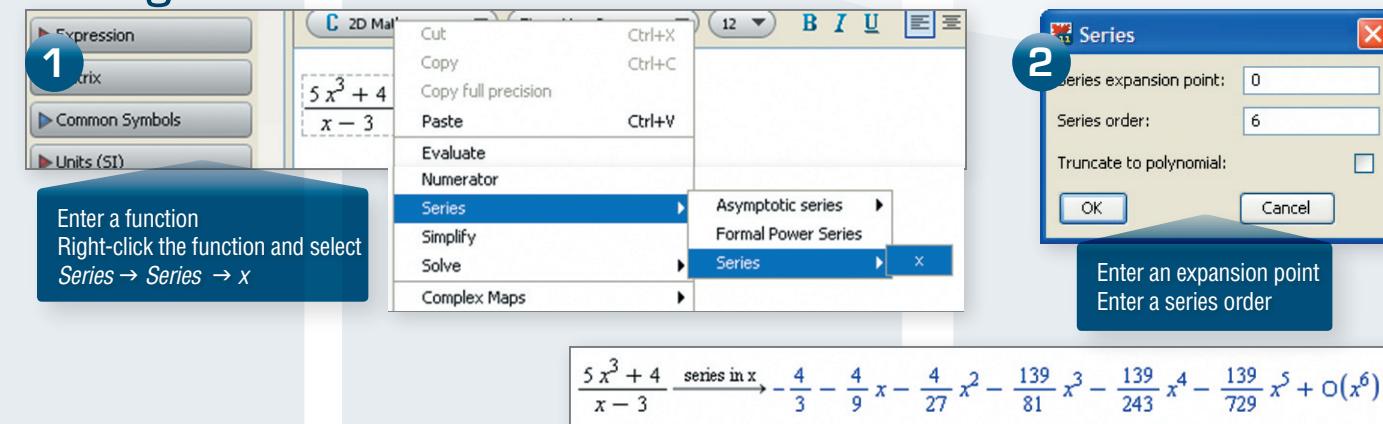


### Natural Math Notation Shortcut



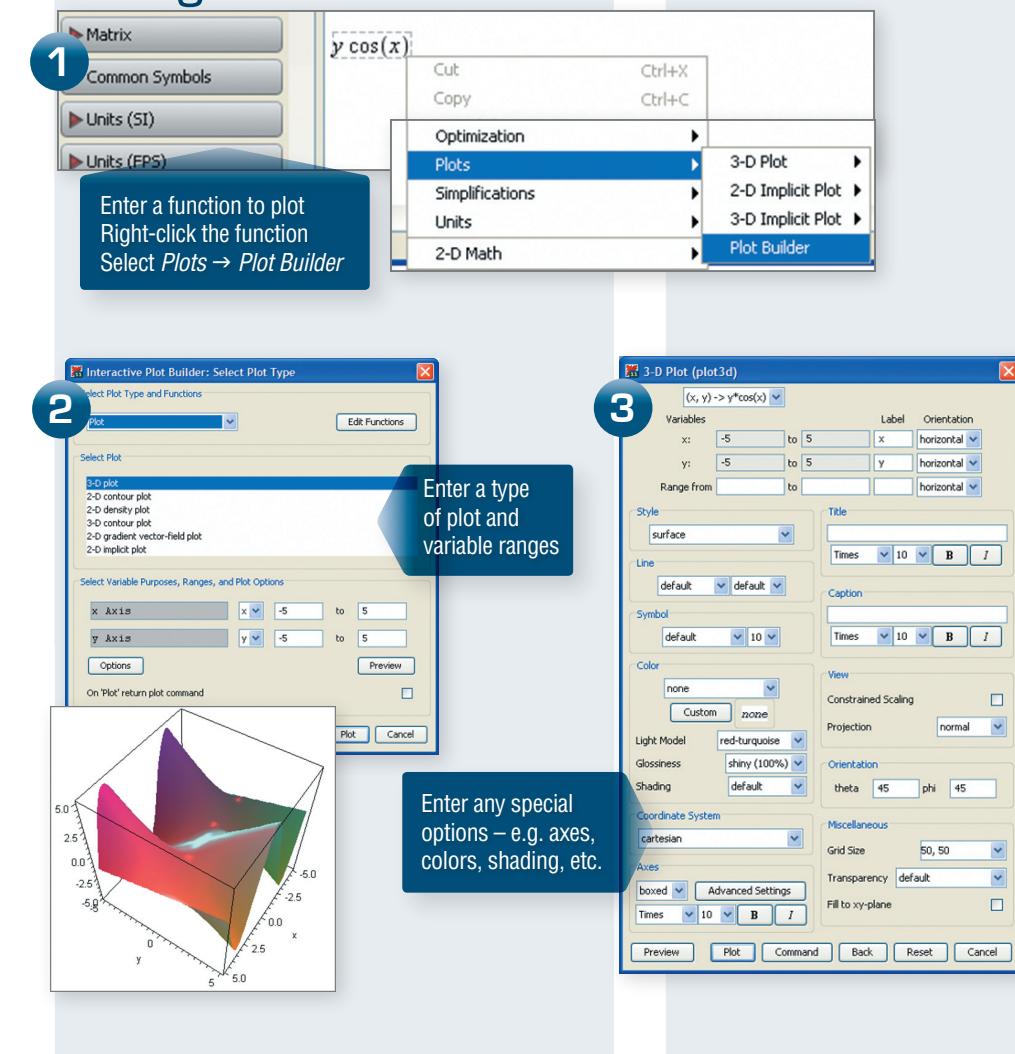
## Series

### Using Context Menus

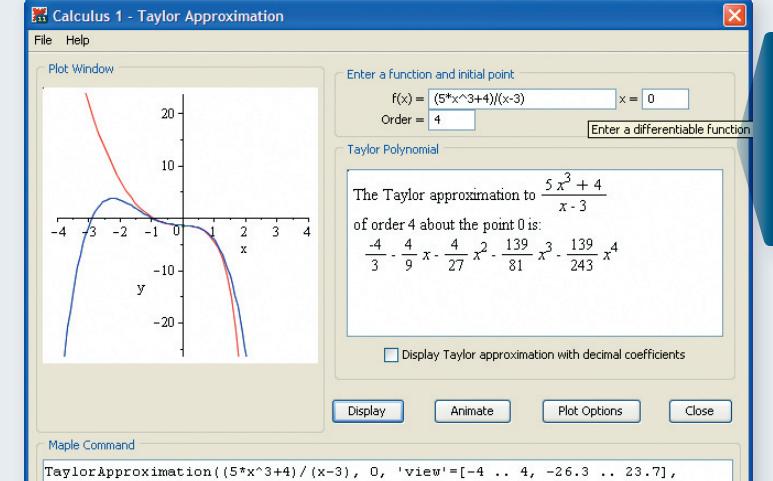


## Plotting

### Using Context Menus



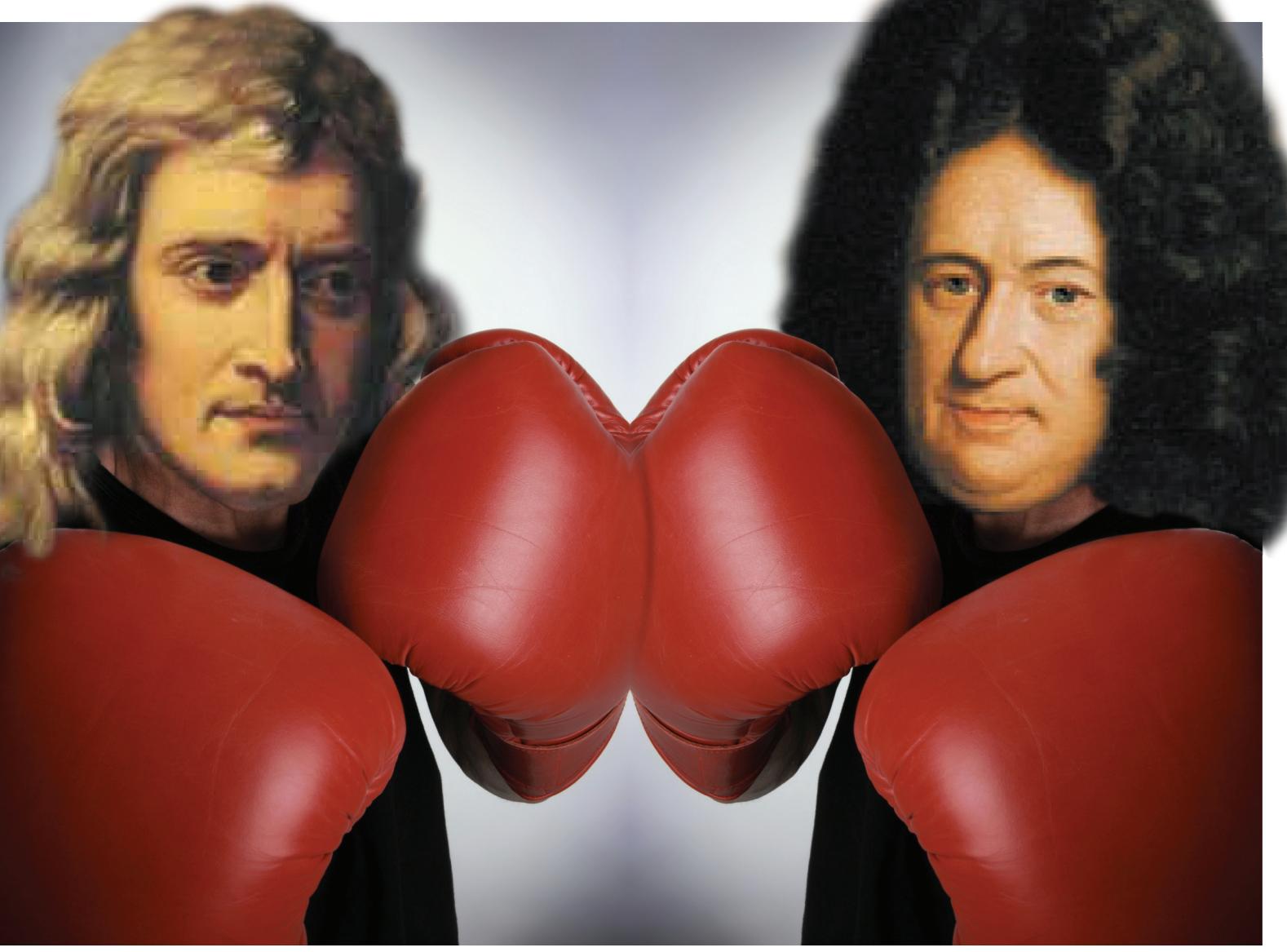
### Interactive Tutors



# Clickable Calculus™

The easiest way to power through your calculus problems

## Who invented calculus?

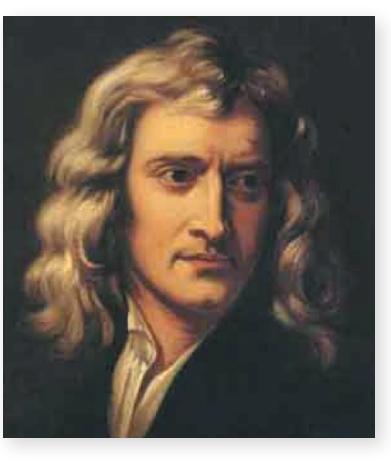


Calculus is among the most important fields of mathematics. In disciplines such as engineering and science, it is arguably the most important.

Calculus deals with describing the way mathematical quantities change and what happens when things get very large (infinite) or very small (infinitesimal). It provides elegant and powerful tools to describe and work with such quantities. It allows mathematicians to "model" real phenomena – i.e. develop equations that predict the behavior of real world systems. Mathematical models are essential for most of the complex technological developments of the modern world.

Two mathematicians are credited as having simultaneously and independently invented calculus: Isaac Newton, the English mathematician, and Gottfried Wilhelm Leibniz, the German mathematician and philosopher. Although Newton is credited with developing the ideas of calculus approximately ten years before Leibniz, it was Leibniz who first published the first paper on calculus (1684), years before Newton (nothing before 1687, full treatment in 1704).

The question of who finished first was a topic of furious and divisive debate in the past and lingers to this day. At that time, most mathematicians on the European continent supported Leibniz, while most in England supported Newton. Unfortunately, this argument impeded everyone's ability to fully benefit from these powerful concepts. Some estimate that the world lost half a century of productive mathematics.



**Isaac Newton (1642-1727)** attended Trinity College in Cambridge where he developed his formal knowledge of mathematics and science. During the years of 1665 to 1666, he retreated to his birthplace, Woolsthorpe, where he developed the foundations of calculus and its applications in physics. Newton introduced the concepts of the *fluxion* and *fluent* for the differential and integral respectively. Today, we continue to use the fluxional notation ( $\dot{x}$  for the derivative of  $x$  with respect to time) in special applications. In 1669, he was elected Lucasian Professor of Mathematics at Cambridge – the same position now held by modern scientific genius Stephen Hawking. Newton's other contributions included the nature and laws of gravity, the fundamental laws of mechanics, and the discovery of the spectral composition of light.

*"If I have seen further, it is by standing on ye shoulders of giants."*

Newton in a letter to Robert Hooke in 1676



**Gottfried Wilhelm Leibniz (1646-1716)** made contributions to mathematics, philosophy, religion, natural science, literature, and more. In 1675, he first established a formal way of computing the area under the function  $y = x$  and in doing so, he introduced the now familiar notation of the elongated "S" or  $\int$  for integration and the "d" for differentiation. These symbols are now the most commonly used notation of modern calculus – perhaps his most visible legacy. One reason for this is the ability for this notation to capture more complex mathematical thought, which is valuable in the ongoing evolution of mathematics. Although most of Leibniz's work was done in various parts of what is now Germany, between the years 1673 and 1676, he toured various parts of Europe, including England, to further develop his ideas through interaction with others. This was a source of accusations of plagiarism by some Newton supporters. Uniquely gifted in several fields, Leibniz's other contributions included tools and methods still used in modern logic and philosophy.

*"The only way to rectify our reasonings is to make them as tangible as those of the mathematicians, so that we can find our error at a glance, and when there are disputes among persons, we can simply say: *calculemus* [let us calculate], without further ado, to see who is right."*

Leibniz in "The Art of Discovery" 1685.

### Shoulders of Giants

Both Newton and Leibniz saw themselves, mathematically, than anyone else before because they stood on the shoulders of giants. These giants include some of the most recognizable names in mathematics. Following Newton and Leibniz were, of course, generations of mathematicians who saw further still as they peered from Newton's and Leibniz's shoulders.



**Zeno of Elea (Ancient Greek, -450 BC)**

Zeno was one of the first to investigate problems dealing with the infinite and minima of functions by considering the case where the tangent at a point on the curve is parallel to the  $x$  axis. Today, we would call this process he introduced as finding when the derivative is zero. Fermat also used this technique as early as 1629. Consequently, others, including the famous mathematician Joseph-Louis Lagrange, consider Fermat to have been the father of calculus.



**René Descartes (French, 1596 to 1650)**

Descartes introduced a technique for finding maxima and minima of functions by considering the case where the tangent at a point on the curve is parallel to the  $x$  axis. Today, we would call this process he introduced as finding when the derivative is zero. Fermat also used this technique as early as 1629. Consequently, others, including the famous mathematician Joseph-Louis Lagrange, consider Fermat to have been the father of calculus. His name also lives on with an important mathematical curve called the "witch of Agnesi" ( $yx^2 = a^2(x-y)$ ).



**Pierre de Fermat (French, 1601 to 1665)**

Fermat introduced a technique for finding maxima and minima of functions by considering the case where the tangent at a point on the curve is parallel to the  $x$  axis. Today, we would call this process he introduced as finding when the derivative is zero. Fermat also used this technique as early as 1629. Consequently, others, including the famous mathematician Joseph-Louis Lagrange, consider Fermat to have been the father of calculus.



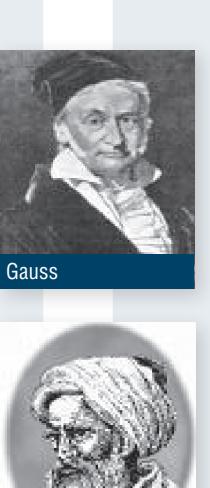
**Maria Gaetana Agnesi (Italian, 1718 to 1797)**

Agnesi was the author of one of the first comprehensive mathematical manuscripts that combines differential and integral calculus in roughly the same way as modern authors would. Her *Instituzioni Analitiche (Analytical Institutions)* – 1748 – is a landmark book in the history of mathematics. Her name also lives on with an important mathematical curve called the "witch of Agnesi" ( $yx^2 = a^2(x-y)$ ).

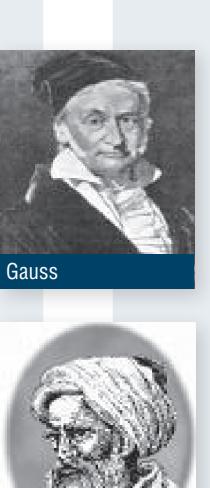


**Augustin Louis Cauchy (French, 1789 to 1857)**

Cauchy introduced the  $\epsilon$ - $\delta$  (delta-epsilon) method that mathematically clarified the concept of the limit. Until Cauchy, mathematicians had been calculating limits and other related operations without a formal, rigorous foundation. Cauchy's work developed the way mathematics was done and how it evolved. His work initiated the modern mathematical field of analysis, which provides a unified algebraic framework for calculus and related mathematics.



**Gauss**



**Bernoulli**



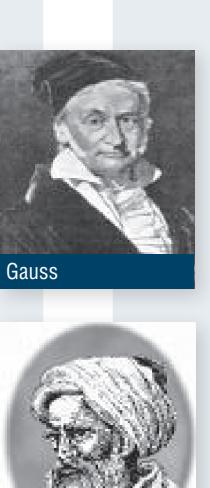
**Weierstrass**



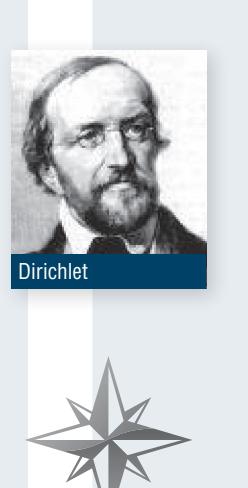
**Laplace**



**Euler**



**Archimedes**



**Dirichlet**

It is impossible to compose a complete list of all of the talented mathematicians who contributed to the development of modern calculus. Gauss, Euler, Archimedes, and Laplace are a few of the names that would immediately come to mind, but there are countless others. In many ways, the story of calculus is the story of modern mathematics itself, and we should look forward to continuing to explore this rich and wonderful story as time goes on.

To view a demo movie and download sample applications, course material, training material, and best practices, visit:

[www.clickablecalculus.com](http://www.clickablecalculus.com)

**Maplesoft**  
command the brilliance

[www.maplesoft.com](http://www.maplesoft.com) | [info@maplesoft.com](mailto:info@maplesoft.com)

Toll-free: (US & Canada) 1-800-267-6583 | Direct: 1-519-747-2373